Review about the physics of neoclassical tearing modes

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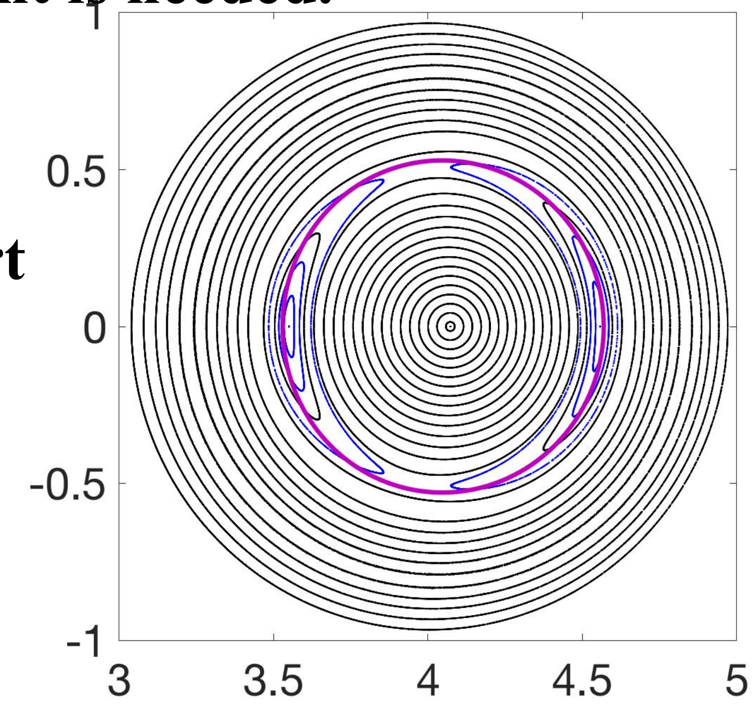
Outline

- Motivation and Background
- Basic physics of classical tearing mode
- Physics of neoclassical tearing mode (NTM)
  - Bootstrap current
  - Neoclassical tearing mode
  - Threshold of neoclassical tearing mode
    - ✓Transport model
    - ✓Neoclassical polarization current model
    - ✓….
  - Other effects on neoclassical tearing mode
- Control of neoclassical tearing mode
- Interaction between energetic ions and neoclassical tearing mode
- Conclusion, Remark and Outlook

Motivation

- NTM is driven by the perturbed helical bootstrap current due to the pressure flattening across the island.
- To achieve high confinement and steady state plasma ⟹ long sustained plasma current (non-inductive current): bootstrap current and auxiliary driven current (i.e. ECCD, LHCD)
- To reduce the cost, high fraction (>50%) bootstrap current is needed.
- Bootstrap current would cause NTM.
- The damage of NTM
  - The formation of island, increase local radial transport
  - Limit the long pulse tokamak maximum beta
  - Island overlap, lead to disruption (about 50% tokamak disruption)
- Must control NTM



Limit the long pulse tokamak maximum beta

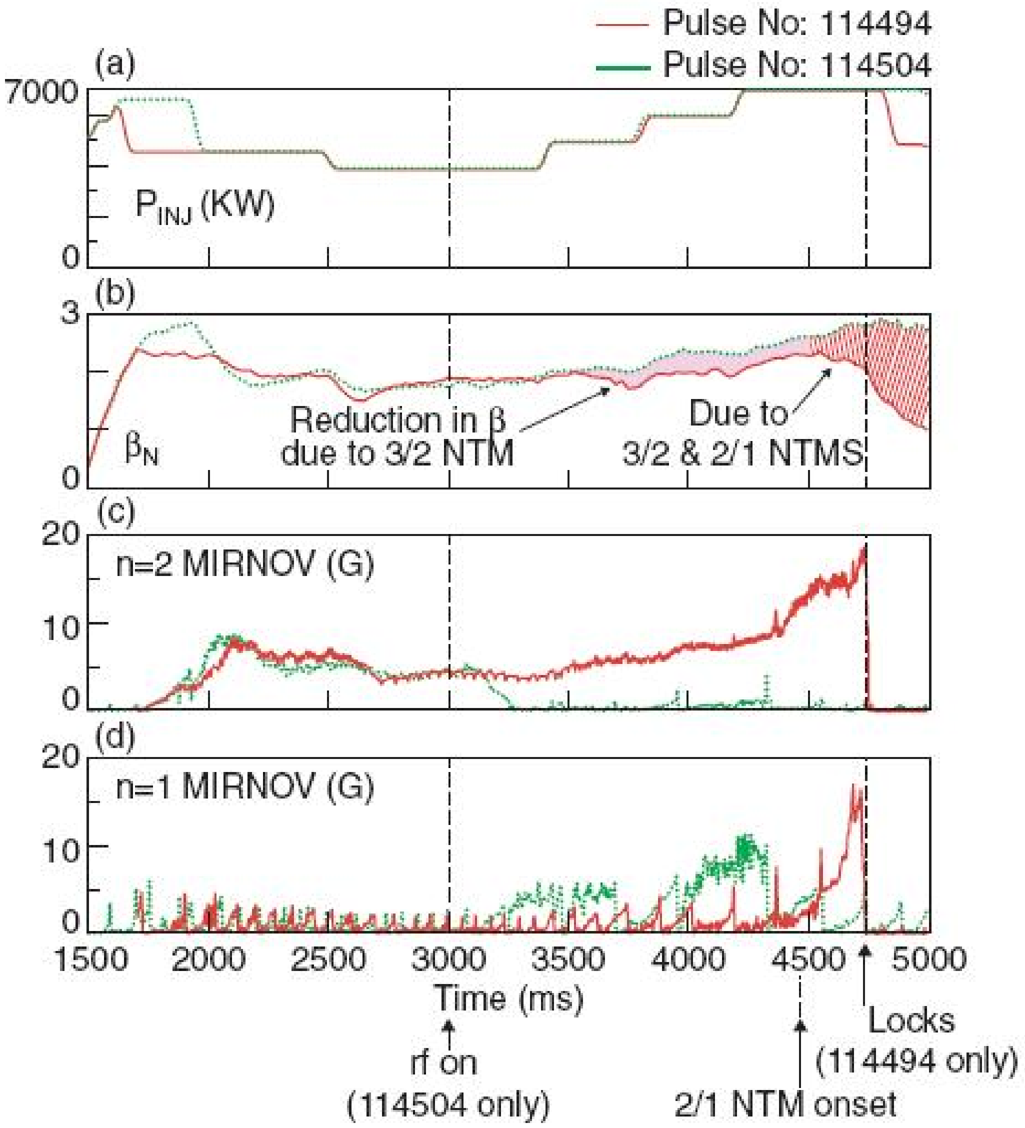


Figure 2. DIII–D discharges with (114504, dotted lines) and without (114494, solid lines) ECCD suppression of an $m/n = 3/2$ neoclassical tearing mode. (*a*) Neutral beam power, (*b*) β_N, (*c*) $n = 2$ Mirnov $|\tilde{B}_\theta|$, (*d*) $n = 1$ Mirnov $|\tilde{B}_\theta|$. The degradation in energy confinement due to the NTM from 3/2 and 2/1 NTMs can be seen in the effect on β_N.

Nucl. Fusion, *Progress in the ITER Physics basis,* (2007)

Island overlap, lead to disruption

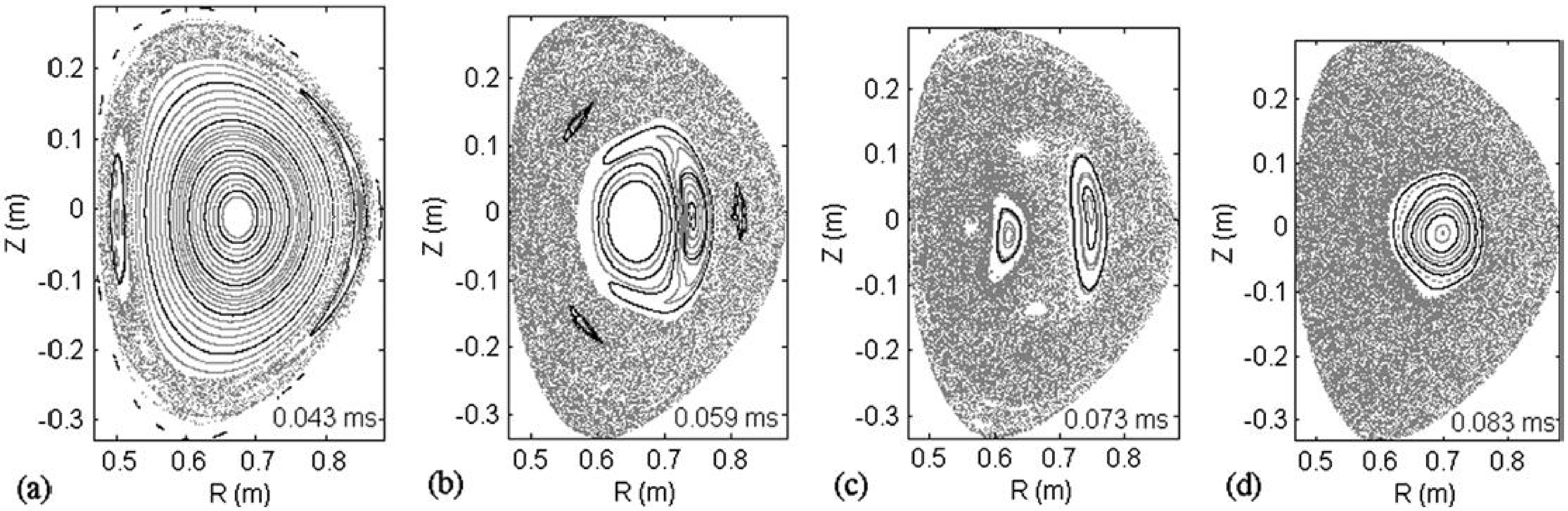


Figure 7. Modelling of C-Mod equilibrium with NIMROD predicts fast growing 2/1 and 1/1 MHD tearing modes which result in ergodic field lines over much of the plasma cross-section and loss of confinement. The NIMROD timescale shown is a factor of ~20 faster than in the actual experiment due to the lower Lundquist number used in the modelling.

R. S. Granetz, Ncul. Fusion 2017

History

- **MHD including bootstrap current contribution:**
  - **W. X Qu, J. D. Callen, Nonlinear Growth of a Single Neoclassical MHD Tearing Mode in a Tokamak. UWPR855 Report of University of Wisconsin (1985)**
  - **J. D. Callen, et al, Plasma Physics and Controlled Nuclear Fusion Research, vol. 2 (Vienna: IAEA) p.157 (1987)**
- **Drift kinetic theory:**
  - **Carrera, R., Hazeltine, R.D., Kotschenreuther, M., Phys. Fluids, 29, 899(1986).**
- **NTM first observed in the TFTR experiment: Z. Chang, et al, Phys. Rev. Lett., 74, 4663(1995)**
- **Polarization current model: A. I. Smolyakov, Sov. J. Plasma Phys., 15, 667 (1989), Plasma Phys. Control. Fusion, 35, 657 (1993)**
- **Transport threshold model: Fitzpatrick, R., Phys. Plasmas, 2, 825 (1995)**
- **Neoclassical polarization current threshold model: A.I. Smolyakov, et al, Phys. Plasmas, 2, 1581 (1995); H.R. Wilson, et al, Phys. Plasmas, 3, 248 (1996)**
- **Magnetic well effect**
- **……**

Basic physics of tearing modes

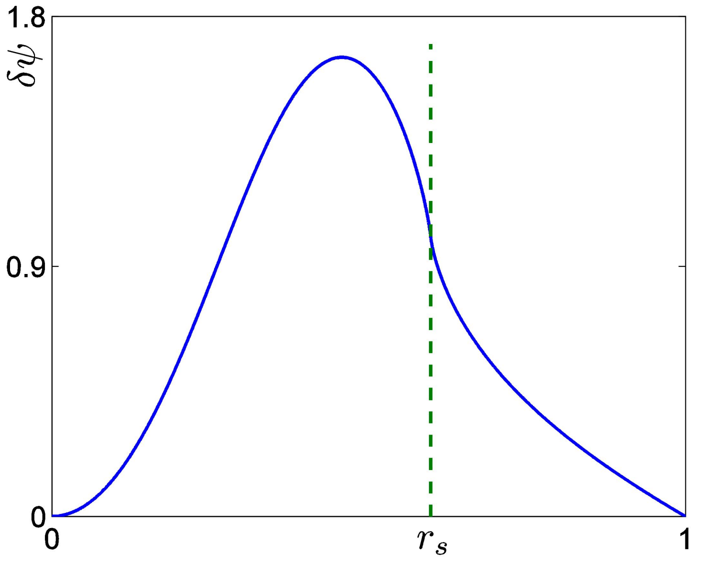
➢ **Away from rational surface, $\delta\psi$ is determined by the ideal MHD equations. It has a discontinuous derivative at rational surface.**

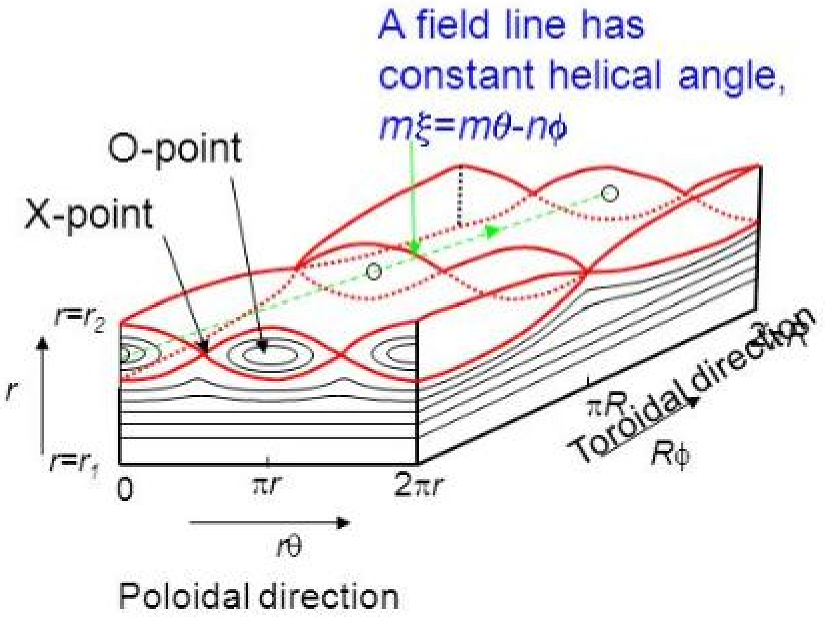
➢ **At the singular layer, $\delta\psi$ is determined by resistive MHD Eqs.**

➢ **By matching the solutions of outer and inner region, the dispersion relation can be obtained.**

$$\Delta'\delta\psi = 2\mu_0 \int_{0^-}^{0^+} dx \oint d\xi\, \delta J_{//} \cos\xi,$$

$$\Delta' = \frac{1}{\delta\psi}\left[\left.\frac{d\delta\psi}{dr}\right|_{r_s^+} - \left.\frac{d\delta\psi}{dr}\right|_{r_s^-} \right].$$





Magnetic island

- **Magnetic field is given as** $\boldsymbol{B} = I\nabla\zeta + \nabla\zeta \times \nabla(\psi + \delta\psi), \delta\psi = \delta\hat{\psi}(0)\cos(m\theta - n\zeta)$
- **The flux surface satisfies** $\boldsymbol{B} \cdot \nabla\Omega = 0$

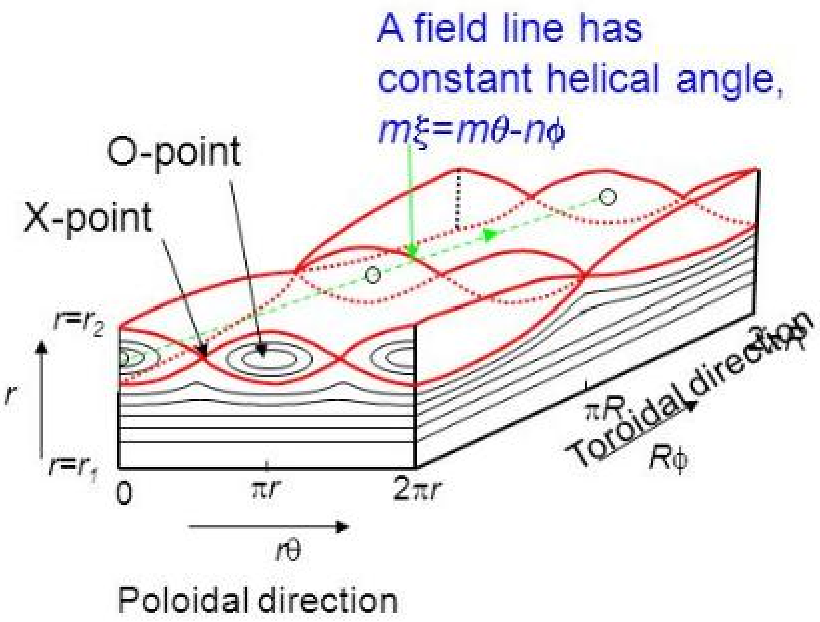
$$\boldsymbol{B} \cdot \nabla\Omega = \frac{1}{\mathrm{J}}\left(\frac{\partial}{\partial\theta} + q\frac{\partial}{\partial\zeta} + \frac{\partial\delta\psi}{\partial\theta}\frac{\partial}{\partial\psi}\right)\Omega = \frac{1}{\mathrm{J}}\left(\frac{\partial}{\partial\theta} + (m - nq)\frac{\partial}{\partial\xi} + m\frac{\partial\delta\psi}{\partial\xi}\frac{\partial}{\partial\psi}\right)\Omega = 0$$

$$\Rightarrow \quad \frac{\partial\Omega}{\partial\psi} = \frac{\mathrm{q}}{\mathrm{q_s}} - 1, \frac{\partial\Omega}{\partial\xi} = \frac{\partial\delta\psi}{\partial\xi}$$

Near the rational surface

$$\frac{\partial\Omega}{\partial\psi} \sim q_s'/q_s(\psi - \psi_s)$$

$$\Rightarrow \quad \Omega = \frac{2}{w_\chi^2}(\psi - \psi_s)^2 - cos\xi = \frac{2}{w^2}(r - r_s)^2 - cos\xi$$



Basic physics of tearing modes

Points:

- **Tearing mode is driven by plasma current.**
- **Plasma current: Ohm current, bootstrap current, current driven by others (auxiliary heat, alpha particle…)**
- **They all can affect the behavior of tearing mode**

Basic physics of neoclassical tearing modes

◆**NTM is driven by the perturbed helical bootstrap current due to the pressure flattening across the island.**

◆**What is bootstrap current?**

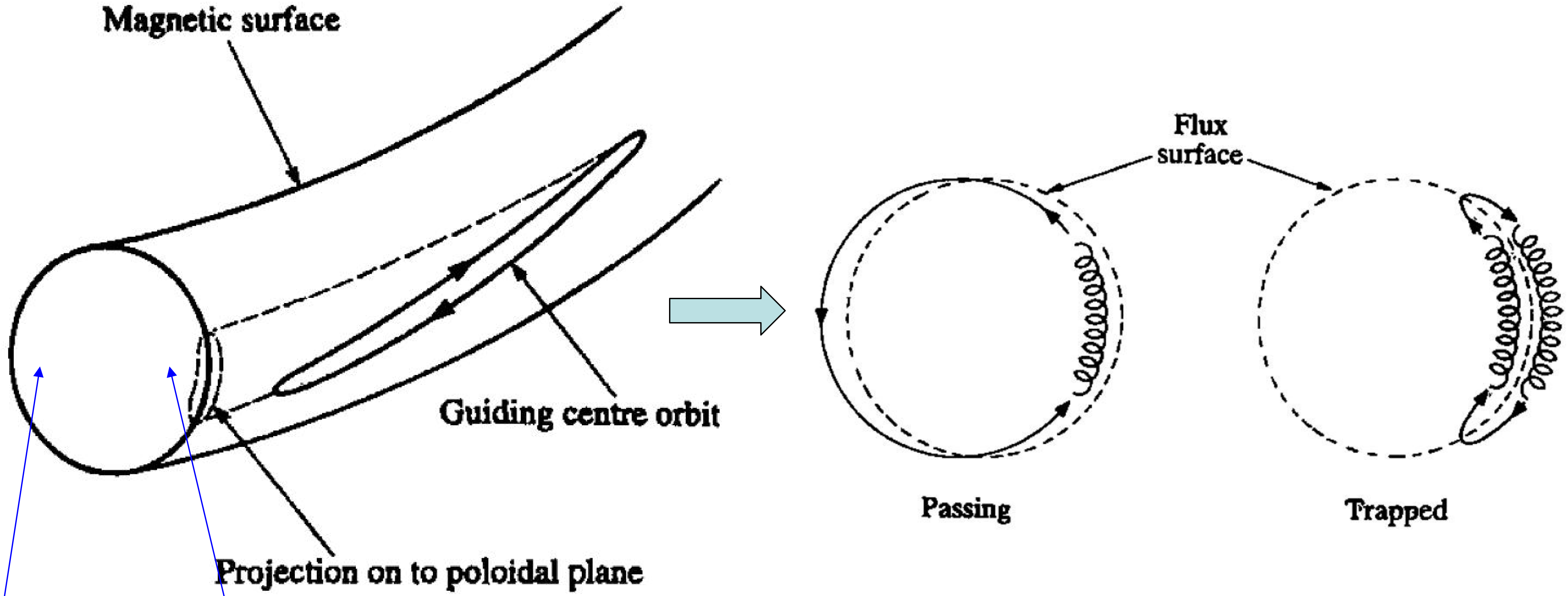
- **Particle orbit: passing particle, trapped particle**

- **Particle drift velocity**

$$\mathbf{v}_{\mathrm{d}} = \frac{\mathbf{B}}{B} \times \left\{ \frac{\nabla \Phi}{B} + \frac{1}{\Omega_{\mathrm{c}}} \left(\mu \nabla B + v_{\parallel}^{2} \boldsymbol{\kappa} \right) \right\},$$

$$\mu = v_{\perp}^{2}/(2B), \ \boldsymbol{\kappa} = (\mathbf{B}/B) \cdot \nabla (\mathbf{B}/B)$$

Particle orbit-*J. Wesson, Tokamaks*



high field side **low field side** ⇦ $B \sim B_0\left(1-\varepsilon\cos\theta\right)$

Particle orbit in tokamak

$$v_{//} = \pm v(1-\lambda B)^{1/2} \sim \pm v(1-\lambda B_0(1-\varepsilon\cos\theta))^{1/2} = \pm v\left(1-\lambda B_0+\varepsilon\lambda B_0-2\varepsilon\lambda B_0\sin^2\frac{\theta}{2}\right)^{1/2}$$

$$= \pm v(1-\lambda B_0+\varepsilon\lambda B_0)^{1/2}\left(1-\frac{2\varepsilon\lambda B_0}{1-\lambda B_0+\varepsilon\lambda B_0}\sin^2\frac{\theta}{2}\right)^{1/2}$$

$\lambda = \dfrac{\mu}{E}$ **is the pitch angle.**

$$\begin{cases} \dfrac{2\varepsilon\lambda B_0}{1-\lambda B_0+\varepsilon\lambda B_0} > 1 \Longrightarrow \lambda B_0 > \dfrac{1}{1+\varepsilon} \\ \lambda B_0 < (\lambda B_0)_{\max} \Longrightarrow \lambda B_0 < \dfrac{1}{1-\varepsilon} \end{cases}$$

Trapped particle: $\dfrac{1}{1+\varepsilon} < \lambda B_0 < \dfrac{1}{1-\varepsilon}$ $\Longleftarrow$ (from the two conditions above)

Passing particle: $0 < \lambda B_0 < \dfrac{1}{1+\varepsilon}$

Fraction of trapped particles with Maxwellian distribution: $f = \left(\dfrac{v_{//}}{v}\right)_{crit} \sim \left(\dfrac{2\varepsilon}{1+\varepsilon}\right)^{1/2}$

Orbit width of particles in tokamak

- **The conservation of toroidal canonical momentum**

$$p_\phi = \frac{Ze}{m}\psi + Rv_\phi \Longrightarrow \hat{p}_\phi = \psi + v_{//}\frac{RB_\phi}{\Omega_c}$$

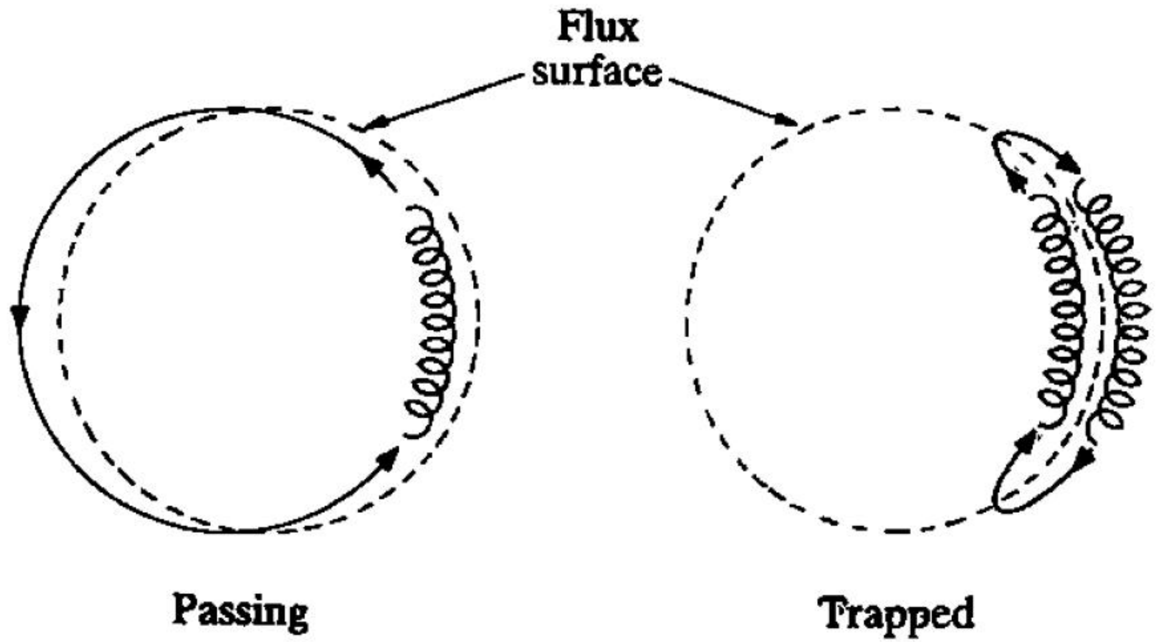
$$\Delta\psi + \Delta\left(v_{//}\frac{RB_\phi}{\Omega_c}\right) = 0 \Longrightarrow d = -\frac{1}{RB_\theta}\Delta\left(v_{//}\frac{RB_\phi}{\Omega_c}\right)$$

- **For trapped particle:**

$$d \sim 2\frac{|v_{//}|}{\Omega_{c0}}\frac{B_\phi}{B_\theta} = 2\frac{q}{\varepsilon}\frac{|v_{//}|}{\Omega_{c0}} = 2\frac{q}{\varepsilon^{1/2}}\frac{v}{\Omega_{c0}} = \frac{2q\rho_L}{\varepsilon^{1/2}}$$

- **For passing particle:**

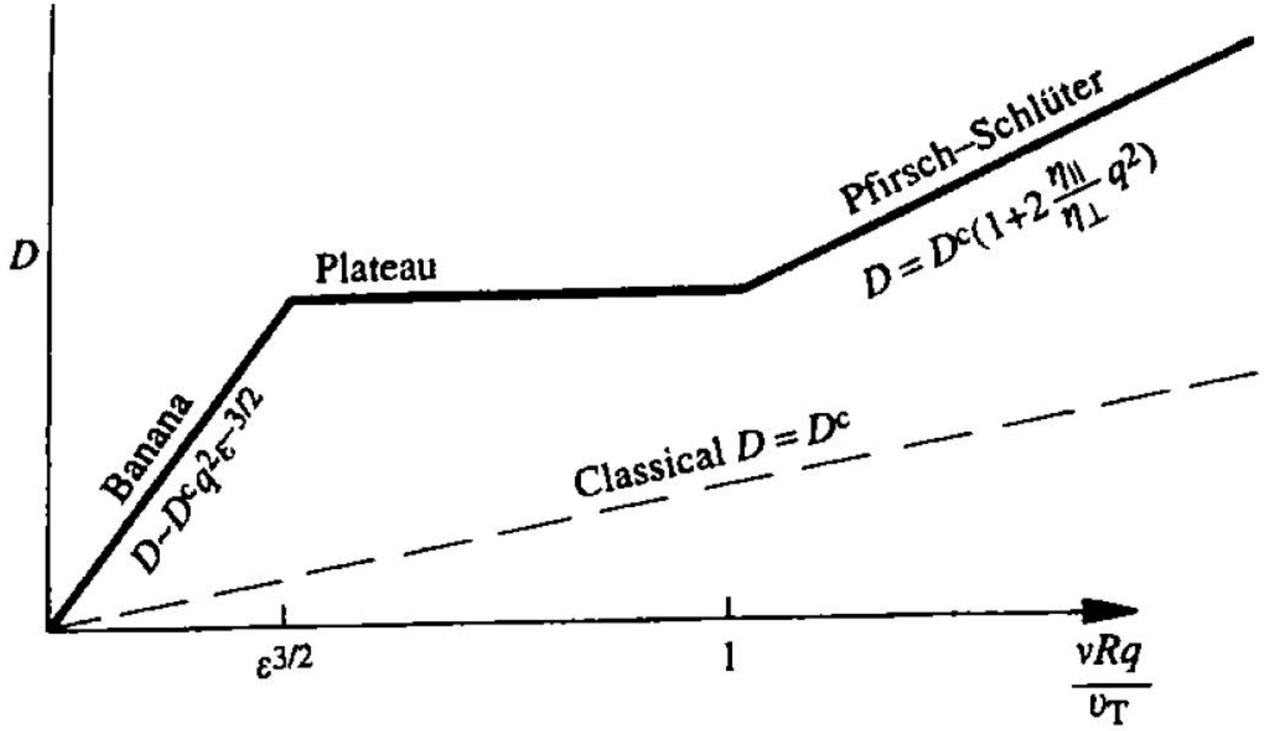
$$d \sim 2\varepsilon\frac{|v_{//}|}{\Omega_{c0}}\frac{B_\phi}{B_\theta} = 2q\frac{|v_{//}|}{\Omega_{c0}}$$



Bootstrap current

- **Banana regime (low collision frequency): particles complete at least one bounce orbit before suffering a collision. Collision cause scattering out of the region of velocity phase where particles are trapped.**

$$\nu_{eff} \sim \frac{\nu}{(\Delta\theta)^2} \sim \nu/\varepsilon,\ \omega_b \sim \frac{\varepsilon^{1/2} v_T}{qR},\quad \nu_{eff} < \omega_b, \Rightarrow \nu < \frac{\varepsilon^{3/2} v_T}{qR}$$



Bootstrap current

- **A heuristic interpretation:**

In the presence of a density gradient, trapped particles carry a parallel current:

$$J_t \sim -e f_{frac} v_{//} w_b \frac{dn}{dr} \sim -e\varepsilon^{1/2}\left(\varepsilon^{1/2} v_T\right)\frac{q\rho_L}{\varepsilon^{1/2}}\frac{dn}{dr} \sim -q\frac{\varepsilon^{1/2}}{B}T\frac{dn}{dr}$$

It is similar to the diamagnetic current except the direction.

Momentum of trapped particles transfer to passing particles.

Bootstrap current: Arising from the difference in velocity between passing ions and passing electrons, parallel to magnetic field .

- **Momentum balance**

  - **Momentum exchange between passing electron and passing ions** $\nu_{ei} m_e J_b / e.$

  - **Momentum exchange between passing and trapped particles** $\nu_{eff} m_e J_t / e.$

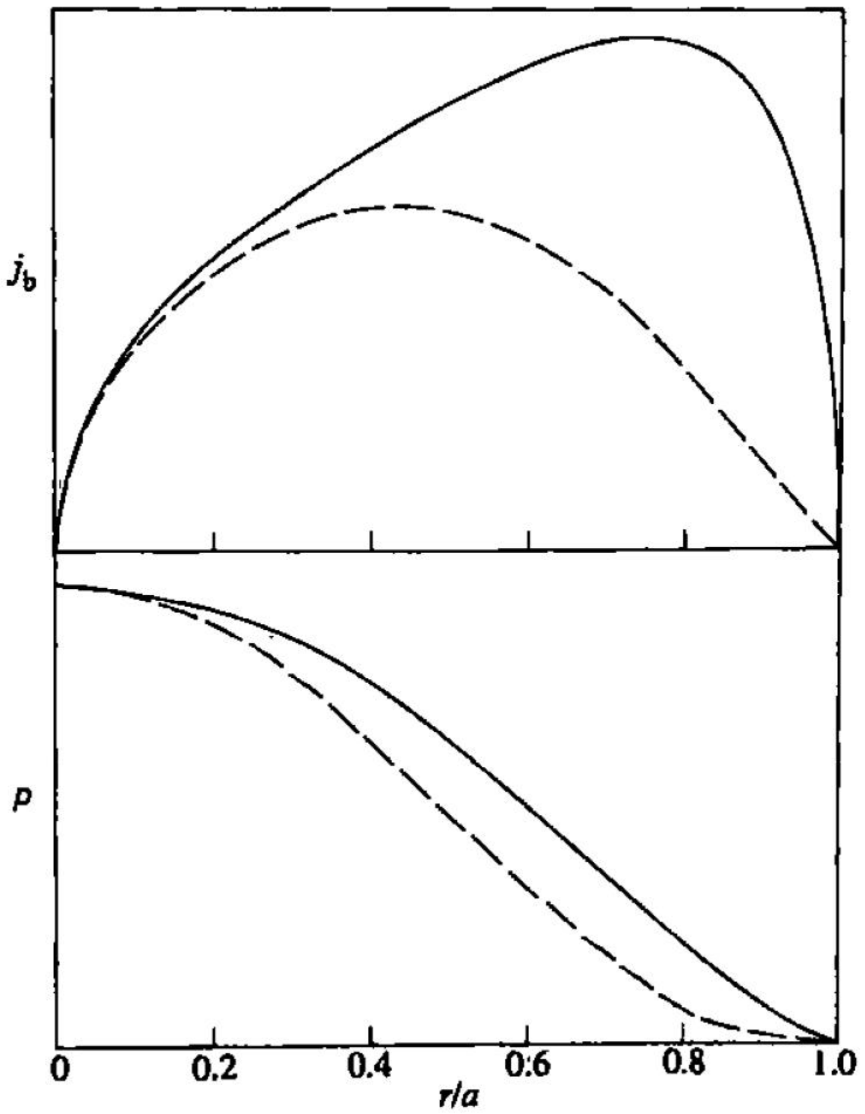
$$\nu_{eff} m_e J_t / e \sim \nu_{ei} m_e J_b / e \;\Rightarrow\; J_b \sim -\frac{\nu_{ee}}{\nu_{ei}}\frac{q}{\varepsilon^{1/2}}\frac{1}{B}T\frac{dn}{dr} \;\Rightarrow\; J_b \sim -\frac{\varepsilon^{1/2}}{B_\theta}T\frac{dn}{dr}$$

Bootstrap current

- **A precise expression:**

$$J_b = -\frac{\varepsilon^{1/2} n}{B_\theta}\left[2.44\left(T_e + T_i\right)\frac{1}{n}\frac{dn}{dr} + 0.69\frac{dT_e}{dr} - 0.42\frac{dT_i}{dr}\right] + O\left(\varepsilon^{1/2}\right)$$

- **One can solve it by the drift kinetic equation.**
  - **P. Helander, et. al., Phys. Plasmas 4,3211(1997)**
  - **F. L. Hinton, et. al., Rev. Modern Physics 48, 239(1976)**
  - **M. N. Rosenbluth, et. al., Phys. Fluids 15, 116(1972).**



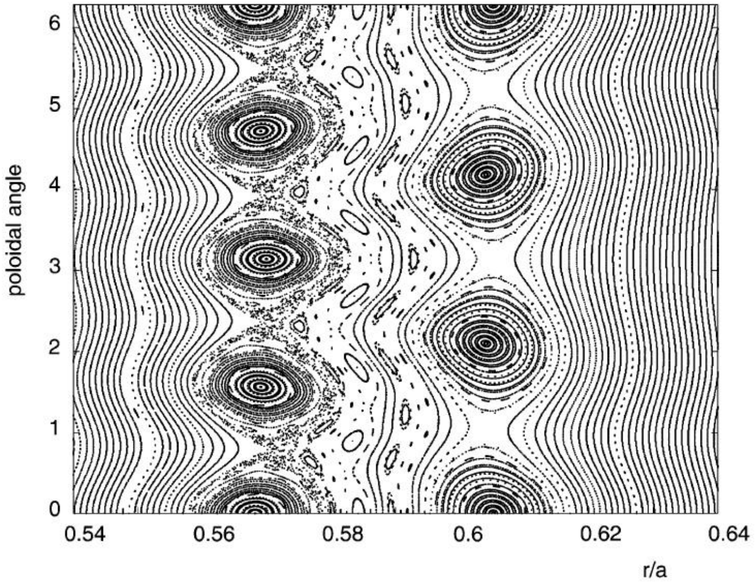
Temperature with island

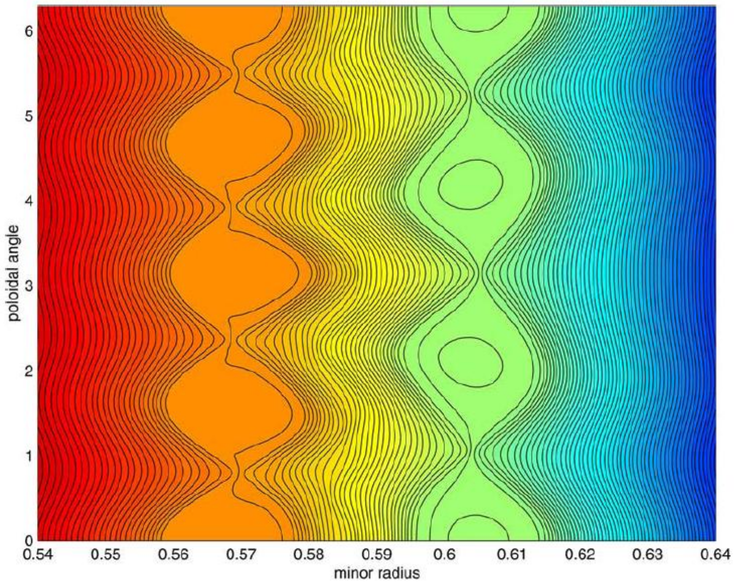
- **The heat transport equation**

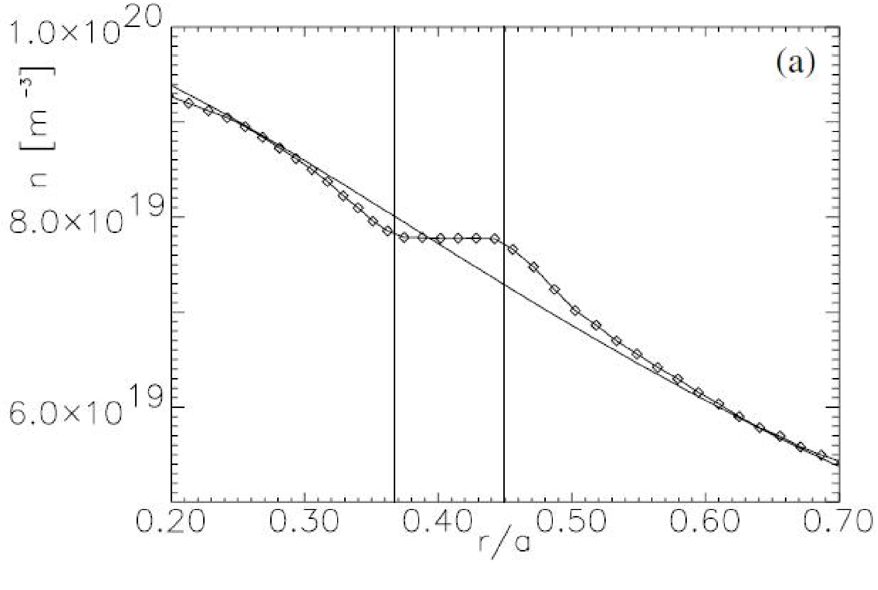
$$\frac{3}{2}n\frac{\partial T}{\partial t}=\nabla_{//}\cdot\left(n\kappa_{//}\nabla_{//}T\right)+\nabla_{\perp}\cdot\left(n\kappa_{\perp}\nabla_{\perp}T\right)+S,\quad \kappa_{//}>>\kappa_{\perp}$$

$$\kappa_{//}\sim n_e v_{te}\lambda_e \quad or\ \kappa_{//}\sim n_e v_{te}\lambda_{//},\lambda_{//}\sim 1/|k_{//}|,\quad \kappa_{\perp}\sim\frac{n_e a^2}{6\tau_E},$$

- **Without source or sink, the gradient along field lines approaches to zero.**
- **Without source or sink, the gradient insides the island approaches to zero.**







Temperature with island

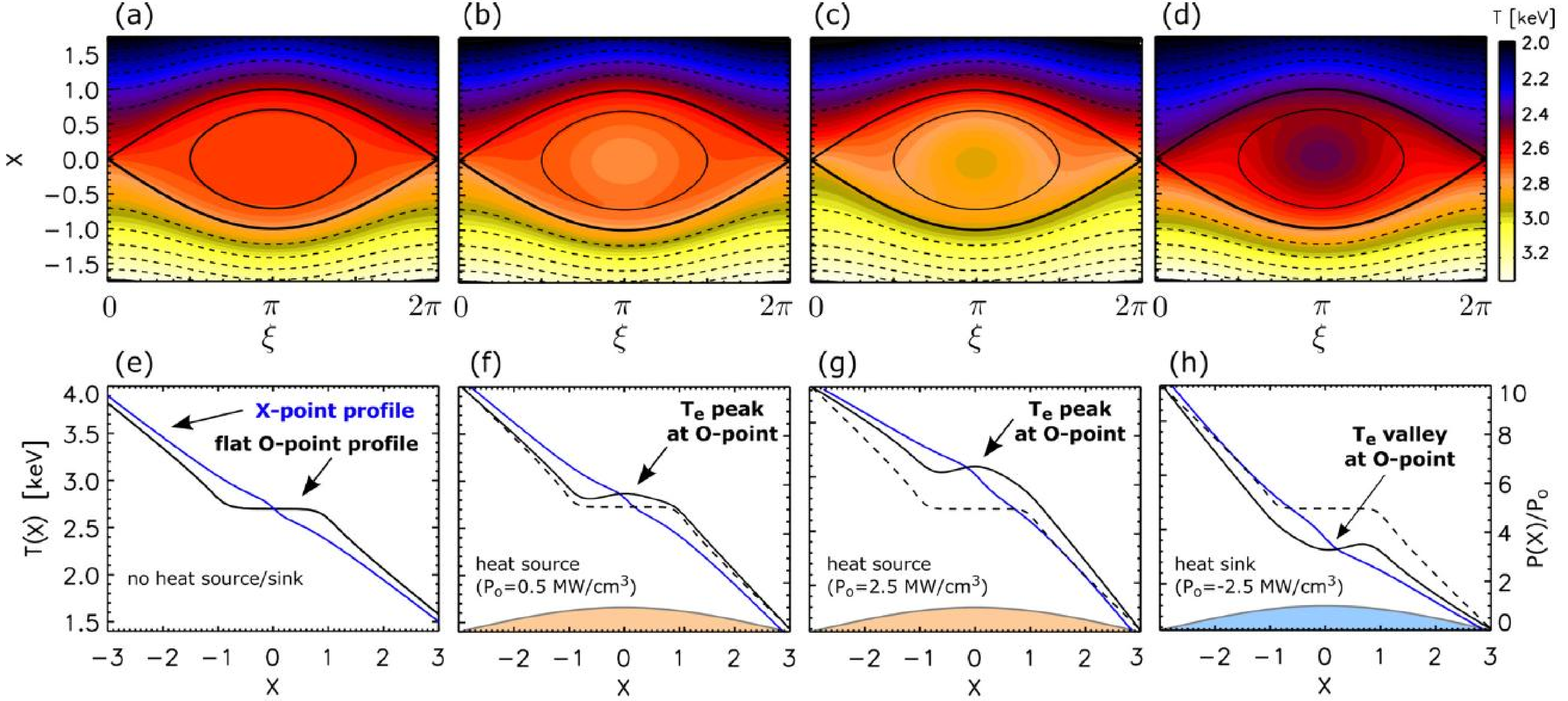


FIG. 1. Stationary solutions of the heat transport model [Eq. (1)]. (a), (e) No heat source ($P_{\circ} = 0\,\mathrm{MW/cm^3}$). (b), (f) small heat source ($P_{\circ} = 0.5\,\mathrm{MW/cm^3}$), (c), (g) large heat source ($P_{\circ} = 2.5\,\mathrm{MW/cm^3}$) and (d), (h) heat sink ($P_{\circ} = -2.5\,\mathrm{MW/cm^3}$). Contours of $T_e(X, \xi)$ and contour lines of the magnetic island flux ($\Omega(X, \xi)$) are shown in (a)–(d) ($\Omega = 0$ thin solid line, $\Omega = 1$ thick solid line, separatrix of island and $\Omega > 1$ dashed lines). Profiles across the X-point and the O-point are shown in (e)–(h) with solid lines. Dashed lines are the O-point profiles without heat source/sink for comparison in (f)–(h).

Basic theory of NTMs

Magnetic island ⟹ Pressure flattened ⟹ Perturbed bootstrap current ⟹ NTM

- **Given magnetic field**

$$\mathbf{B} = I\nabla\zeta + \nabla\zeta \times \nabla(\psi + \delta\psi),\ \delta\psi = \delta\hat{\psi}(0)\cos\xi,\ \xi = m(\theta - \zeta / q_s)$$

- **Basic equations:**

$$E_{//} = \eta J_{//,ohm} = \eta(J_{//} - J_b)$$ **Modified Ohm's law including bootstrap current**

$$\mathbf{B}\cdot\nabla\frac{J_{//}}{B} = -\nabla\cdot\mathbf{J}_\perp$$ **Quasi-neutrality equation/current conservation**

For simplicity, the (neoclassical) polarization, diamagnetic current are not considered, then

$$\mathbf{B}\cdot\nabla\frac{J_{//}}{B} = 0. \Longrightarrow J_{//} = J_{//}(\Omega), \quad \mathbf{B}\cdot\nabla\Omega = 0.$$

$$\Omega = -\delta\psi / \delta\hat{\psi} + \frac{2}{w^2}x^2 = -\cos\xi + \frac{2}{w^2}x^2, \quad x = r - r_s, w = 2\left(\left(\frac{\partial\psi}{\partial r}\right)^{-1}\frac{q_s\delta\hat{\psi}}{q_s'}\right)^{1/2}$$

Basic theory of NTMs

- **Introducing the island coordinate** (Ω, ξ),
- **The matching between outer region and inner region:**

$$\Delta' = \frac{1}{\delta\hat{\psi}}\int_{0^-}^{0^+}\frac{\partial^2\delta\hat{\psi}}{\partial x^2}dx = \frac{2}{\delta\hat{\psi}}\frac{4\pi R}{c}\oint\frac{d\xi}{2\pi}\int_{0^-}^{0^+}dx\,\delta J_{//}\cos\xi$$

$$\Delta' = \frac{16\mathrm{q_s}}{w^2\psi' q_s'}\frac{4\pi R}{c}\frac{w}{2\sqrt{2}}\int_{-1}^{\infty}d\Omega < \delta J_{//}\cos\xi >$$

where

$$< \ldots > = \oint\frac{d\xi}{2\pi}\frac{(\ldots)}{\sqrt{\Omega + \cos\xi}}$$

denotes the flux surface average.

Basic theory of NTMs

◆ Ohm's Law

$$E_{//} = -\mathbf{b}\cdot\nabla\delta\phi - \frac{1}{c}\frac{\partial \delta A_{//}}{\partial t} = -\mathbf{b}\cdot\nabla\delta\phi + \frac{1}{cR}\frac{\partial\delta\psi}{\partial t} = \eta\left(\delta J_{//} - \delta J_b\right)$$

Making flux average,

$$< J_{//} >= \frac{1}{cR}\frac{1}{\eta} < \frac{\partial\delta\hat{\psi}}{\partial t}\cos\xi > + < \delta J_b > .$$

$$J_{//} = J_{//}(\Omega) =< J_{//} > / < 1 >,$$

$$\Rightarrow J_{//} = \frac{1}{cR}\frac{1}{\eta} < \frac{\partial\delta\hat{\psi}}{\partial t}\cos\xi > / < 1 > + < \delta J_b > / < 1 >,$$

$$\Delta' = \frac{4\sqrt{2}q_s}{q_s'\psi_s' w}\frac{4\pi R}{c}\{\int_{-1}^{\infty} d\Omega \frac{1}{\eta}\frac{1}{c}\frac{1}{R} < \frac{\partial\delta\psi}{\partial t} >< \cos\xi > + \int_{-1}^{\infty} d\Omega \frac{1}{2}\left(< J_b > (\sigma_x) + < J_b > (-\sigma_x)\right)\frac{< \cos\xi >}{< 1 >}\}$$

$$\Rightarrow \frac{8\pi}{c^2}\frac{1}{\eta} I_1 \frac{dw}{dt} = \Delta' + \Delta'_b , \qquad \Delta'_b = -\frac{4\pi R}{c}\frac{4\sqrt{2}\mathrm{q_s}}{w\psi' q_s'}\int_{-1}^{\infty} d\Omega \frac{< \delta J_b >}{< 1 >} < \cos\xi >$$

Contribution from bootstrap current

$$\mathrm{I}_1 = \sqrt{2}\int_{-1}^{\infty} \frac{< \cos\xi >^2}{< 1 >} d\Omega = 0.8277,$$

Contribution from bootstrap current

◆ Perturbed bootstrap current

$$\delta J_b = -1.46 \frac{q_s}{\sqrt{\varepsilon} B} \frac{\partial \delta p_e}{\partial x}$$

◆ From heat transport equation without source or sink

$\nabla_{//} \cdot \left(n \kappa_{//} \nabla_{//} T \right) \sim 0,$ **The contribution from perpendicular transport is not considered.**

Making flux average,

$$\pm \frac{\partial}{\partial \Omega} \left(\frac{1}{2\pi} \oint d\xi \sqrt{\Omega + \cos\xi} \frac{\partial T}{\partial \Omega} \right) = 0 \quad \Longrightarrow \quad \frac{dT}{d\Omega} = \pm T_s' \frac{\sqrt{2} w}{4} \frac{H(\Omega - 1)}{\oint \sqrt{\Omega + \cos\xi} / (2\pi)} = \pm T_s' \frac{\sqrt{2} w}{4} \frac{\pi}{2} \frac{H(\Omega - 1)}{(\Omega + 1)^{1/2} E(2/(\Omega + 1))}$$

$H(\Omega - 1)$ **is the Heaviside function.**

It denotes temperature is flattened in the island.

$$\delta J_b = J_{bs} \frac{\pi}{2} \frac{1}{(\Omega + 1)^{1/2} E(2/(\Omega + 1))} \sqrt{\Omega + \cos\xi} \quad \Longrightarrow \quad \Delta'_b = -2.31 \sqrt{\varepsilon} \frac{d\beta_\theta}{dr} \frac{q_s}{q_s'} \frac{1}{w}$$

The evolution of NTMs

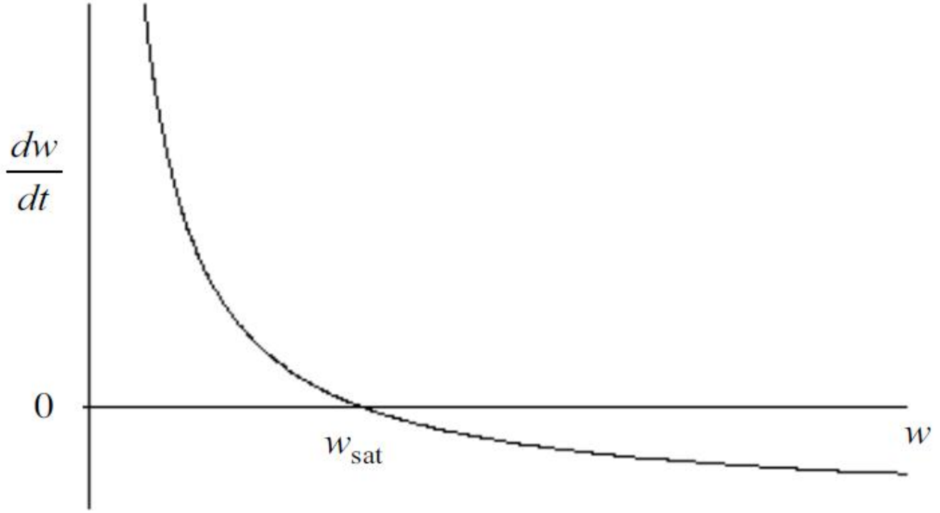
◆ **The island evolution equation is**

$$\frac{8\pi}{c^2}\frac{1}{\eta}I_1\frac{dw}{dt}=\Delta'\text{-}2.31\sqrt{\varepsilon}\frac{d\beta_\theta}{dr}\frac{q_s}{q_s'}\frac{1}{w},$$

➢ **The contribution of bootstrap current is destabilizing**

$$\Delta'_b\propto\frac{1}{w},\quad if\quad w\rightarrow 0?$$

➢ **The rotation frequency of island is not considered?**



Threshold physics of neoclassical tearing modes

- **Observation of NTM in TFTR**
- **Mode is initiated at a finite amplitude**
- **When the island starts to decay, the fit is not good**
- **Suggesting a threshold mechanism is important for small island width**

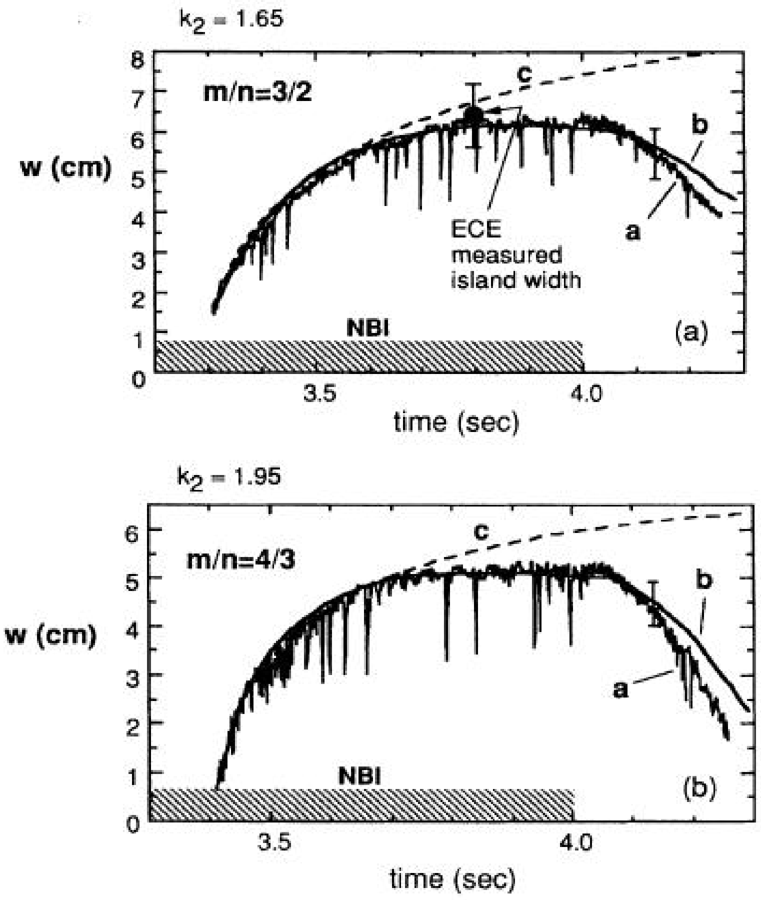


FIG. 2. Comparison of the "measured" magnetic island (curves labeled a) with the neoclassical ∇p-driven tearing mode theory. (a) $m/n = 3/2$ island. Also shown is the island width measured by the multichannel ECE diagnostics. (b) $m/n = 4/3$ island. Curves labeled b use the time-dependent parameters (see Fig. 3). Curves labeled c use fixed parameters [for 3/2 (4/3) mode: $\eta_{nc}/\mu_0 = 120$ (110) cm^2/s, $w_c = 0.40$ (0.38), and $\Delta' = -0.07$ (-0.11) cm^{-1}].

Models to describe the threshold physics

- A number of physics issues may impact an accurate prediction of the nonlinear island width threshold:
  - Transport model: Incomplete flattening of the pressure gradient across the island region
    - ✓Standard $\chi_\perp / \chi_{//}$ model (Fitzpatrick, 95; Gorelenkov,et. al., 96) ⇨ $w_c \propto (\chi_\perp / \chi_{//})^{1/4}$
    - ✓Convective transport model (Gates, et. al., 97) ⇨ $w_c \propto \chi_\perp^{1/3}$
    - ✓Rotation transport model (Konovalov, et. al., 02) ⇨ $w_c \propto (\chi_\perp / \omega)^{1/2}$
    - ✓Finite ion orbit model (Poli, et. al., 02; Cai, et. al., 15; Imada, et. al, 2019,2020)
  - Neoclassical polarization currents (Wilson, et. al., 96; Kuvshinov, et. al., 98; Connor, et. al., 01)
  - Resistive interchange physics (Lutjens, et. al.,02)
  - Current profile evolution (Brennan, et. al.,02)
  - +Others?

Transport model

- In steady state, in the absence of any source, heat flux:

$$\nabla \cdot \mathbf{Q} = n\kappa_{//}\nabla_{//}^2 T + n\kappa_{\perp}\nabla_{\perp}^2 T = 0$$

  - If the perpendicular transport can be neglected, temperature is constant along the field line, namely it must be constant inside the island.

  - If the perpendicular transport can be neglected, then there is a critical island that the perpendicular and parallel transport terms are comparable, as

$$\frac{m^2 w^2}{R^2 q^2 L_q^2}\chi_{//} \sim \frac{\chi_{\perp}}{w^2},$$

  - The critical island width (below which pressure is not flattened across the island) can be obtained

$$w_{\chi} = \sqrt{\frac{RqL_q}{m}}\left(\frac{\chi_{\perp}}{\chi_{//}}\right)^{1/4}$$

Transport model

◆ **In tokamak, perpendicular transport is larger than neoclassical prediction because of turbulence. If we assume the perpendicular transport has a gyro-Bohm scaling, collision model for parallel transport, as**

$$\chi_{\perp} \sim \rho_i^2 v_{thi} / r, \chi_{//} \sim v_{the}^2 / \nu_e,$$

Then

$$w_{\chi} \sim \sqrt{\frac{L_q \rho_i}{m}} \nu_{*e}^{1/4} q^{1/4} \left(\frac{\varepsilon m_e}{m_i} \right)^{1/8}$$

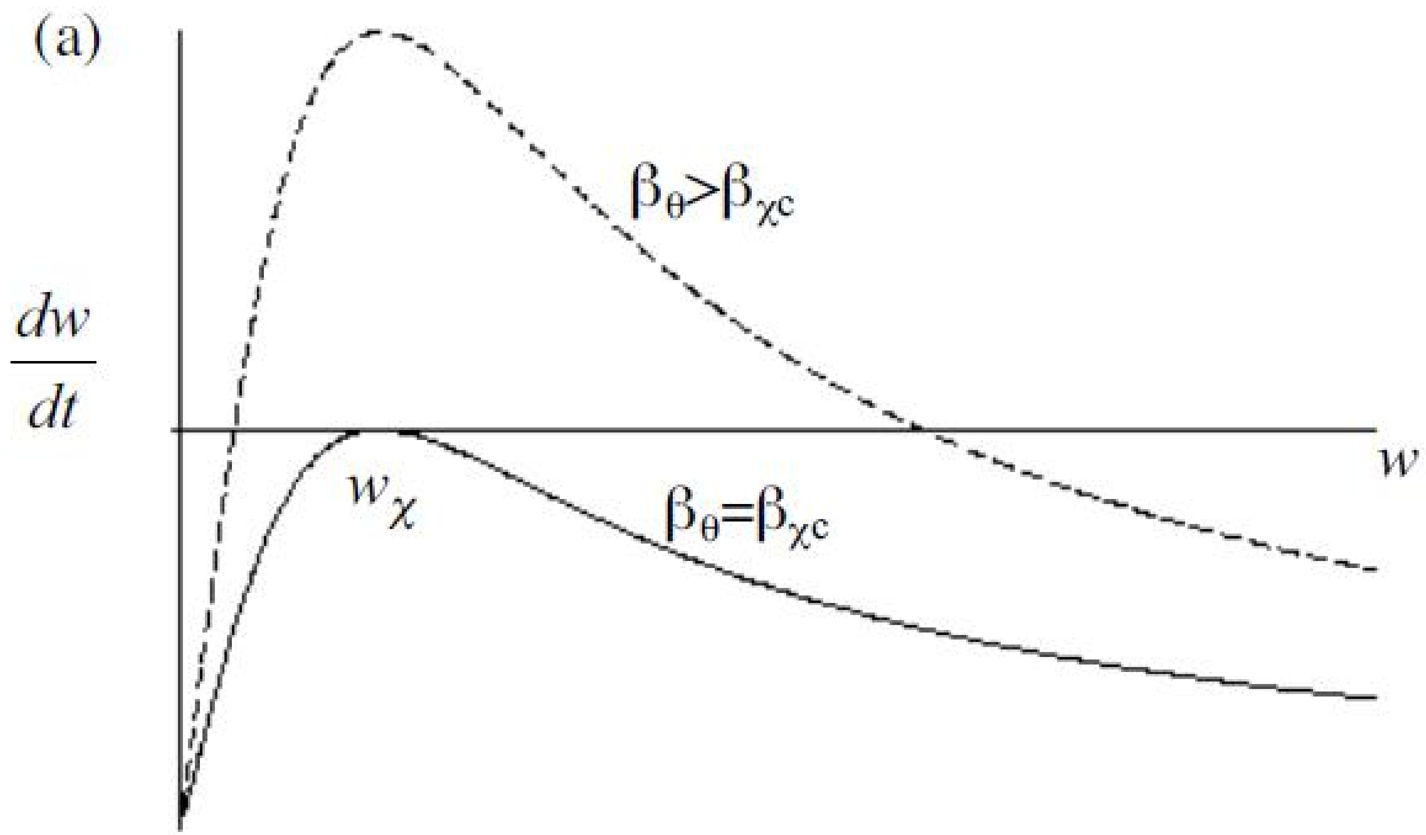
For typical tokamak parameters, it is about 1cm.

➢ **Remark: the perpendicular transport coefficient?**

Transport model

- **The island evolution is**

$$\frac{8\pi}{c^2}\frac{1}{\eta}I_1\frac{dw}{dt}=\Delta'-2.31\sqrt{\varepsilon}\frac{d\beta_\theta}{dr}\frac{q_s}{q_s'}\frac{1}{w}\frac{1}{1+w_\chi^2/w^2}$$



- **The excitation of NTMs must be above an onset threshold**

$(\beta_p^{onset}, w_{seed})$ $(\beta_N=\beta_T/(I_p/(aB_t)))$

Influence of ion orbit width on the threshold of NTMs

- **Theory of NTM are all based on the assumption of small ion orbit width.**
- **In typical tokamak, the onset island width is compared with ion orbit width**

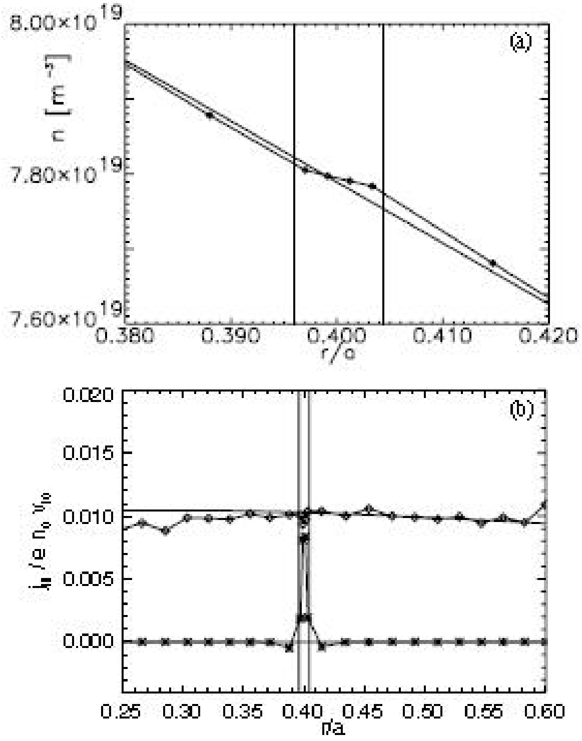


FIG. 2. Density (a) and ion bootstrap current (b) profiles in the presence of a small island $w_b/W = 1.1$. Solid lines again show the analytical calculation; diamonds and stars refer to the simulated ion bootstrap current and its helical component, respectively.

Poli, et. al.,PRL,02

- Simulation have shown the ion orbit width effect is important

Influence of ion orbit width on the threshold of NTMs

- **Here, we focus on the effect of ion orbit width in the standard transport model:**
  - In the standard transport model, critical island width w_c is determined by $\chi_\perp / \chi_{//}$.
  - If $w < w_c$, perpendicular transport dominates, pressure is not flattened.
    perturbed bootstrap current decreases.
- **When the ion orbit width is comparable with island width, the effect of finite ion orbit is important.**
  - Due to the orbit average, the effective perturbation felt by ions is smaller than inherent perturbation, namely the effective island width felt by ion is smaller than island width.
  - Correspondingly, the amplitude of perturbed bootstrap current is reduced.
  - Ion orbit effect tends to increase the critical island width.

Influence of ion orbit width on the threshold of NTMs

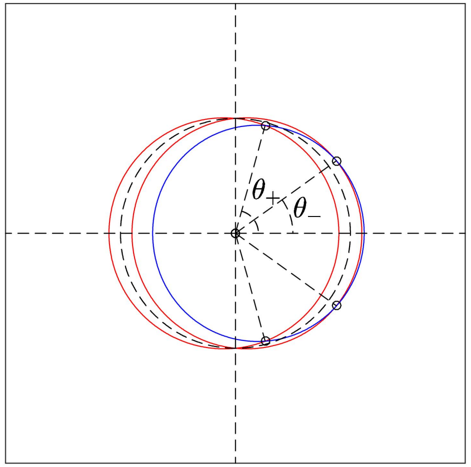
One can obtain $\delta\bar{f}_0 = \delta\bar{f}_0\left(\bar{\psi}, \bar{\xi}, \lambda, E, \sigma_v\right)$, then making orbit average,

$$\left((m - nq(\bar{\psi}))\frac{\partial \delta\bar{f}_0}{\partial \bar{\xi}} - m\frac{\partial}{\partial \bar{\xi}} < \delta\psi(\psi,\xi) >_\theta \frac{\partial \delta\bar{f}_0}{\partial \bar{\psi}}\right) - m\frac{\partial}{\partial \bar{\xi}} < \delta\psi(\psi,\xi) >_\theta \frac{\partial F_M}{\partial \bar{\psi}} + \omega_t^{-1}\chi_\perp \frac{\partial^2 \delta\bar{f}_0}{\partial \bar{\psi}^2} = \omega_t^{-1}\bar{C}\left(\delta\bar{f}\right), \quad (3)$$

In the island, $< \delta\psi(\psi,\xi) >_{\bar{\theta}} \sim \tau^2 \delta\psi(0)$, denotes the effective perturbed flux felt by ions, the constant $\delta\psi$ approximation is made for TM.

$\tau^2 = (\theta_+ - \theta_-)/\pi, \theta_\pm$ are the intersect angles between ion orbit and island, satisfying

$$w_b \cos\theta_\pm = r_s - \bar{r} \pm w\sqrt{1+\cos\xi}\,/(2\sqrt{2}).$$



τ^2 represents the ratio of overlap area of island and ion's orbit to the island, depending on w_b / w.

- If $w_b / w << 1, \tau^2 \sim 1$, ions are almost in the island.
- If $w_b / w >> 1, \tau^2 \sim 0$, ions are almost in the outer region.

Influence of ion orbit width on the threshold of NTMs_*Cai, et. al.,POP,2015*

- the evolution of island width can be written as

$$\frac{8\pi}{c^2}\frac{I_1}{\eta}\frac{dw}{dt}=\Delta'+\frac{1.155\sqrt{\varepsilon}L_q}{L_p}\frac{\beta_{\theta,s}}{w}\left[\frac{1}{1+w_\chi^2/w^2}+\frac{1}{1+w_\chi^2/(\tau^2w^2)+5.63\mathrm{w}_b^2/(\tau^2w^2)}\right],$$

where $L_q=(d\ln q_s/dr)^{-1}, L_p=(d\ln\beta_{\theta,s}/dr)^{-1}, \beta_\theta=\beta_{\theta,e}+\beta_{\theta,i}, w_\chi=0.63w_0,$
$w_{0,i}=w_{0,e}=w_0$ is assumed.

Eq.(13) is an interpolation formula, which approximates the limits.

✓ion orbit effect is similar to the finite transport effect, and tends to reduce the flatten effect of ion pressure in the island.

- The dependence of $\beta_{\theta,s}^{onset}$ (marginal stability, given by $dw/dt=0$), on the critical seed island width w_{seed}

$$\beta_{\theta onset}=-\frac{\mathrm{r_s}\Delta' L_p}{1.155\sqrt{\varepsilon}L_q}\frac{w_b}{r_s}\left[\frac{1}{\hat{w}_{seed}^2+w_\chi^2/w_b^2}+\frac{\tau^2}{\tau^2\hat{w}_{seed}^2+w_\chi^2/w_b^2+5.63}\right]^{-1}\frac{1}{\hat{w}_{seed}},$$

where $\hat{w}_{seed}=w_{seed}/w_b$.

Influence of ion orbit width on the threshold of NTMs_*Cai, et. al.,POP,2015*

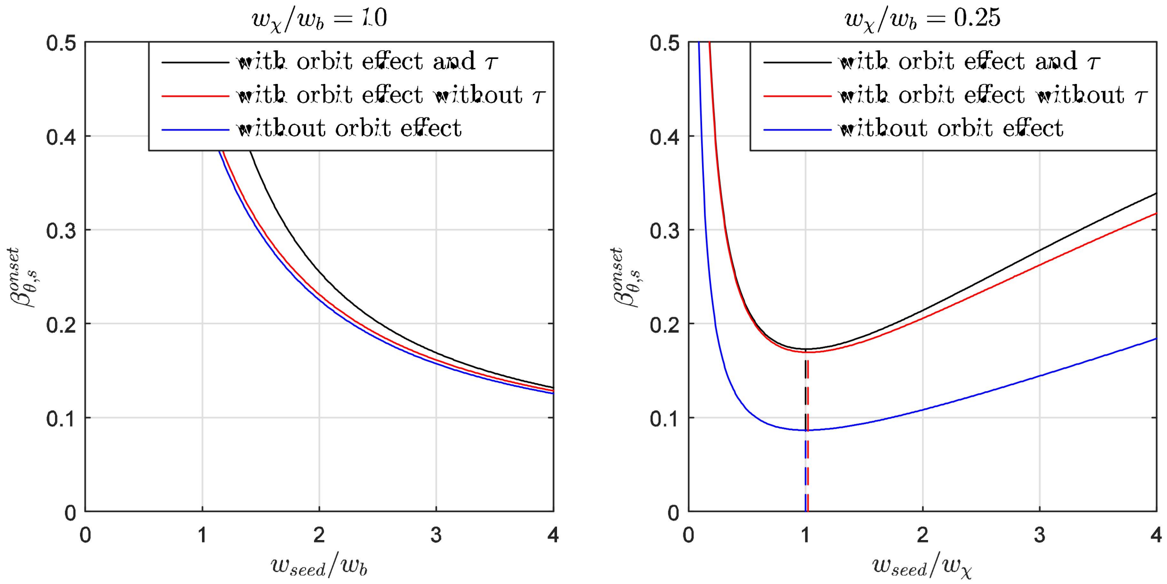
- For a typical tokamak, given $r_s\Delta' = 2, L_p / L_q = 1, \sqrt{\varepsilon} = 0.5,$

  For m=2,n=1 island, $\tau^2 \propto w^2 / w_b^2$ can be obtained if $w << w_b$.

  Then, $\tau^2 = \hat{w}_{seed}^2 / (1 + \hat{w}_{seed}^2)$ is assumed, which approximates to the two limits

  $w << w_b, w >> w_b$.

Influence of ion orbit width on the threshold of NTMs_*Cai, et. al.,POP,2015*

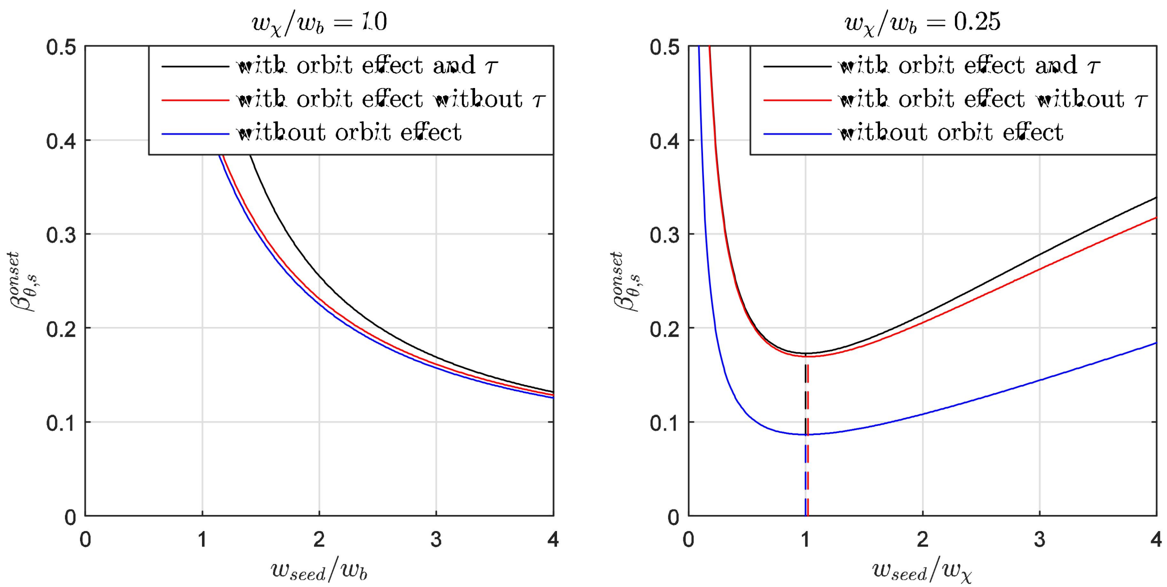


✓ For $w_\chi >> w_b$, finite transport effect dominates, and the ion orbit effect mainly reflects in effective island modification τ.

if $w < w_b$, effective island modification tends to increase $\beta_{\theta,s}^{onset}$;

if $w >> w_b$, $\beta_{\theta,s}^{onset}$ keeps almost unchanged.

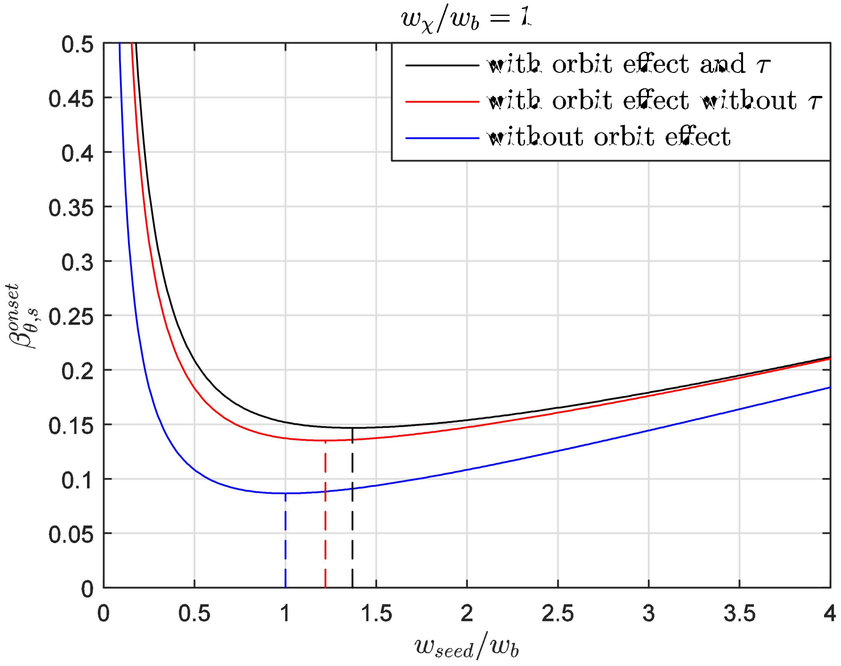
Influence of ion orbit width on the threshold of NTMs_*Cai, et. al.,POP,2015*



- ✓ For $w_\chi << w_b$, ion orbit effect dominates, and tends to increase $\beta_{\theta,s}^{onset}$ for all w. $\beta_{\theta,\min}$ is also enhanced, while w_c changes little and $w_c \sim w_\chi$, since the ion contribution can be neglected in the range of $w << w_b$. For $w \sim w_b$, effective island modification is important, and increases $\beta_{\theta,s}^{onset}$ further.

$\beta_{\theta,s}^{onset} \propto \rho_{th} / r_s$ if $w \sim w_b$ for $w_b >> w_\chi$. It is in agreement with experiment in ASDEX-U.

Influence of ion orbit width on the threshold of NTMs_*Cai, et. al.,POP,2015*



✓ For $w_\chi \sim w_b$, ion orbit effect tends to increase $\beta_{\theta,s}^{onset}$ for a given w_{seed}, effective island modification is important if $w_{seed} \sim w_b$.

ion orbit effect increases the lowest threshold $\beta_{\theta,\min}, w_c$

Neoclassical polarization

- The motion of trapped particles:

$$\mathbf{v} = v_{//}\mathbf{b} + \mathbf{v}_d + \mathbf{v}_E, \mathbf{v}_d = \frac{1}{\Omega_c}\mathbf{b}\times\left(\mu\nabla B + v_{//}^2\boldsymbol{\kappa}\right), \mathbf{v}_E = \frac{c\mathbf{E}\times\mathbf{B}}{B^2}$$

$$v_\theta = v_{//}\frac{B_\theta}{B} - v_d\cos\theta - \frac{cE_r}{B},$$

Making orbit average (taking the time average over a bounce period of trapped particles),

$$0 = \langle v_{//}\rangle\frac{B_\theta}{B} - \langle v_d\cos\theta\rangle - \frac{cE_r}{B}, \Rightarrow \langle v_{//}\rangle = \frac{B}{B_\theta}\langle v_d\cos\theta\rangle + \frac{cE_r}{B_\theta},$$

The conservation of toroidal canonical momentum without toroidal electric field,

$$p_\phi = \frac{Ze}{m}\psi + Rv_\phi \Rightarrow \frac{dr}{dt} = \frac{1}{\omega_{c\theta}}\frac{d}{dt}\left(v_{//}\frac{B_0}{B}\right) \Rightarrow \left\langle\frac{dr}{dt}\right\rangle \sim \frac{1}{\omega_{c\theta}}\frac{d}{dt}\left(\langle v_{//}\rangle\right) = \frac{1}{\omega_{c\theta}}\frac{c}{B_\theta}\frac{dE_r}{dt}$$

Neoclassical polarization drift

Neoclassical polarization current: $J_{npol,r} = \frac{n_{i0}m_ic^2}{B_\theta^{\ 2}}\frac{dE_r}{dt}.$

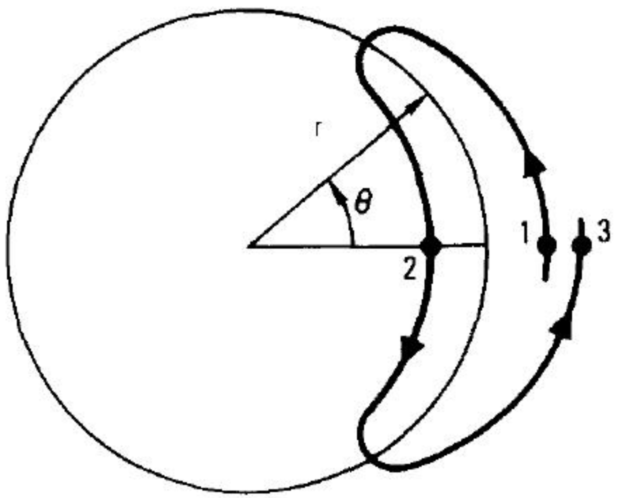
polarization current: $\mathbf{v}_{pol} = \frac{c}{\omega_c B}\frac{\partial\mathbf{E}_\perp}{\partial t}, \Rightarrow \mathbf{J}_{pol} \sim \frac{n_{i0}m_ic^2}{eB^2}\frac{\partial\mathbf{E}_\perp}{\partial t},$

$$\Rightarrow |J_{npol,r}/J_{pol}| \sim \frac{B^2}{B_\theta^{\ 2}}$$

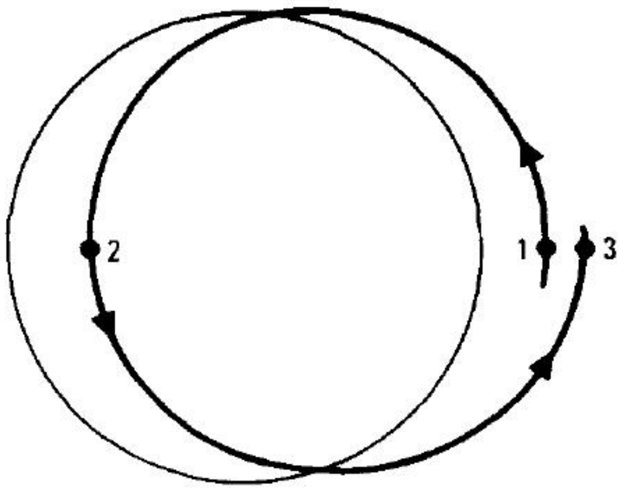
Neoclassical polarization

- One also can calculate the change of kinetic energy over banana orbit as shown in figs. *(Hinton, PF1984)*
- Note: neoclassical polarization current is related to finite orbit width.
- Neoclassical polarization current depends on collision frequency: momentum transfer from the trapped particles to the passing particles
  - In the fluid regime, poloidal flows are strongly damped due to high poloidal viscosity. A parallel flow to maintain torque balance.
  - For low collision frequency regime, the poloidal ion viscosity does not dominate the poloidal force balance and the poloidal ion flow is not necessarily zero. This modifies parallel ion flow.

(a)



(b)



Neoclassical polarization

- **Neoclassical polarization current in the island:**

$$J_{npol,r} \sim \frac{n_{i0} m_i c}{B_\theta} \frac{du_{//}}{dt}, \quad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_E \cdot \nabla$$

Note: The island propagation frequency dominates its growth, the operator on a flux surface quantity is zero.

- **In the fluid regime, the flow varies along the flux surface,**

$$\frac{du_{//}}{dt} \sim \omega c E_r / B_\theta$$

- **For low collision, the flow in the leading order is a flux quantity,** $u_{//} = < cE_r / B_\theta >_f$,

It do not contribute to the neoclassical polarization current. The contribution comes from the trapped-particle parallel flow $\frac{du_{//}}{dt} \sim \varepsilon^{3/2} \omega c E_r / B_\theta$

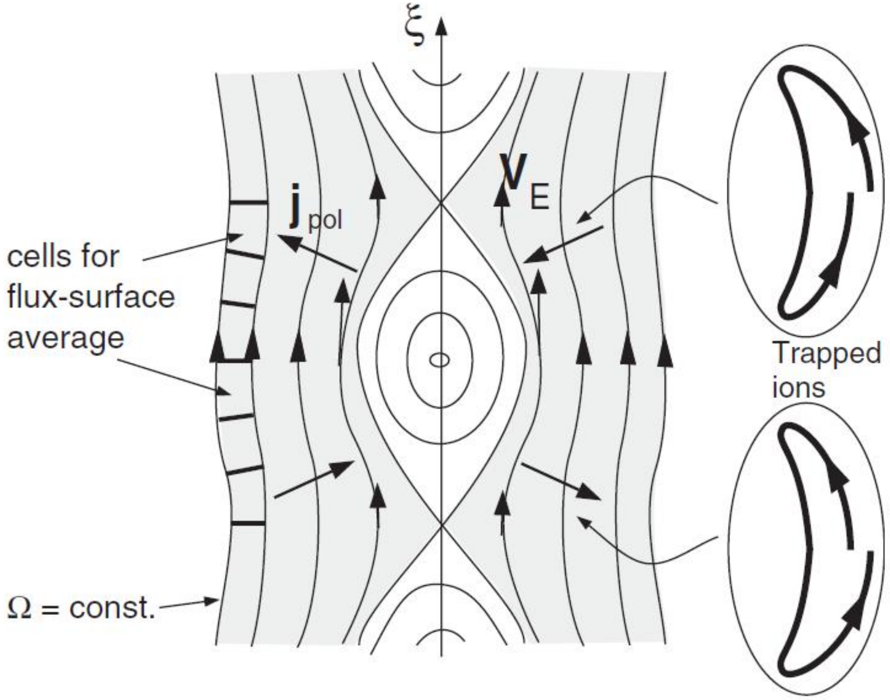


Figure 1. Polarization current j_{pol} in the presence of a magnetic island.

Fluid regime

- For simplicity, the diamagnetic drifts are not considered.
- Given magnetic field

$$\mathbf{B} = I\nabla\zeta + \nabla\zeta\times\nabla(\psi+\delta\psi),\ \delta\psi = \delta\hat{\psi}(0)\cos\xi,\quad \xi = m(\theta-\zeta/q_s) - \int\omega(t')dt$$

- Basic equations:

$$E_{//} = \eta(J_{//} - J_{bs}),\quad \mathbf{B}\cdot\nabla\frac{J_{//}}{B} + \nabla\cdot\mathbf{J}_{pol} = 0,\qquad E_{//} = -\mathbf{b}\cdot\nabla\phi - \frac{1}{c}\frac{\partial\delta A_{//}}{\partial t}$$

- Coordinate: (Ω,ξ,t'), $\Rightarrow \dfrac{\partial}{\partial t} = \dfrac{\partial}{\partial t'} + \dfrac{\partial\xi}{\partial t}\dfrac{\partial}{\partial\xi} = \dfrac{\partial}{\partial t'} - \omega\dfrac{\partial}{\partial\xi}$

The growth rate is much smaller than island propagation frequency, electrons flow rapidly along the oscillating magnetic field lines of the island to short out the parallel electric field. To leading order, $E_{//} = 0$.

$$E_{//} = -\mathbf{b}\cdot\nabla\phi^0 + \frac{1}{c}\omega\frac{\partial\delta A_{//}}{\partial\xi} \Rightarrow \frac{n}{qR}\frac{dq_s}{dx}x\frac{\partial\phi}{\partial\xi} + \frac{1}{c}\omega\frac{\partial\delta A_{//}}{\partial\xi} = 0 \Rightarrow \phi^{(0)} = \frac{\omega}{nc}\left(x - \lambda(\Omega)H(\Omega-1)\right).$$

The first term leads to the $E\times B$ rotation of the whole plasma, and the second term describes the modification of this potential outside the island.

Fluid regime

$$< J_{//} > = \frac{1}{cR}\frac{1}{\eta} < \frac{\partial \delta\hat{\psi}}{\partial t}\cos\xi > + < \delta J_b > .$$

$$J_{//} = J_{//,pol} + < J_{//} > / < 1 >, \ < J_{//,npol} > = 0.$$

$$\frac{8\pi}{c^2}\frac{1}{\eta} I_1 \frac{dw}{dt} = \Delta' + \Delta'_b + \Delta'_{npol} \,, \ \Delta'_{npol} = \frac{G_3}{w^3} L_s^2 \frac{(\omega - \omega_E)(\omega - \omega_{*i})}{k_\theta^2 v_{A\theta}^2}$$

- **Depends on the island propagation frequency**
- **Proportional to** $1/v_{A\theta}^2$, **contrast with** $1/v_A^2$ **for polarization current.**

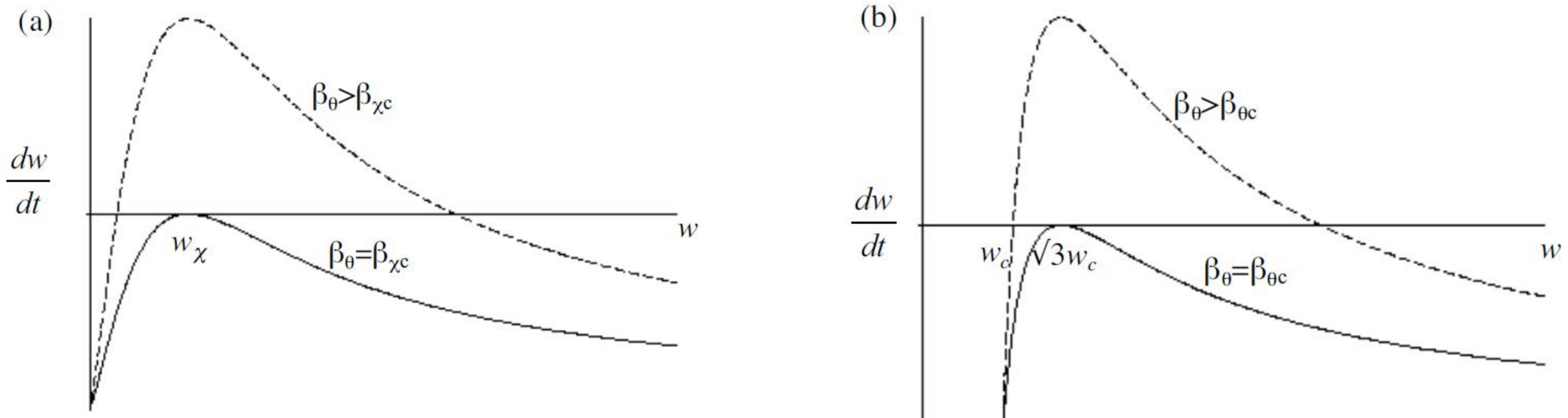
Neoclassical polarization model

- In toroidal geometry, trapped particles drifting along the island under the influence of electric drift flow (difference in the O and X point). This drift is transferred collisionally to the passing particles, yielding the neoclassical polarization current. For low collision,

$$|\mathbf{J}_{npol}/\mathbf{J}^{c}_{pol}| = q^2/\sqrt{\varepsilon} \Longrightarrow \mathbf{B}\cdot\nabla\frac{J_{//,np}}{B} = -\nabla\cdot\mathbf{J}_{npol}$$

- The island evolution including neoclassical polarization current:

$$a_1\frac{\tau_r}{r_s^2}\frac{dw}{dt} = \Delta' + a_2\sqrt{\varepsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w^2}{w^2+w_\chi^2} - a_3\frac{\omega(\omega-\omega_{*i})}{\omega_{*e}^2}g(\varepsilon,\nu_i)\left(\frac{\rho_{bi}}{w}\right)^2\left(\frac{L_q}{L_p}\right)^2\frac{\beta_\theta}{w}, \qquad g(\varepsilon,\nu_i) = \begin{cases} 1.64\varepsilon^{1/2} & \nu_i/\varepsilon\omega << 1 \\ \varepsilon^{-1} & \nu_i/\varepsilon\omega >> 1 \end{cases}$$



(a) the transport threshold model, and (b) the polarization current model.

Island propagation frequency

$$a_1 \frac{\tau_r}{r_s^2}\frac{dw}{dt} = \Delta' + a_2\sqrt{\varepsilon}\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w^2}{w^2+w_\chi^2} - a_3\frac{\omega(\omega-\omega_{*i})}{\omega_{*e}^2} g(\varepsilon,\nu_i)\left(\frac{\rho_{bi}}{w}\right)^2\left(\frac{L_q}{L_p}\right)^2\frac{\beta_\theta}{w},$$

- **Remark: this polarization term proportional to island propagation frequency, and can be stabilizing or destabilizing. The problem is additional physics related to dissipation process needed to be introduced to determine the frequency. Still an open question.**
- **The island propagation frequency is determined by the momentum/torque balance.**

$$m_i n_0 \frac{d}{dt}\left(\mathbf{B}_\phi \cdot \mathbf{V}\right) = \frac{1}{c}\mathbf{B}_\phi \cdot \mathbf{J}\times\mathbf{B} - <\mathbf{B}_\phi \cdot \nabla \cdot \mathbf{\Pi}_{//}> + \mu_a m_i n_0 B_0 \nabla_\perp^2 V_\zeta$$

- **Or by matching condition:**

$$\Delta_c' = \int_{0^-}^{0^+} dx \oint J_{//} \cos\xi d\xi /(2\pi)$$ **determine island width evolution**

$$\Delta_s' = \int_{0^-}^{0^+} dx \oint J_{//} \sin\xi d\xi /(2\pi)$$ **determine island propagation frequency**

⇧

Resulting from external field, such as error field, RMP…

Experimental evidence

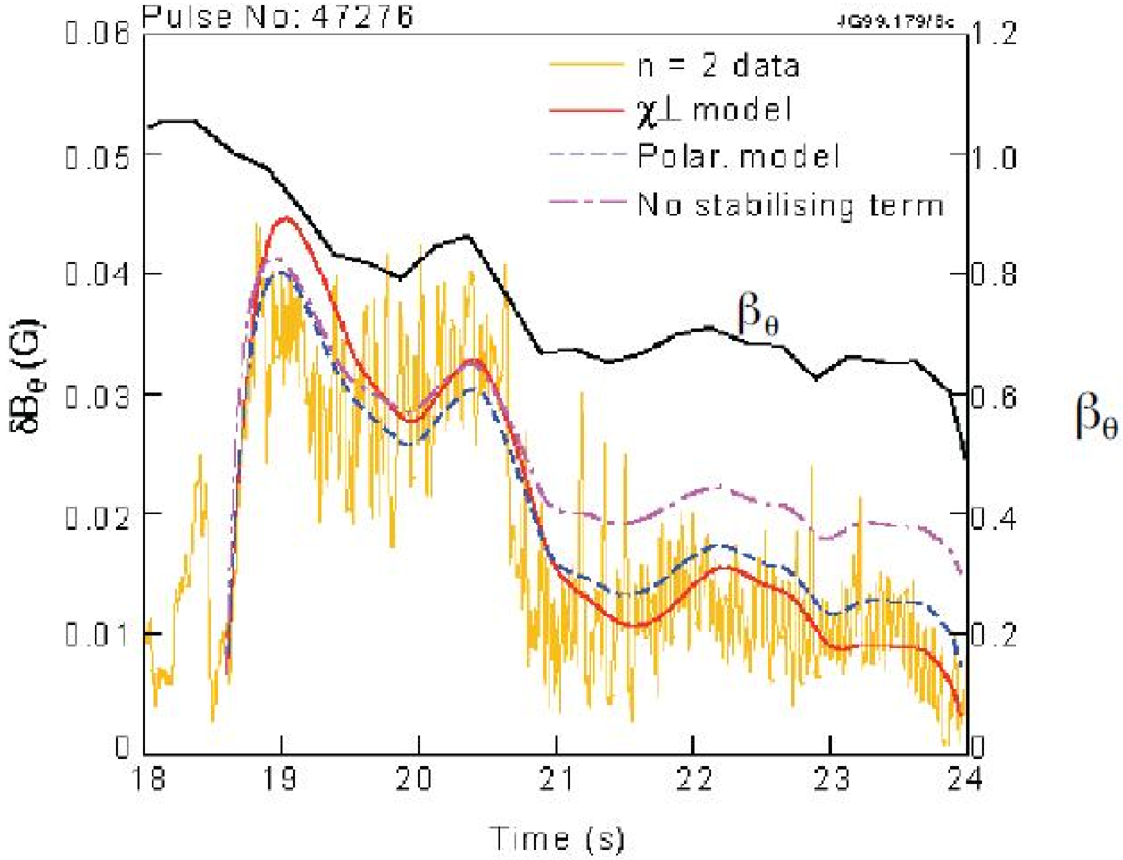
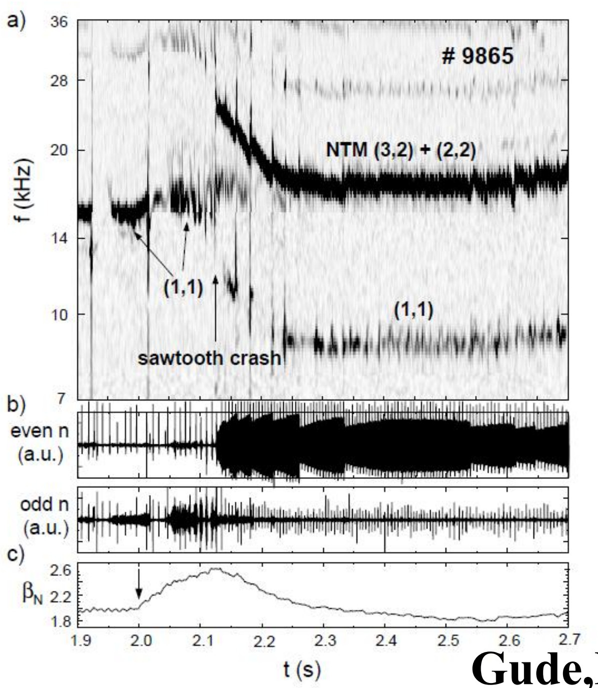


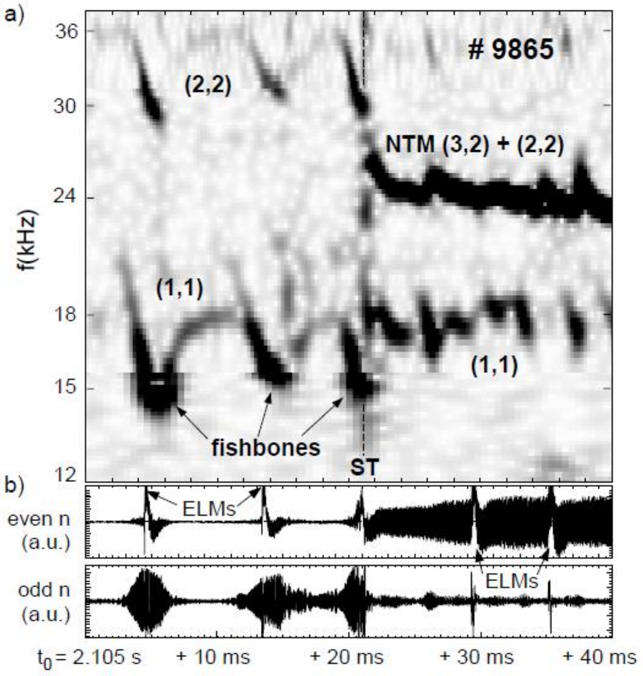
Fig. 5. Tracking the island evolution as the heating power is reduced on JET, we see that inclusion of either of the threshold effects improves the agreement with the measured amplitude of the magnetic perturbation, δB (from Ref 18)

18. R J Buttery et al "Neoclassical tearing modes", *Plasma Phys Control Fusion* 42 B61 (2000)

Seed island

- The onset threshold of NTM $(\beta_p^{onset}, w_{seed})$ $(\beta_N = \beta_T/(I_p/(aB_t)))$
  - Spontaneous tearing mode provides the seed island. $(\Delta' > 0)$
  - Seed island is generated by other mechanisms to it above the threshold. $(\Delta' < 0)$
    - Instabilities: sawtooth; fishbone; ELMs…
    - RMP…





Gude,NF1999

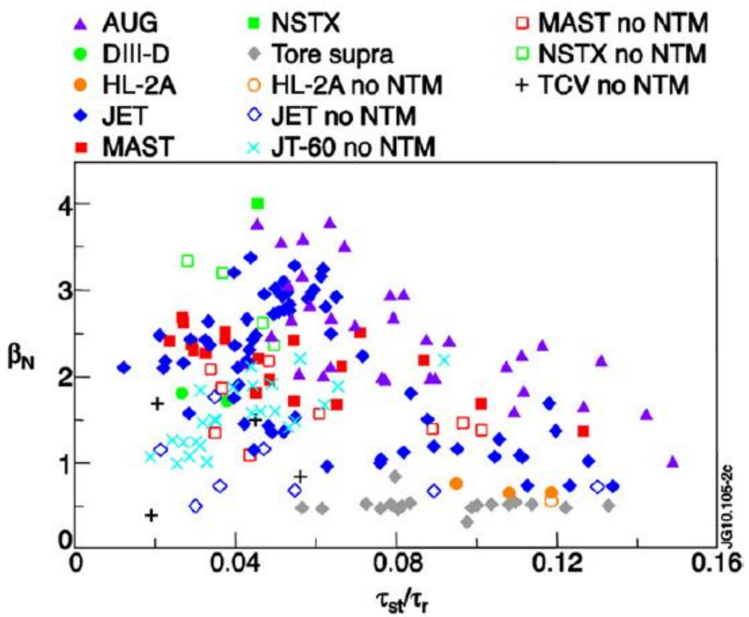


Figure 2. β_N at the NTM onset with respect to the sawtooth period normalized to the resistive diffusion time (here all (m, n) NTMs are included).

Chapmon,NF2010

Seed island formation due to geometry coupling

- **External magnetic field perturbation plays a role like forced magnetic reconnection .**
- **External source magnetic field perturbation:** $\psi_S(t) = \Psi_s \left(\frac{t}{\tau_G} \right)^\alpha e^{iM\theta - in\zeta}$
- **Ideal MHD equation for** $\psi_m e^{im\theta - in\zeta}$:

$L_{m,n}\psi_m(r) = -\Lambda_s^m \psi_s(r)$, Λ_s^m **an operator describing geometry coupling** ψ_s, ψ_m.

⇩

$$\Delta' = \Delta'_{\mathrm{mode}} + \Delta'_c \frac{\psi_s(t)}{\Psi(r_s)}$$

- **Perturbation current at the rational surface:**

$$\delta J_{//} \sim \delta(r - r_s)\Delta'\Psi_s = \delta(r - r_s)\left(\Delta'_{\mathrm{mode}}\Psi_s + \Delta'_c \psi_S(t)\right) \Longrightarrow \Delta'\Psi(r_s) = \int_{0^-}^{0^+} dx\,\delta J_{inner}$$

$$\Longrightarrow \Psi(t) = -\psi_s(t) \frac{\Delta_c'}{\Delta'_{\mathrm{mode}}} \frac{t^2}{\tau_{sp}^2} \frac{2m^2}{\pi(\alpha+1)(\alpha+2)}$$

➢ **Time scale**

- **Linear theory should be extended to nonlinear theory** *(Hegna, Phys. Plasmas, 1999)*

Seed island formation due to geometry coupling

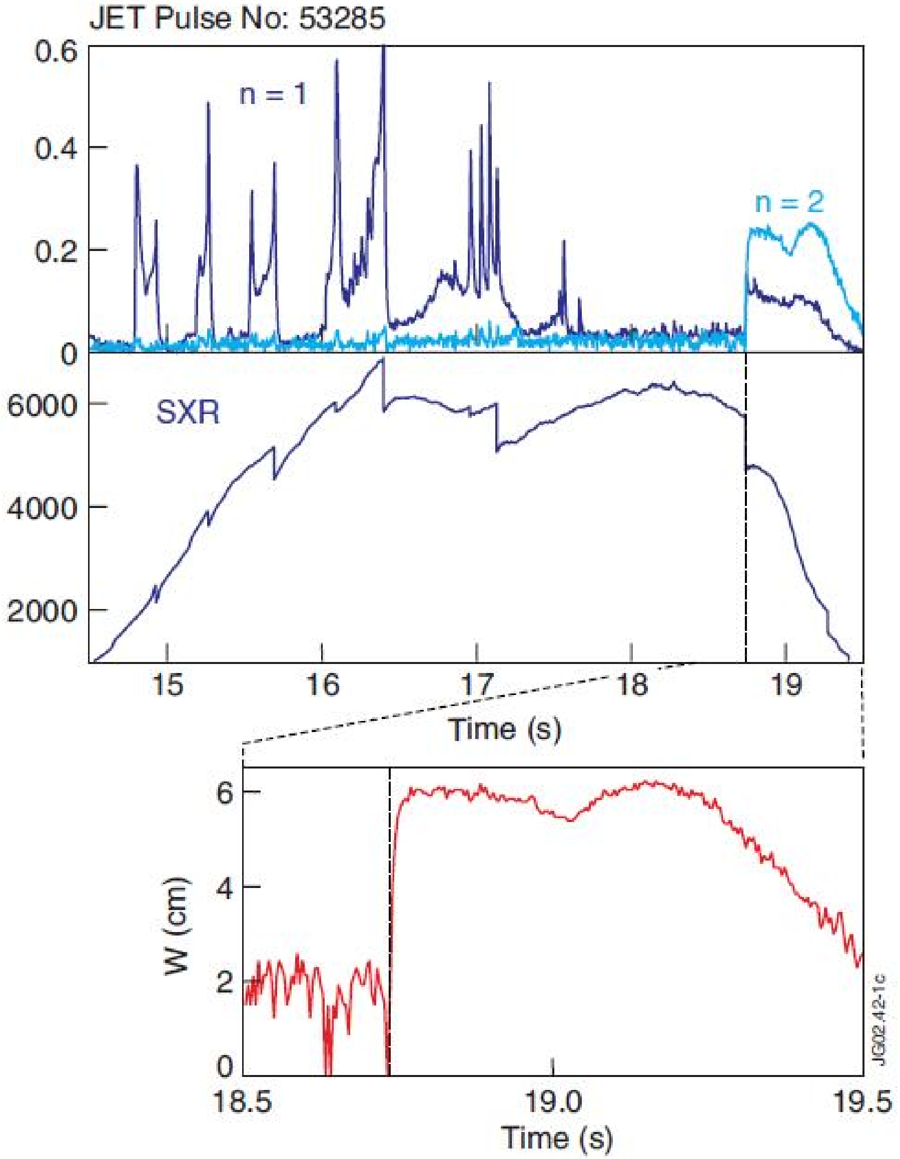


Figure 7. $m = 3/n = 2$ NTM triggered by a sawtooth crash being delayed (achieving a sawtooth period of 1.5 s) by the presence of ICRF fast particles at $I_p = 3.3$ MA, $B = 3.3$ T using minority H heating in the centre with $f = 51$ MHz. The island is directly triggered at a saturated size of 6 cm. Note that there is no significant $n = 1$ activity before the crash, and the relative SXR crash of about 15% is not much larger than the previous sawtooth crashes.

Control of NTMs

◆ **Generalized dispersion:**

$$\Delta'\delta\psi = 2\mu_0 \int_{0^-}^{0^+} dx \oint d\xi \, \delta J_{//} \cos\xi,$$

◆ **Two control schemes:**

➢ **To reduce the free energy available in the equilibrium current profile, so that the stability criterion becomes more negative. i.e. LHCD**

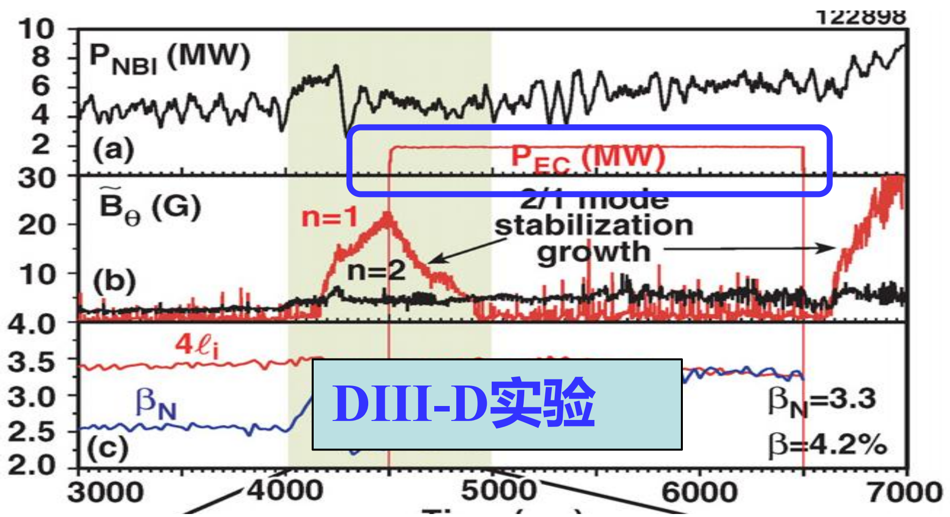
➢ **To drive current directly at the island O-point (to replace the missing bootstrap current). i.e. ECCD (valid method so far, but it needs large power)**

$$a_1 \frac{\tau_r}{r_s^2}\frac{dw}{dt} = \Delta' + a_2\sqrt{\varepsilon}\,\frac{\beta_\theta}{w}\frac{L_q}{L_p}\frac{w^2}{w^2+w_\chi^2} - a_3\frac{\omega(\omega-\omega_{*i})}{\omega_{*e}^2} g(\varepsilon,\nu_i)\left(\frac{\rho_{bi}}{w}\right)^2\left(\frac{L_q}{L_p}\right)^2\frac{\beta_\theta}{w} + \Delta'_{ECCD} + \ldots,$$

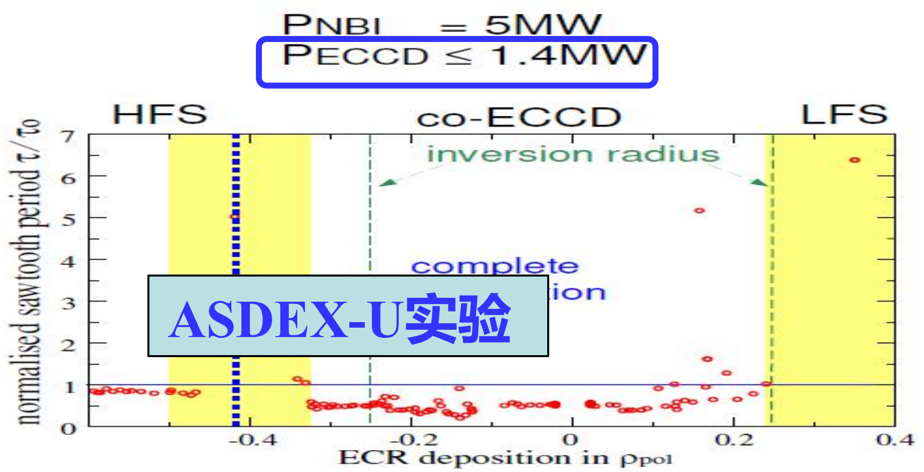
➢ **External coils for repositioning of locked island**

Control of NTMs-ECCD

◆ **Radio frequency waves at the electron-cyclotron resonance, which drives current (ECCD) in a much narrower radial region at the O-point. The deposition width of ECCD must be smaller than island width.**



Nucl. Fusion 47, 371(2007)



Plasma Phys. Control. Fusion 47, 1633(2005)

Control of NTMs-LHCD

◆ **Radio frequency waves in the lower hybrid frequency range have been used to drive current close to the rational surface in COMPASS-D. Modelling indicates the stabilization consistent with the reduction in Δ'.**

The radial width of the current deposition was typically much wider than the island width.

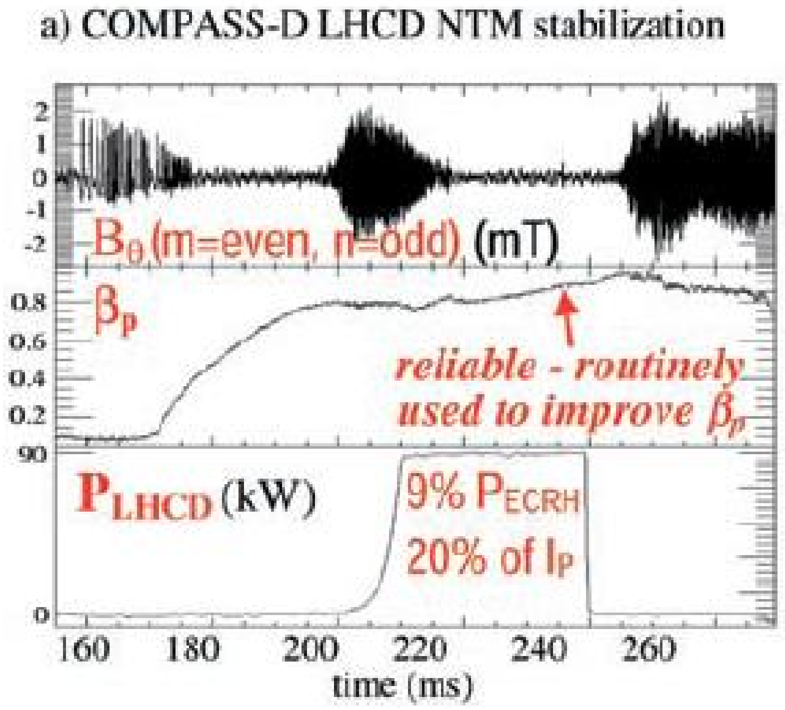


Fig. Stabilization of NTMs using LHCD

Buttery, et. al., PPCF 2000.

Control of NTMs-ICRH

- **In principle also ICRH has the capability of driving current in a plasma. Due to the applied frequencies, a narrow localization, comparable to the ECRH systems, is not possible.**

- **The more important role of the ICRH lies in tailoring the sawtooth stability via a modification of the fast particle.**

- **With large crash time and large sawtooth crashes NTMs could be triggered at very low β values, whereas for small crash time and small sawteeth NTMs could be avoided almost reaching the ideal limit before triggering an NTM during a ramp of the external NBI heating power.**

External coils for repositioning of locked modes

- **If an NTM, grows to a sufficiently large size, it eventually locks to the intrinsic error field in a characteristic position. In such a situation the mode stops rotating and remains in a fixed position and eventually stops the entire plasma rotation. The plasma can no longer be described as a 2D equilibrium, but has to be treated as a full 3D equilibrium containing the (2/1) locked mode as a perturbation.**

- **Applying ECCD on the resonant surface with a locked mode is more complicated. The ECCD deposition spots have to be aligned with the O-point. The adjustment of the deposition above and below the midplane allows for some adjustment, but does not in general assure pure current drive in the O-point.**

- **A possible way to resolve this problem lies in the modification of the locking position of the mode itself. This can be achieved by controlling the intrinsic error field of the experiment. At DIII-D the total error field can be modified by additional internal coils.**

Outline

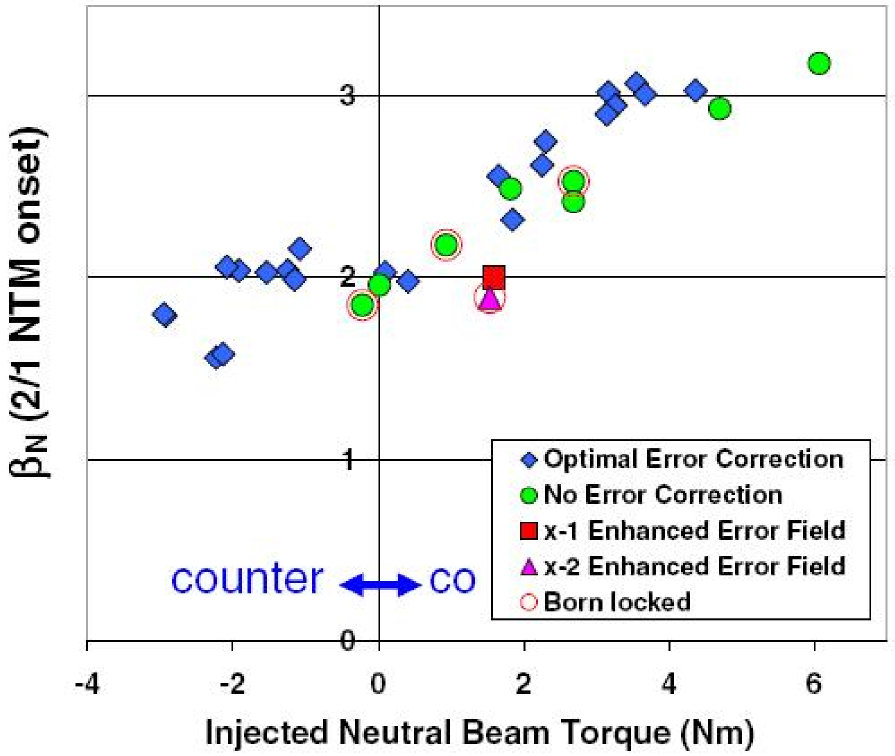
- ◆ Motivation and Background
- ◆ Basic physics of classical tearing mode
- ◆ Physics of neoclassical tearing mode (NTM)
  - ➢ Bootstrap current
  - ➢ Neoclassical tearing mode
  - ➢ Threshold of neoclassical tearing mode
    - ✓Transport model
    - ✓Neoclassical polarization current model
    - ✓….
  - ➢ Other effects on neoclassical tearing mode
- ◆ Control of neoclassical tearing mode
- ◆ **Interaction between energetic ions and neoclassical tearing mode**
- ◆ Conclusion, Remark and Outlook

Background of energetic ions

- **Influences of energetic ions:**
  - **affect the heating efficiency, sustainment of high confinement capability of 'burning' plasmas and the output of high fusion energy.**
  - **The loss of fast particles leads to excessive heat loading and damage to plasma facing components**
  - **The interaction between fast particles and plasma waves (e. g. Alfvén waves) and instabilities (e. g. kink mode) will lead to the redistribution and loss of fast particles and bring about some new plasma phenomena (e. g. fishbone).**
  - **……**

Interaction between energetic ions and tearing modes

◆ The experiments showed that the onset threshold of NTMs with NBI.



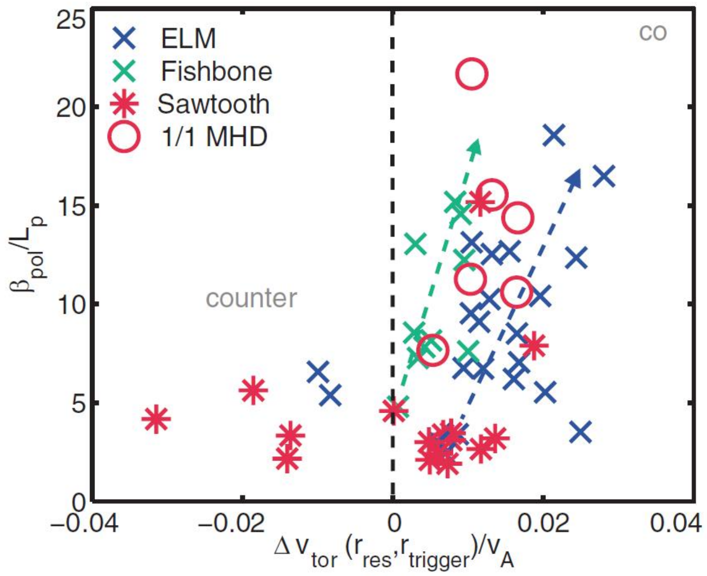
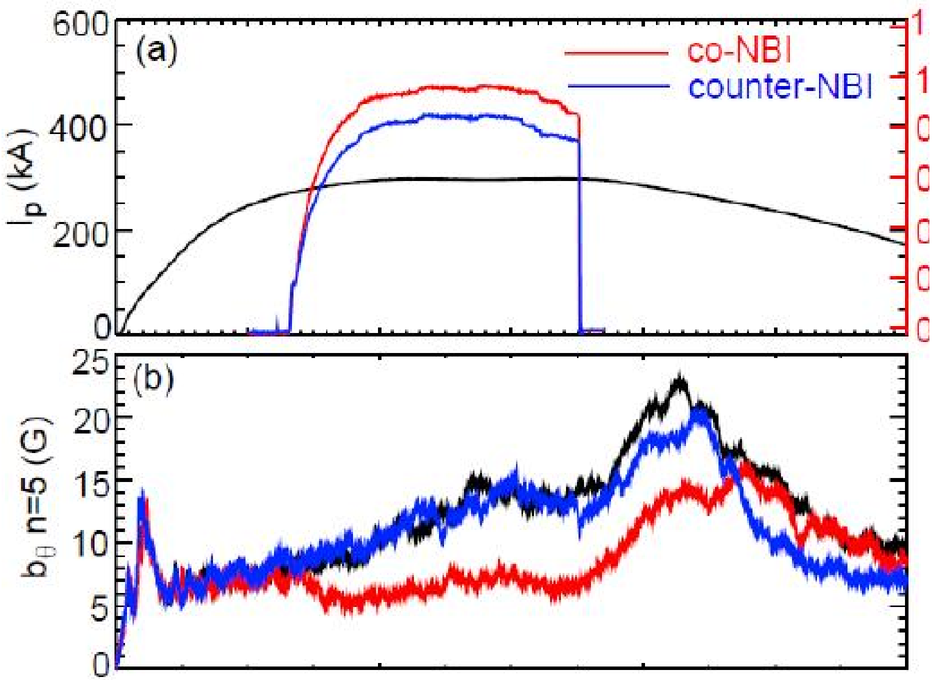


Figure 8. (3, 2) NTM onset threshold versus the differential rotation between the trigger surface (either pedestal top for ELMs or $q = 1$ surface for fishbones, sawtooth or other 1/1 activity) and island surface normalized to the Alfvén velocity.

Experiment in DIIID, Pop *15, 056115(2008)*

Experiment in ASDEX-U, PPCF *(2013)*

Interaction between energetic ions and tearing modes



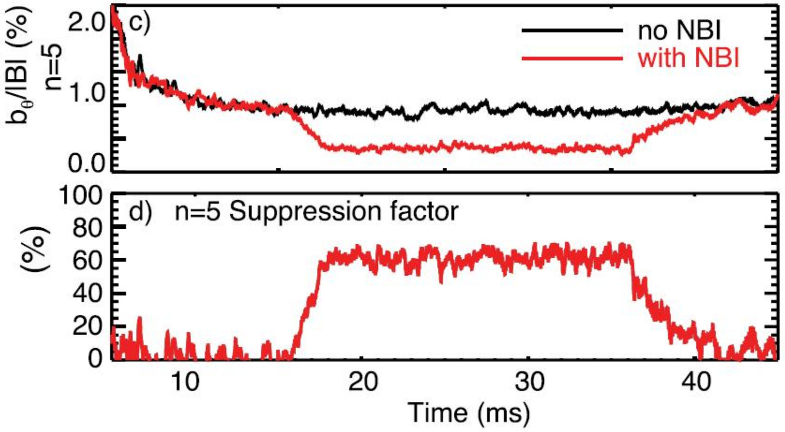


FIG. 5. NBI-induced suppression of the core-most ($n = 5$ in this case) tearing mode. Panel (a) is the plasma current and NB injection versus time, (b) is the line-averaged electron density and neutron flux versus time, and panel (c) is the measure of the magnetic fluctuation amplitude of the core-most tearing mode with and without NBI. Also defined (d) is the mode suppression factor, used as a proxy of core fast ion content.

Experiment in MST,
(IAEA 12th s7-o19)

Experiment in MST,
(pop2013)

Interaction between energetic ions and NTMs

- **Some experiments have shown that fast ions can interact effectively with NTMs. (Experiment in ASDEX-U)**

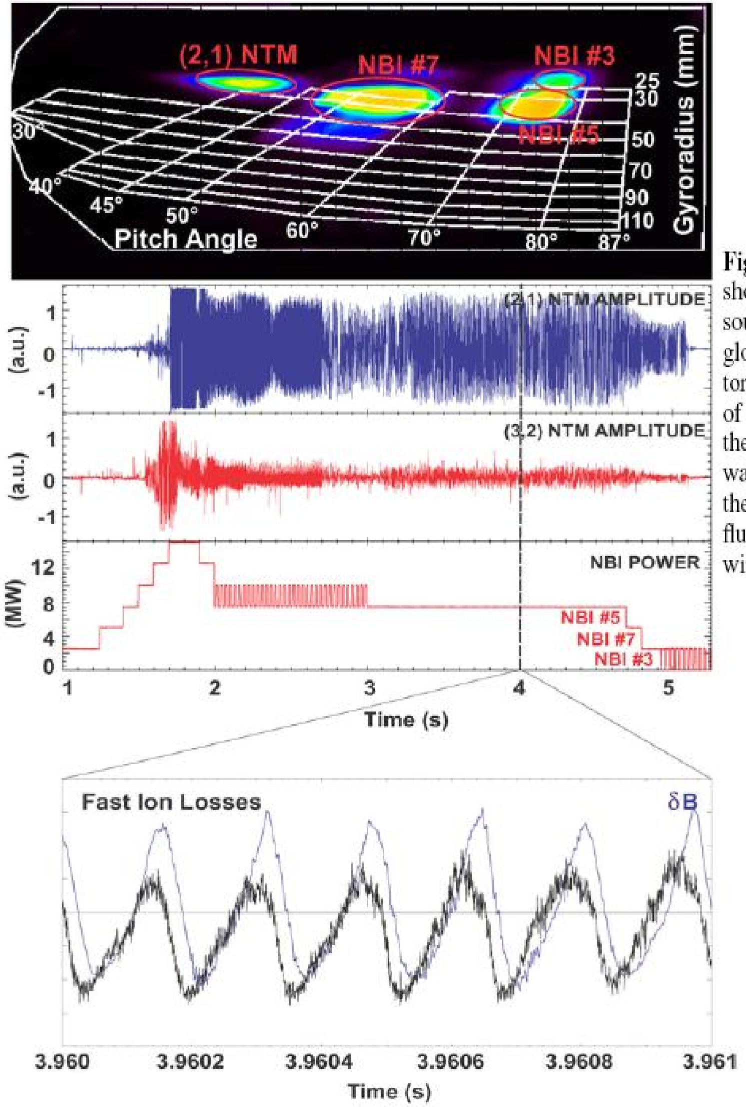


Figure 2. AUG discharge #21089. *Top frame*: CCD picture showing the three spots produced by first orbit losses from NBI sources #3, #5 and #7 and (2,1) NTM induced FIL. *Mid frames*: global magnetic fluctuation amplitude $\delta B(t)$ for the modes with odd toroidal mode number (this signal is dominated by the contribution of the (2,1) mode) and with even toroidal mode number (in this case the signal is dominated by the contribution of the (3,2) mode); waveform of the total NBI power. *Bottom frame*: time evolution of the amplitude of a FIL signal (black curve) and of the magnetic fluctuation due to the (2,1) mode (blue curve) in a short time window including a few mode oscillations.

M.García-Muñoz, *et. al.*, Nucl. Fusion(2007)

The basic physics of NTMs

◆ The basic physics of tearing modes

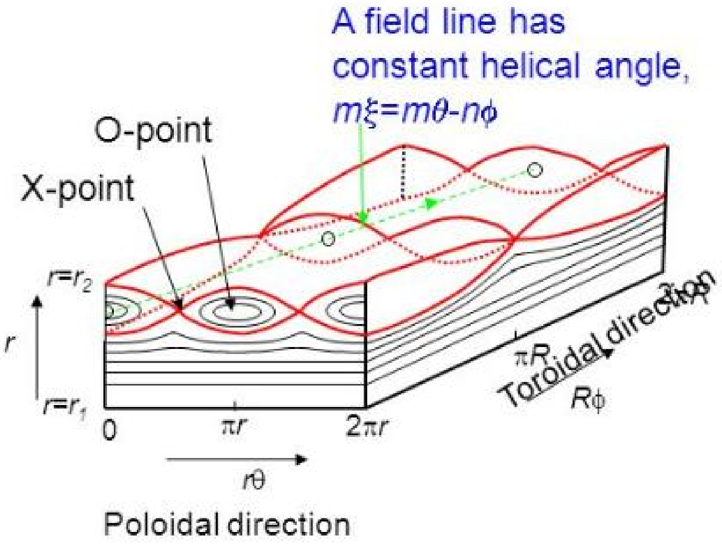
- ➢Away from rational surface, $\delta\psi$ is determined by the ideal MHD equations. It has a discontinuous derivative at rational surface.
- ➢At the singular layer, $\delta\psi$ is determined by resistive MHD Eqs.
- ➢By matching the solutions of outer and inner region, the dispersion relation can be obtained.

$$\Delta'\delta\psi = 2\mu_0 \int_{0^-}^{0^+} dx \oint d\xi J_{//} e^{i\xi},$$

$$\Delta' = \frac{1}{\delta\psi}\left[\left.\frac{d\delta\psi}{dr}\right|_{r_s^+} - \left.\frac{d\delta\psi}{dr}\right|_{r_s^-}\right].$$

$$\Delta_c'\delta\psi = 2\mu_0 \int_{0^-}^{0^+} dx \oint d\xi \left(J_{//,R}\cos\xi - \delta J_{//,I}\sin\xi\right),$$

$$\Delta_s'\delta\psi = 2\mu_0 \int_{0^-}^{0^+} dx \oint d\xi \left(J_{//,R}\sin\xi + \delta J_{//,I}\cos\xi\right)$$



◆EPs can influence TMs through interaction with the outer region and island region

- **The influence of energetic ions on NTMs**
- **The resonance between energetic ions and NTMs**

Heuristic picture of energetic ion

- **Energetic ions: energy much larger than thermal particles; deviated Maxwellian distribution (isotropic or anisotropic)**
- **characteristic frequencies: precessional frequency, bounce/transit frequency; trapped/passing particle**
- **Described by gyrokinetic theory**
- **Understanding its effects on instabilities**
  - **Non-resonant effect: energetic ions can be imaged like a steel wire, have a stiff response to the perturbation**
  - **Resonant effect**

Influence of energetic ions on NTMs

- **Magnetic island much smaller than orbit width of energetic ions (satisfied for linear, early nonlinear phase, and the onset threshold of NTMs) :**

  - orbits of EPs are mainly in the outer region, i.e., Eps mainly interact directly with TMs in the outer region, and changes the stability criterion of tearing modes by changing the perturbed parallel current. (PRL, 106, 075002(2011); Pop(2012);PoP(2019))

  - In the island region, owing to quasineutrality, EPs will provide an uncompensated current because their response is significantly reduced by the orbital averaging effect in the limit of large orbital width.

Influence of circulating energetic ions in the outer region

- Heuristic interpretation:

  - **In the outer region of classical tearing modes:**

$\mathbf{B}\cdot\nabla J_{//}=0 \Longrightarrow J_{//}=J_{//}(\Omega)$ **is flux function.** $\Longrightarrow \delta J_{//}=(dJ_{//0}/dQ)\delta\psi$

where $\Omega=Q(\psi)+\delta\psi$, **satisfying** $\mathbf{B}\cdot\nabla\Omega=0, dQ/d\psi=1-q/q_s$.

$\psi,\delta\psi$ **are the equilibrium and perturbed poloidal magnetic flux, respectively.**

q **is the safety factor,** $q_s=m/n$ **is the value at the resonant surface.**

  - **Due to the magnetic drift of energetic ions,**

$\mathbf{B}\cdot\nabla J_{//}=-\nabla_\perp\cdot\mathbf{J}_{\perp,h} \Longrightarrow J_{//}$ **is not a flux function.**

$J_{//}=J_{//,1}+\delta J_{//,2} \Longrightarrow \mathbf{B}_0\cdot\nabla\delta J_{//,2}=-\nabla_\perp\cdot\delta\mathbf{J}_{\perp,h}, J_{//,1}=J_{//.1}(\Omega)$

$\Longrightarrow \delta J_{//}=(dJ_{//0}/dQ)\delta\psi+\delta J_{//,2}$

where $\mathbf{J}_{\perp,h}$ **is the diamagnetic current due to the kinetic part of energetic ions.**

Heuristic interpretation

Without energetic ions, $J_{//} = J_c(\psi_h + \delta\psi) \sim J_{c0}(\psi_h) + \delta J_c(\psi_h) + \frac{dJ_c}{d\psi_h}\delta\psi \Rightarrow \delta J_c(\psi_h) = -\frac{dJ_c}{d\psi_h}\delta\psi$

For energetic particles, if we stand in the particle orbit,

$$J_h = J_h(\psi_{hd} + a\delta\psi_d) \sim J_{h0}(\psi_{hd}) + \delta J_h(\psi_{hd}) + \frac{dJ_{h0}}{d\psi_{hd}} a\delta\psi_d,$$

where the coefficient a **is the effective perturbation for particles in its orbit.**

Then transfer to real space coordinate, and making the flux average, one can obtain

$$< J_h > \sim J_{h0}(\psi_h) + < \delta J_h(\psi_{hd}) > + \frac{dJ_{h0}}{d\psi_h} a^2 \delta\psi_d$$

Thus, the total perturbation current is

$$< \delta J > = \delta J_c + < \delta J_h > = -\frac{dJ_{c0}}{d\psi_h}\delta\psi - \frac{dJ_{h0}}{d\psi_h} a^2 \delta\psi = -\frac{dJ_0}{d\psi_h}\delta\psi + (1 - a^2)\delta\psi \frac{dJ_{h0}}{d\psi_h}$$

So the effect of energetic particles is determined by its direction.

Influence of energetic ions on tearing modes_*PRL* 106, 075002(2011)

Stability criterion of tearing modes:

$$\Delta' = -\frac{\pi(\alpha_0+\alpha_h)}{r_s}\cot\left[\pi\left(\sqrt{m^2+\alpha_0+\alpha_h}-m\right)\right],$$

$$\alpha_0 = -q^2(r_s)[\varepsilon dq(r_s)/dr]^{-1} dJ_{//0}/dr,$$

$$\alpha_h = \chi_0 q^2(r_s)[\varepsilon dq(r_s)/dr]^{-1} dJ_{//0}/dr *$$
$$*\sum_\sigma \sigma(\rho_{hm}^\sigma)^{-1} d\beta_h^\sigma/dr.$$

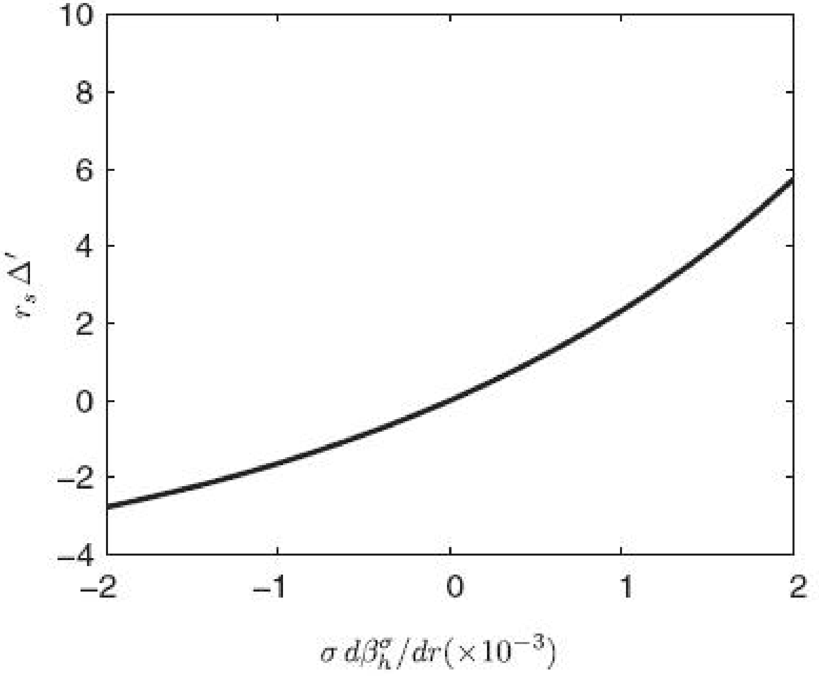
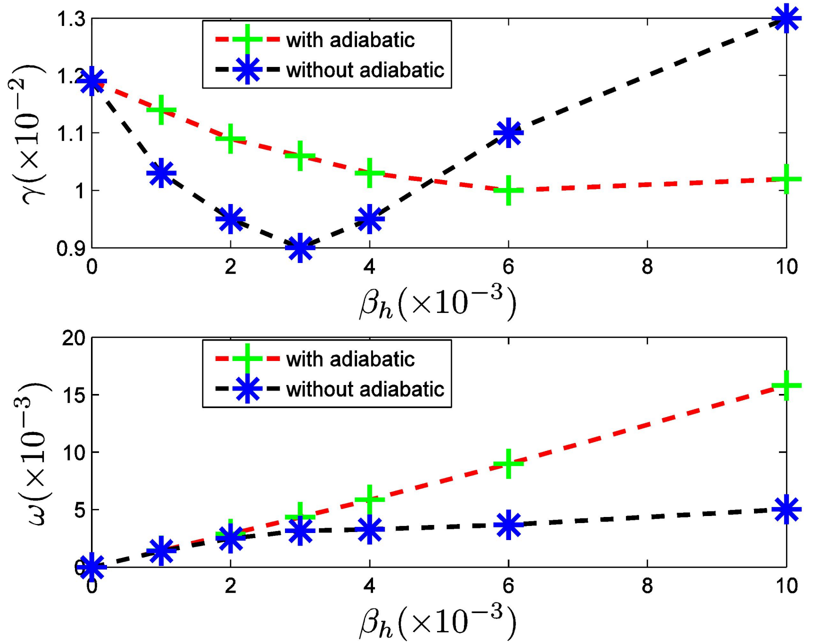


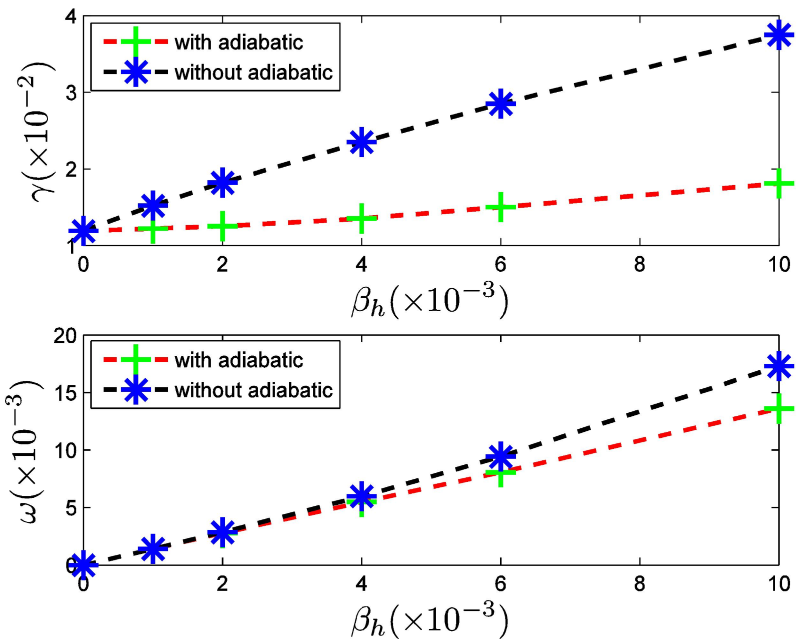
FIG. 2. The stability criterion $r_s\Delta'$ against $\sigma d\beta_h/dr$, where $d\beta_h/dr < 0$ and $d\beta_h/dr \sim -\beta_h/r$ are assumed.

- **For counter-CEI, Δ' increases with β_h and can become positive. Counter-CEI enhances the growth rate and reduces onset threshold.**
- **For co-CEI, Δ' decreases with β_h and can be reduced negative. Co-CEI reduces the growth rate and enhances onset threshold.**

Simulation results for CEI-*Phys. Plasmas* 19,072506(2012)



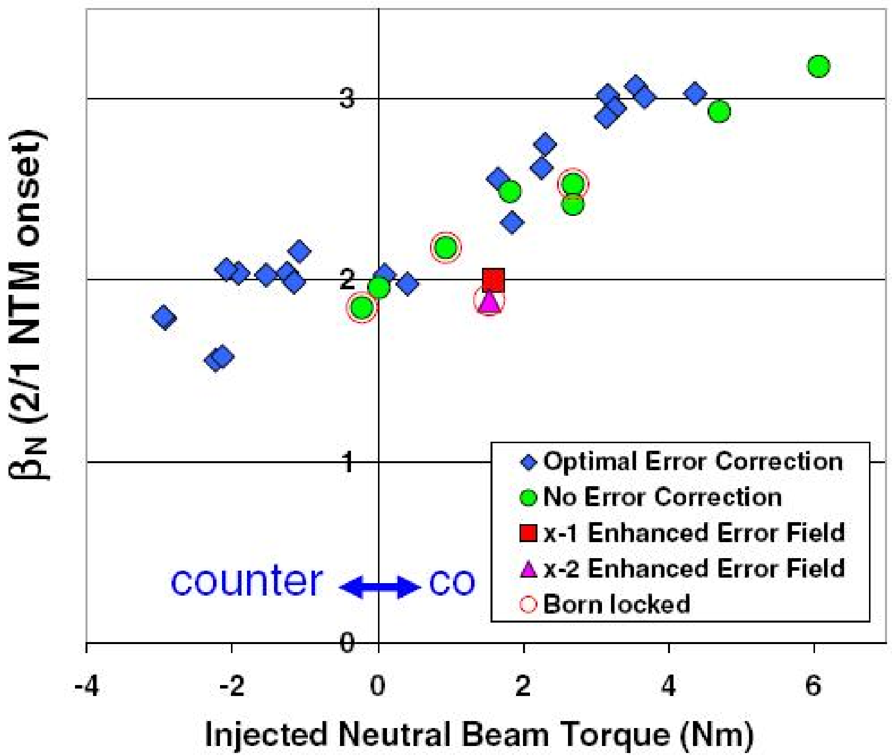
Co-CEI



Counter-CEI

- **For low beta and large drift orbit, the simulation results are consistent with our analytical results (*Phys. Rev. Lett.* 106, 075002(2011)).**
- **The kinetic effect of counter-CEI is strongly destabilizing, while the adiabatic effect is stabilizing. The net effect is destabilizing.**
- **The kinetic effect of co-CEI is weakly stabilizing, while the adiabatic effect is weakly destabilizing. The net effect is stabilizing.**

◆ Experiments showed different dependence of the onset threshold of NTMs on NBI



Experiment in DIIID, Pop *15, 056115(2008)*

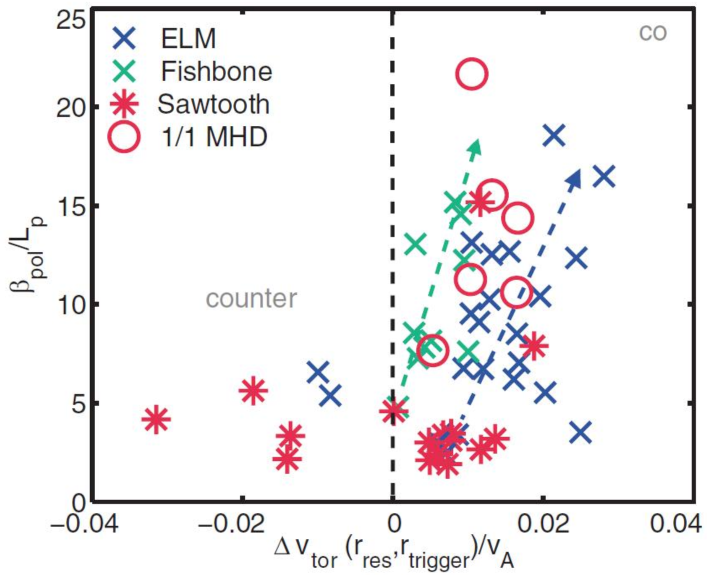


Figure 8. (3, 2) NTM onset threshold versus the differential rotation between the trigger surface (either pedestal top for ELMs or $q = 1$ surface for fishbones, sawtooth or other 1/1 activity) and island surface normalized to the Alfvén velocity.

Experiment in ASDEX-U, PPCF *(2013)*

Discussion about the experimental and theoretical results

- Unbalanced-NBI drives the toroidal rotation which plays a stable role on NTMs (Cai, NF 2016,2017).
- For co-NBI, both energetic ions and rotation plays a stable role, so the onset threshold increases with the power of co-NBI.
- For counter-NBI, whether it is stabilizing or destabilizing depends on which effect dominates among the destabilizing effect of counter-CEIs and the stabilizing effect of toroidal rotation.
- To resolve this issue, self-consistent simulations are required to calculate the dependences of the driven current and the toroidal rotation on the power of NBI consistently.
- Unfortunately, the appropriate codes have yet to be developed.

Influence of trapped energetic ions in the outer region

◆ Heuristic interpretation:

- **In contrast to CEI, TEI orbits in the (r, θ) plane are banana-shaped and concentrated in the low field side.**
- **They thus have strong poloidal asymmetry.**
- **Their orbits are less likely to be changed by perturbations in comparison with CEIs.**
- **They do not produce parallel current directly without considering bootstrap current.**

◆ **Equation including TEI in the outer region:**

$$\mathbf{B}\cdot\nabla\left(\frac{\delta J_\parallel}{B}\right)+\delta\mathbf{B}\cdot\nabla\left(\frac{J_\parallel}{B}\right)$$
$$=-\nabla\cdot\left[\frac{\mathbf{B}}{B^2}\times\nabla\left(\delta p_c+\delta p_{h,a}\right)\right]-\nabla\cdot\left[\frac{\mathbf{B}}{B^2}\times\left(\nabla\cdot\delta\mathbf{p}_{h,N-a}\right)\right]$$

Influence of trapped energetic ions in the outer region_*Phys. Plasmas 26, 062505(2019)*

◆ Equation in the outer region:

$$\frac{1}{q}\frac{1}{R_0}\left[\frac{d}{dr}\left(r\frac{d\delta\hat{A}_\parallel}{dr}\right)-\frac{m^2}{r}\delta\hat{A}_\parallel\right]-\frac{m}{(m-nq)}\frac{d}{dr}\left(\frac{\mu_0 J_{\parallel 0}}{B_0}\right)\delta\hat{A}_\parallel$$

$$-\frac{8\pi(-\bar{\kappa}_r)}{B_0 B_\theta}\frac{m^2}{(m-nq)^2}\frac{d(P_t-P_h)}{dr}\delta\hat{A}_\parallel=0$$

◆ **The fluid and kinetic effects are comparable. They tends to cancel each other out.**

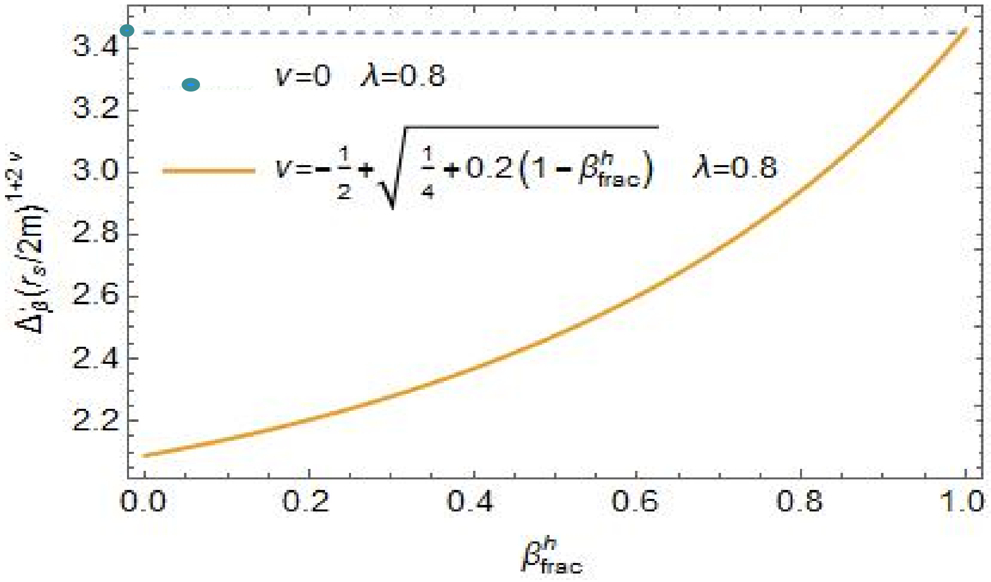
◆ **Instability criterion:**

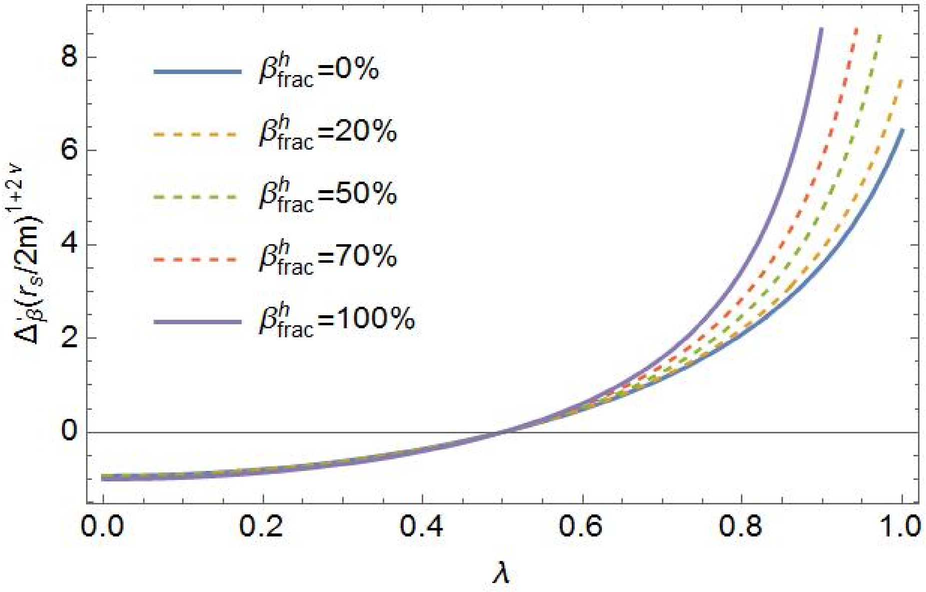
$$\Delta'_\beta=\left(\frac{2m}{r_s}\right)^{2\nu+1}\frac{1}{2\nu(1+2\nu)}\frac{\Gamma(1-2\nu)}{\Gamma(1+2\nu)}\left[\frac{\Gamma(1-\lambda+\nu)}{\Gamma(-\lambda-\nu)}+\frac{\Gamma(1+\lambda+\nu)}{\Gamma(\lambda-\nu)}\right]$$

$$\nu=-\frac{1}{2}+\sqrt{D_I},\ \ D_I=\frac{1}{4}+\frac{1}{8\pi}\frac{q^2}{s^2}2R_0\beta_t\left(\frac{1}{L_t}-\frac{\beta^h_{frac}}{L_h}\right),$$

$$\lambda=-\frac{rq}{2m\left(\frac{dq}{dr}\right)}\frac{1}{B_\theta}\frac{dj_\parallel}{dr},\ \ L_p^{-1}=-(1/p)(dp/dr)$$

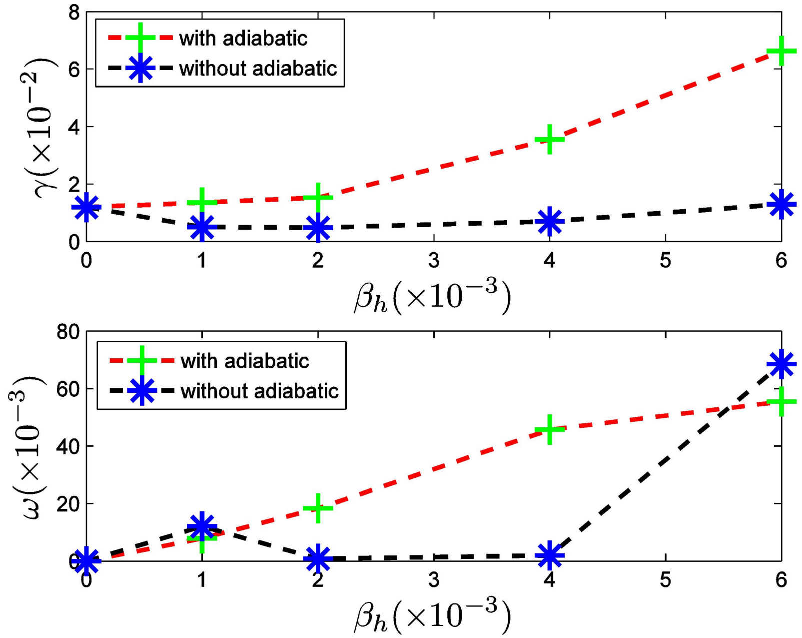
Influence of trapped energetic ions in the outer region_*Phys. Plasmas 26, 062505(2019)*





- **The value of instability criterion increases with the fraction of TEIs.**
- **TEIs plays an unstable role.**
- **TEIs can not suppress tearing modes.**

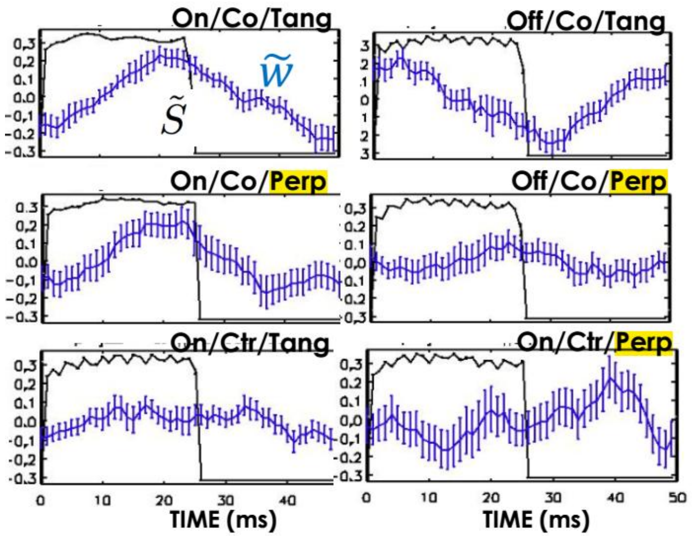
Simulation results for trapped ions-*Phys. Plasmas* 19,072506(2012)



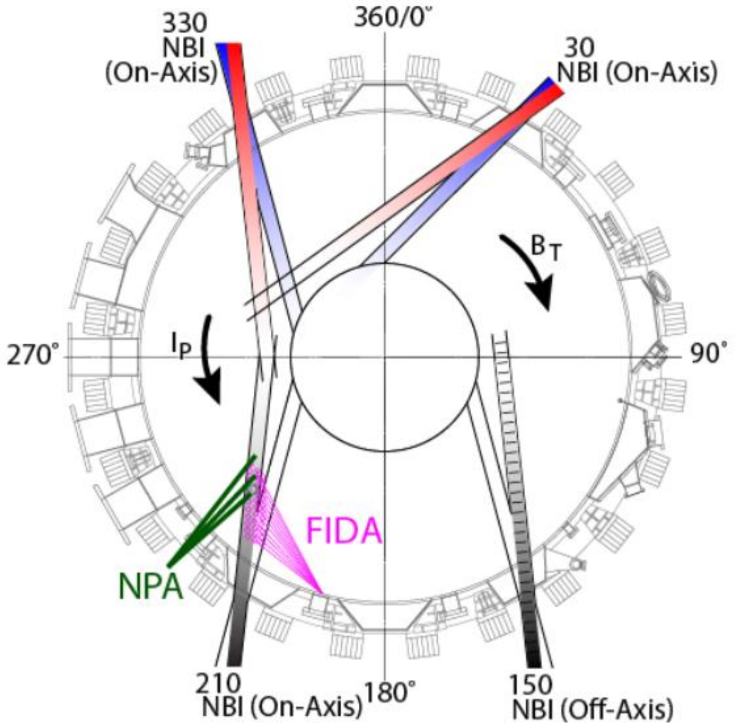
- **The kinetic effect of TEIs is stabilizing, while the adiabatic effect is strongly destabilizing. The net effect is destabilizing.**

- **For TEIs, if the magnitude of beta becomes large enough, resonance may happen, and trigger EPM.**

Experimental result_*DIII-D, W.W. Heidbrink, et. al., (NF2017)*



Source	Effect
On/Co/Tang	Destabilizing
Off/Co/Tang	Stabilizing
On/Co/Perp	Weakly destabilizing
Off/Co/Perp	Small
On/Ctr/Tang	Small
On/Ctr/Perp	Small



- TEIs from On/Co/Perp NBI play weakly destabilizing effect.
- The theoretical results agree with experimental results in DIII-D.

Influence of energetic ions in the island region

◆ Heuristic interpretation:

➢ **Due to the orbit average, response of energetic ions to the perturbation in the inner region are reduced dramatically, and can be neglected. Based on the quasi-neutrality, the cross field current of ions and electrons can not cancel completely, an uncompensated current is yielded $\boldsymbol{J}_u = -n_h e \boldsymbol{v}_E$**

Influence of energetic ions in the island region

- **The conservation law of current with fast ions:**

$$\nabla\cdot\left(n_e\mathbf{v}_E-\mathbf{\Gamma}_i-\mathbf{\Gamma}_h\right)=\frac{1}{e}\nabla_{//}J_{//}$$

where only electric drift is considered, for simplicity. The ion flow is

$$\mathbf{\Gamma}_i=n_i<\mathbf{v}_E>,\mathbf{\Gamma}_h=n_h<\mathbf{v}_E>$$

The quasi-neutrality is

$$n_i+n_h=n_e.$$

Considering the ion larmor radius much smaller than island width,

$$\mathbf{\Gamma}_i=n_i\mathbf{v}_E$$

If the fast ion larmor radius is much larger than island width, due to the orbit average effect,

$$\mathbf{\Gamma}_h\sim 0, \Longrightarrow \nabla\cdot\left[\left(n_e-n_i\right)\mathbf{v}_E\right]=\nabla\cdot\left(n_h\mathbf{v}_E\right)=\frac{1}{e}\nabla_{//}J_{//}$$

Influence of energetic ions in the island region

◆ **The conservation law of current:**

$\mathbf{J}_\mathrm{I}, \mathbf{J}_u$

$$\mathbf{B}\cdot\nabla\frac{J_{//}}{B} = -\nabla_\perp\cdot\mathbf{J}_\mathrm{I} - \nabla_\perp\cdot\mathbf{J}_u,$$

where J_I, J_u are the polarization current and the uncompensated current due to energetic ions.

Comparing:

$$\delta = \frac{|\nabla_\perp\cdot\mathbf{J}_u|}{|\nabla_\perp\cdot\mathbf{J}_\mathrm{I}|} \sim \frac{n_h}{n_i}\frac{w^2}{r_s^2}\frac{R_0}{d_i}\left(\frac{\omega}{\omega_A}\right)^{-1},$$

- **In the linear phase, $\delta \ll 1$**

$\delta << 1;$

- **In the nonlinear phase, although the density of energetic ions is much smaller**

Influence of energetic ions on NTMs

◆ **The evolution of NTM**

$$\frac{8\pi}{\eta c^2} I_1 \frac{dw}{dt} = \Delta' + \Delta_b' + \Delta_i' + \Delta_u', \quad \Delta_b' = 1.46\sqrt{\varepsilon} G_1 \frac{r}{sL_n} \frac{\beta_\theta}{w},$$

$$\Delta_i' = -G_2 \left(\frac{\rho_{\theta i}}{w}\right)^2 \left(\frac{r}{sL_n}\right)^2 \frac{\beta_\theta}{w} \frac{\omega'(\omega' - \omega_{*i})}{\omega_{*i}^2}, \quad \Delta_u' = -G_3 \left(\frac{r}{sL_n}\right)^2 \frac{\beta_\theta}{w} \frac{\omega'}{\omega_{*i}} \frac{n_h}{n_i} \frac{L_n}{L_h},$$

➢**The effects of energetic ions are reflected both in Δ' and Δ_u'**

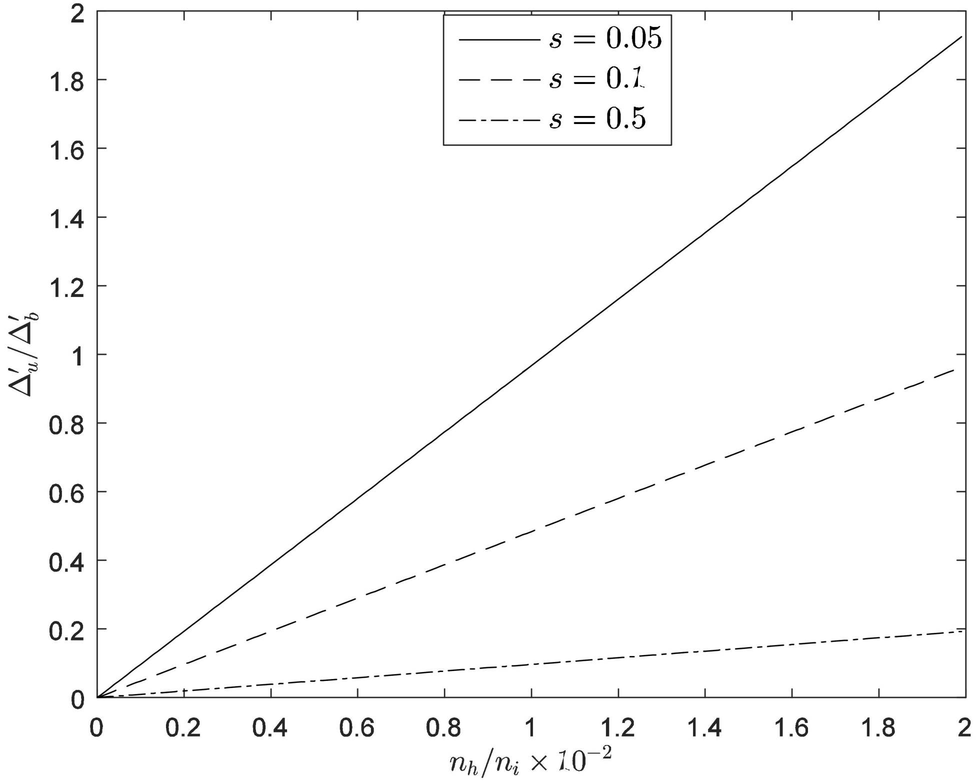
➢**The effect of uncompensated current Δ_u' depends on the propagation frequency of island, and the density gradient of energetic ions and ions at the resonance surface.**

➢**If $(\frac{dn_h}{dr_s})/(\frac{dn_i}{dr_s}) > 0, \omega' > 0, \Delta_u' < 0$, namely, the uncompensated current plays a stable role. It is different from the effect of polarization current, which plays a stable role if $\omega' < 0$ or $\omega' > \omega_{*i}'$.**

Influence of energetic ions on NTMs

For the typical tokamak, like JT-60U, $R_0 = 3.2m, a = 0.8m, \quad n_h / n_i \le 0.02,$

Given $r_s = 0.4m, L_h = 0.16m,$

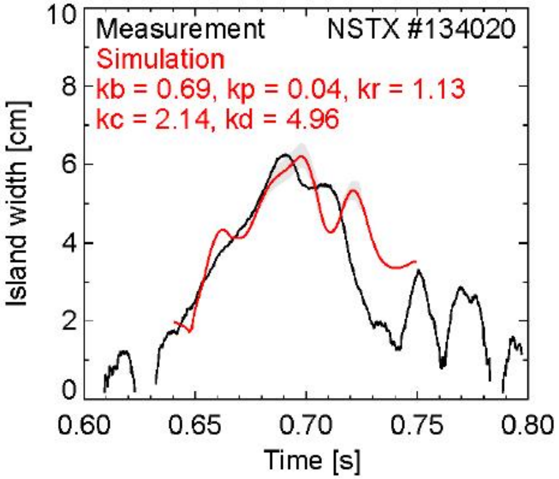


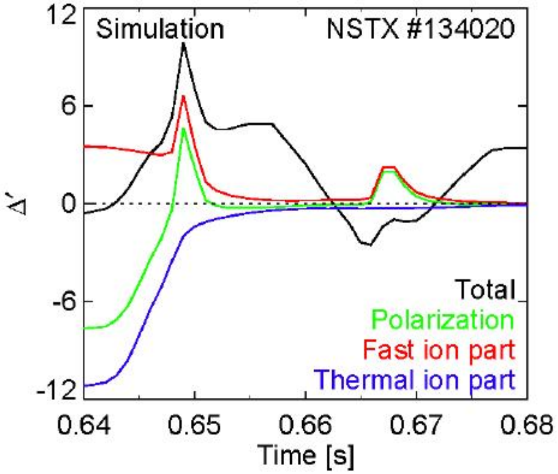
- **The ratio** $\left|\frac{\Delta'_u}{\Delta'_b}\right|$ **increases with** n_h/n_i **increasing and** s **decreasing.**
- **For the weak magnetic shear and large fraction of energetic ions(like stability operation scenario of ITER),** $\left|\frac{\Delta'_u}{\Delta'_b}\right| \sim 1$**, namely, the stabilizing effect of uncompensated current partially cancels or overcomes the destabilizing effect of bootstrap current.**

Experimental result in NSTX_*James, APS talk 2022*

Fast ion term is essential for GRE modeling of island width

- Island width simulated by GRE with fast ion term agrees with the measurement
  - Free parameters are determined by numerical optimization
- Fast ion term contribution is significant at island onset phase
  - Initial value problem solution is impossible without inclusion of fast ion part





$$\Delta'_{pol} = -\left[\varepsilon^{3/2}\frac{\rho_{\theta i}^2}{w^2} - \frac{L_{n_i}}{L_{n_h}}\frac{n_h}{n_i}\right]\frac{\beta_\theta}{w}\left(\frac{L_q}{L_p}\right)^2$$

Thermal ion part (Polarization current) — Fast ion part (Uncompensated current)

NSTX-U — J. Yang, "EP effect on NTM stability" (APS-DPP 2021, UO8.000004) — 8/14

- The influence of energetic ions on NTMs
- **The resonance between energetic ions and NTMs**

◆**Early observation of frequency chirping of NTMs/tearing modes in TFTR, ASDEX-U.**

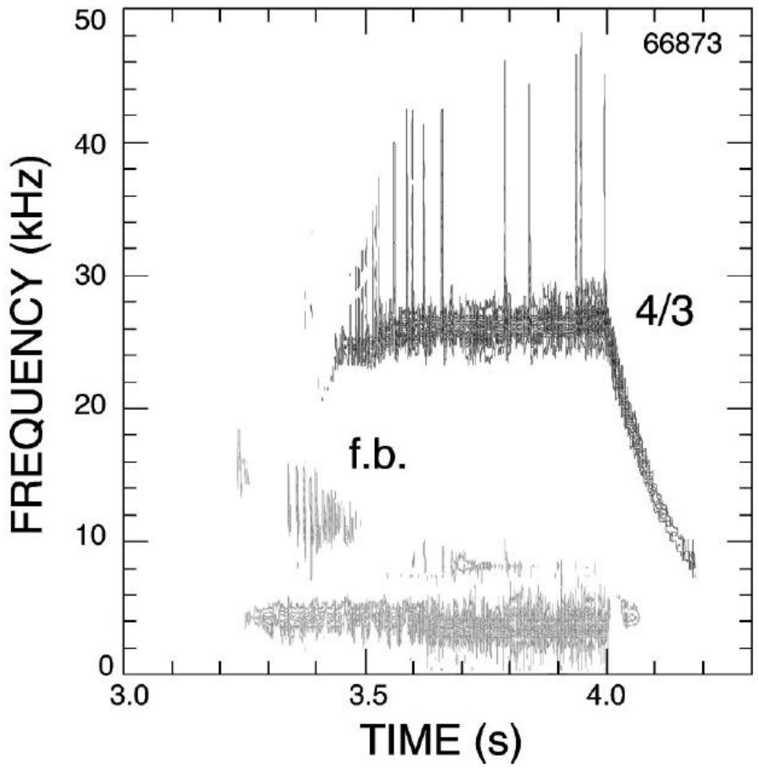


FIG. 4. Spectrogram of Mirnov coil data for shot 66873 showing evolution of 4/3 NTM and presence of bursting, fishbonelike modes.

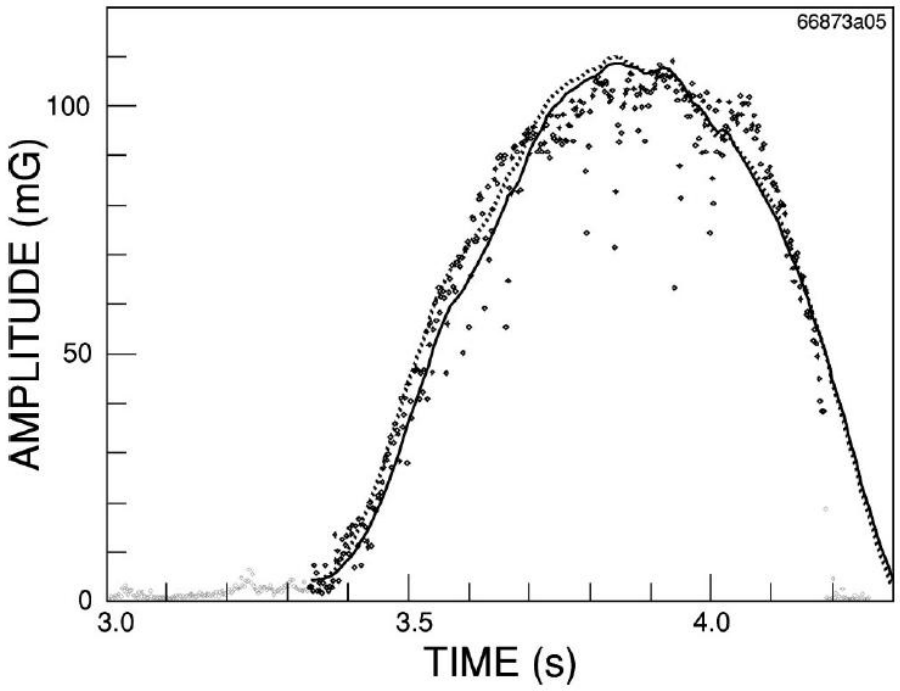
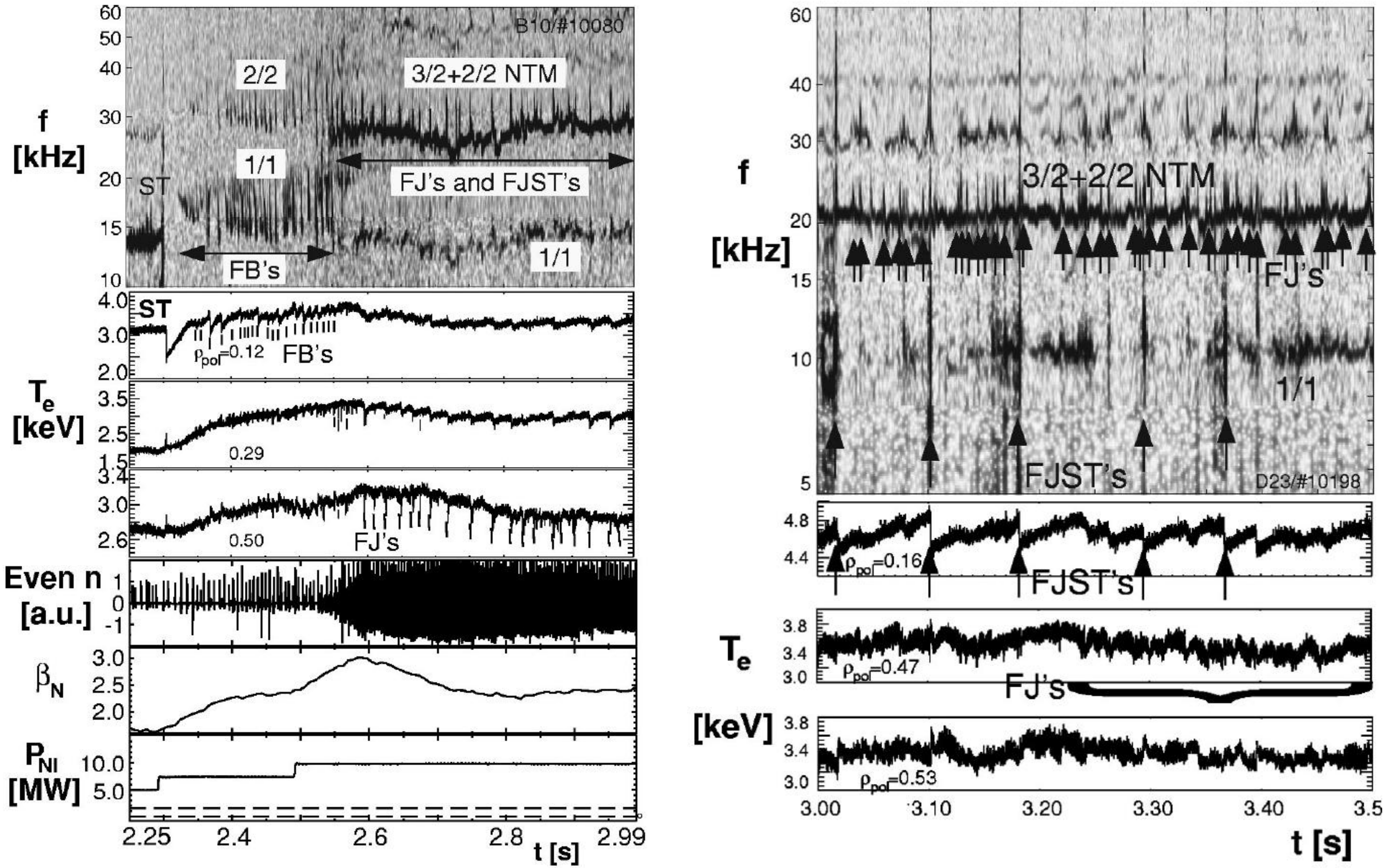


FIG. 6. Comparison of measured (dots) magnetic fluctuation amplitude for the shot shown in Figs. 3–4. The solid and dashed lines are simulations as described in the text. (Gray dots before 3.33 s indicate "noise" level.)

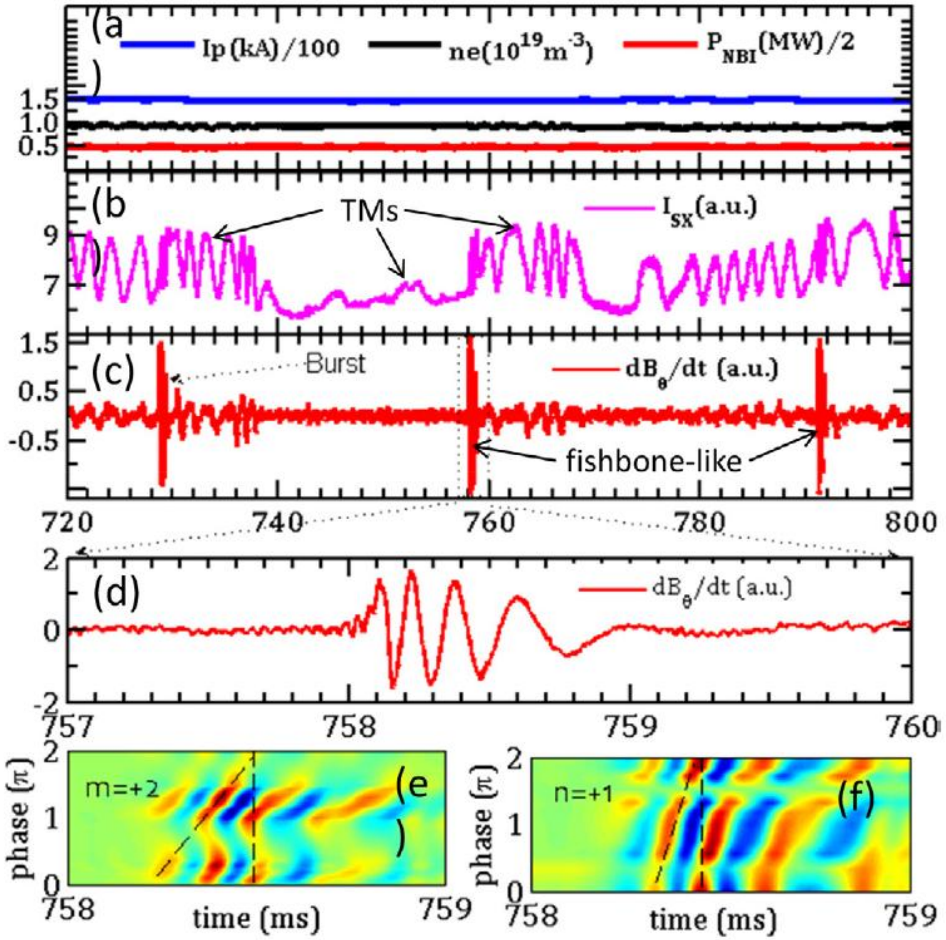
TFTR, (Fredrickson, Phys. Plasmas *2002*)

Experiment observations



ASDEX-U, (Sesnic, et. al., Phys. Plasmas *2000*)

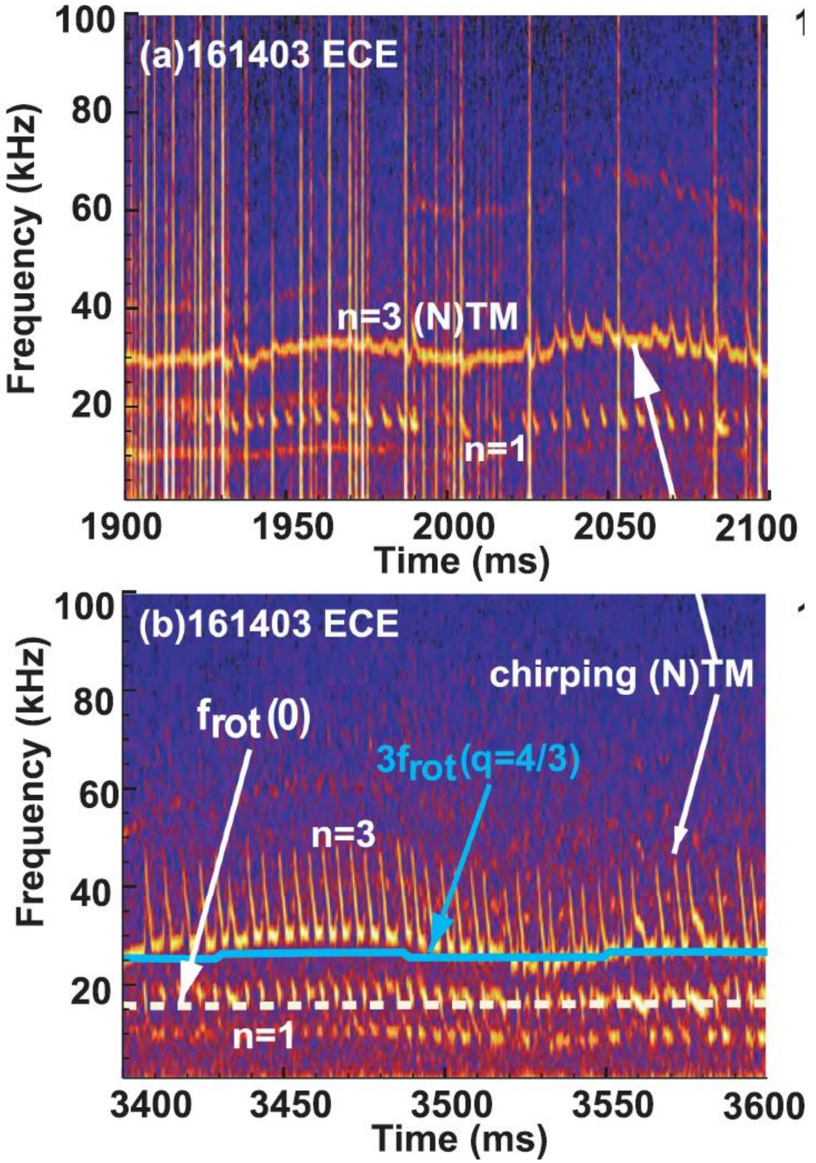
◆Similar observation in recent experiments: HL-2A, DIII-D



HL-2A, (Chen, et. al., Nucl. Fusion *2019*)

- Frequency fast downward within 1ms;
- Propagates in ion diamagnetic frequency direction;
- Similar to fishbone behavior;
- Occurs when the rotation direction of tearing mode changes from electron to ion diamagnetic drift.

Experiment observations



DIII-D (Liu, et. al, Nucl. *Fusion 2020*)

- Frequency jumps up from steady NTM, and chirps down back to steady NTM;
- The process occurs within 1ms;
- Coexist with 1/1 fishbone;
- No frequency correlation between NTM and fishbone;
- NTM propagates in ion diamagnetic drift direction;
- a measurable reduction in the neutron rate (up to 1%).

Resonance between energetic ions and NTMs

- **The common characteristics:**
  - **Frequency jumps up from steady NTM/tearing mode frequency, and chirps down back to steady NTM/tearing mode frequency;**
  - **The process occurs within a few milliseconds;**
  - **NTM propagates in ion diamagnetic drift direction;**
  - **a measurable reduction in the neutron rate (up to 1%).**
- **Implies a strong interaction between energetic ions and NTM/tearing modes:**
  - **A fishbone-like mode?**
  - **So-called chirping-NTM/tearing modes?**

Theoretical works

- **Some theories are tried to understand the physics:**

  - **Chirping NTM (Marchenko, *et. al.*, Phys. Plasmas(2001); Cai, Nucl. Fusion(2021))**
  - **A fishbone-like mode (Zhang, *et. al.* PPCF(2020), Zhu *et. al.*, Nucl. Fusion(2020))**

- **They both result from the resonant interaction with energetic ions.**
- **Whether chirping NTM or fishbone-like mode is still unclear.**

Fishbone-like mode

Fishbone-like mode

- The physics is similar to fishbone mode except mode number.
- Based on energy principle:

$$\delta\widehat{W}_{min} + \delta\widehat{W}_{hk} + \delta\widehat{W}_{R}=0,\ \delta\widehat{W}_{min} = \delta\widehat{W}_{c}+\delta\widehat{W}_{hf}$$

- $\delta\widehat{W}_{c}$ indicates the minimized ideal MHD potential energy, $\delta\widehat{W}_{hf}$ is the adiabatic contribution of energetic ions, $\delta\widehat{W}_{hk}$ is the kinetic contribution of energetic ions, $\delta\widehat{W}_{R}$ arises from the resistive layer.
- Energetic ions can affect the instabilities.
  - Energetic ions affect the unstable mode
  - Energetic ions trigger a new mode or drive stable mode to be unstable

Fishbone-like mode by trapped energetic ions_*Zhang, et. al, PPCF(2020)*

- Equilibrium profiles and parameters based on HL-2A experiment (Chen, Nucl. Fusion (2019))

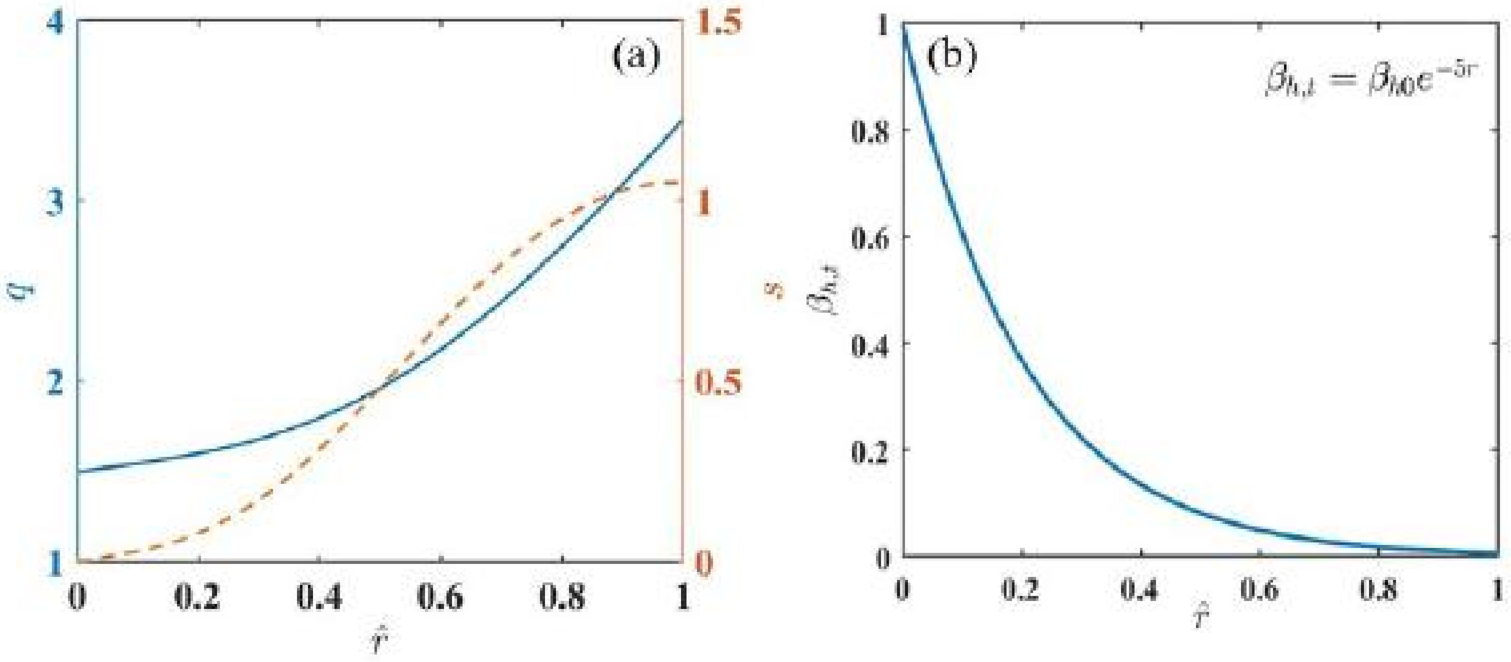


Figure 1. The (a) q-profile (solid blue curve-left vertical axis) and s-profile (orange dotted line-right vertical axis)and (b) $\beta_{h,t}$ profile based on the experimental data in Ref. [24].

Fishbone-like mode by trapped energetic ions_*Zhang, et. al, PPCF(2020)*

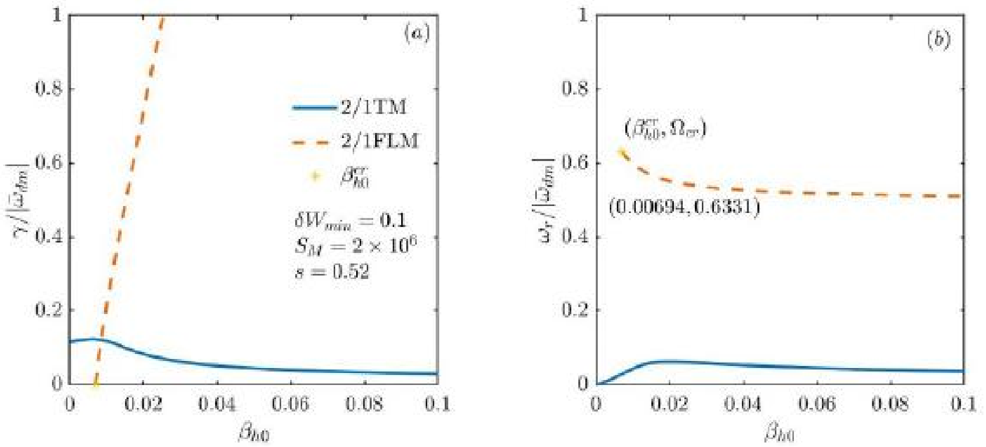


Figure 2. The (a) growth rate and (b) real frequency (normalized by the precession frequency of TEIs) of 2/1 tearing mode (solid blue line) and 2/1 FLMs (dotted orange line) versus β_{h0} with $S_M = 2 \times 10^6$, $s = 0.52$ and $\delta W_{min} = 0.1$. Tearing mode is abbreviated to TM in the above figure.

Zhang, et. al, Plasma Phys. Control. Fusion

- **Energetic ion β_{h0} is small, only tearing mode (TM) is unstable.**
- **When β_{h0} exceeds a critical value, a fishbone-like mode (m/n=2/1) is excited. TM and fishbone-like mode coexist.**
- **The growth rate of tearing mode decreases with β_{h0} increases.**
- **The growth of fishbone-like mode increases with β_{h0}.**
- **The fishbone-like mode is triggered by TEIs.**

Fishbone-like mode by TEIs_*Zhang, et. al, PPCF(2020)*

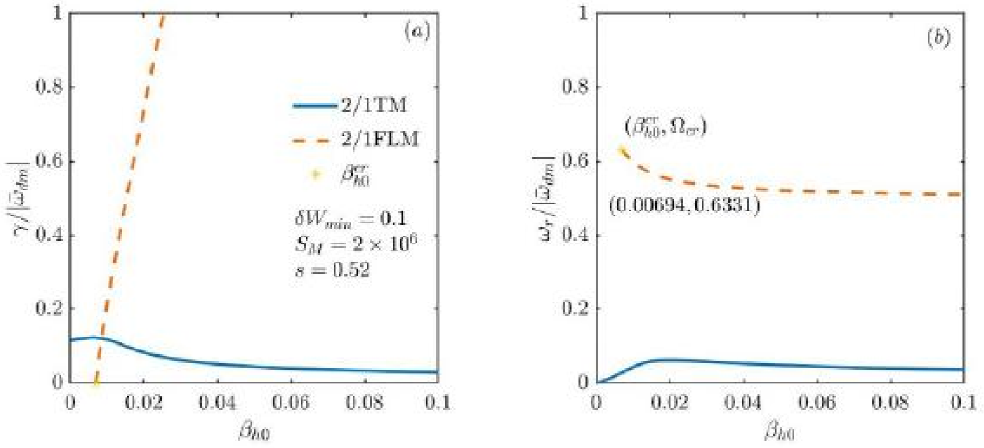
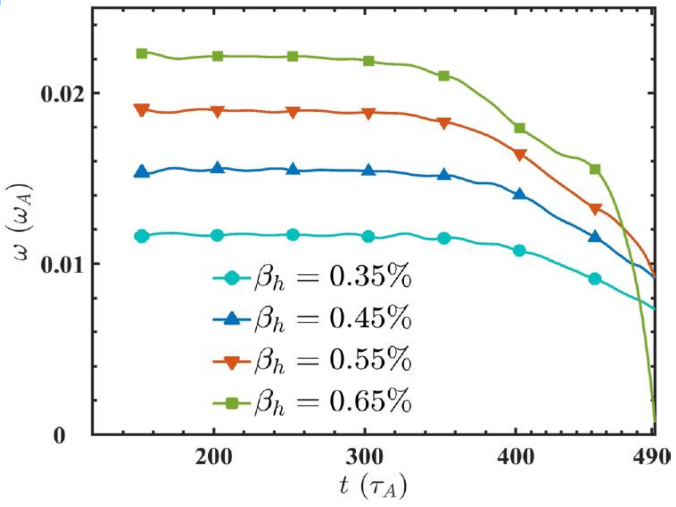


Figure 2. The (a) growth rate and (b) real frequency (normalized by the precession frequency of TEIs) of 2/1 tearing mode (solid blue line) and 2/1 FLMs (dotted orange line) versus β_{h0} with $S_M = 2 \times 10^6$, $s = 0.52$ and $\delta W_{min} = 0.1$. Tearing mode is abbreviated to TM in the above figure.

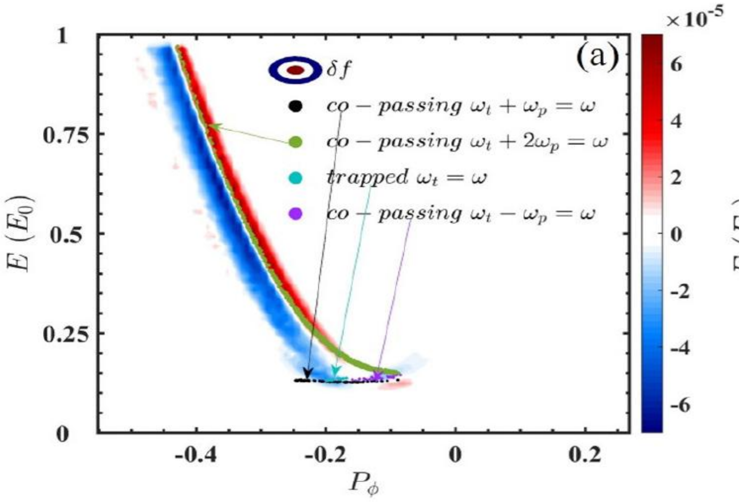
Zhang, et. al, Plasma Phys. Control. Fusion, (2020)

- **The calculated frequency of fishbone-like mode (8.9-11kHz) is consistent with that in HL-2A experiment (2-10kHz).**
- **For passing energetic ions, The resonance occurs only in a very narrow region radially near the rational surface. In the limit of small orbit width, this resonance effect is small.**
- **The resonance effect of passing energetic ions increases with orbit width.**

Fishbone-like mode by CEIs_*Zhu, et. al, Nucl. Fusion (2020)*



Evolution of TM frequency against β_h *and fixed* $\beta_{MHD} = 0.05\%$



The perturbed distribution around $\mu = 0.555$ *for* $\beta_h = 0.65\%$

- **When energetic ion β_h exceeds a critical value, a fishbone-like mode is triggered.**
- **The time of chirping down is consistent with that in HL-2A experiment.**
- **Caused by the resonance with co-passing energetic ions.**
- **Based on the above works, whether passing or trapped energetic ions plays a dominant role is still unclear.**

So-called chirping NTM

Frequency chirping of NTMs by energetic ions

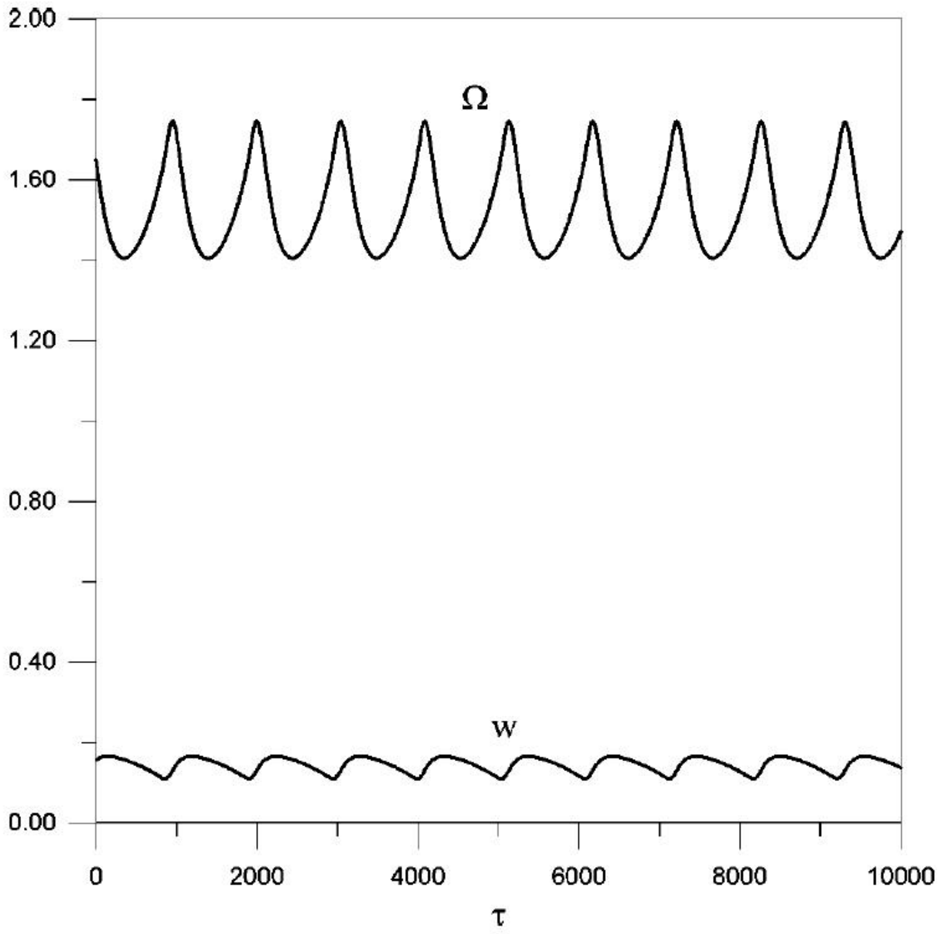


FIG. 1. The time evolution of the NTM frequency (Ω) and the magnetic island width (w) for the set of parameters: $T_\alpha=0.038$, $\Omega_0=3$, $\nu\simeq 2.76\times 10^{-4}$, $r_s\Delta'\simeq -m=-3$, $(\beta_p/2)(a_{bs}-a_{GGJ})\simeq 0.75$ [which is consistent with the interdevice database (Ref. 21)], $P=0.009$, $\Omega_1=2(\eta_i\simeq 4)$.

Marchenko, *et. al.*, Phys. Plasmas(2001)

- **Physics Interpretation: the resonance between trapped energetic ions and rotating NTMs provides a torque to accelerate the rotation of NTMs;**
- **Chirping time calculated is much longer (about 100 times) than it in the experiments.**
- **Particle model used in it.**
- **We use a drift kinetic theory self-consistently.**

Chirping NTM

- **Separating into**

$$\Delta'_c = \frac{16q_s}{w^2\psi' q_s'}\frac{4\pi R}{c}\frac{w}{2\sqrt{2}}\int_{-1}^{\infty} d\Omega < \delta J_{//,R}\cos\xi - \delta J_{//,I}\sin\xi >$$

(determines the island evolution)

$$\Delta'_s = \frac{16q_s}{w^2\psi' q_s'}\frac{4\pi R}{c}\frac{w}{2\sqrt{2}}\int_{-1}^{\infty} d\Omega < \delta J_{//,R}\sin\xi + \delta J_{//,I}\cos\xi >$$

(determines the evolution of island rotation frequency)

$\Delta'_c = \mathrm{Re}(\Delta')$ **provides the instability criterion of tearing mode**

$\Delta'_s = \mathrm{Im}(\Delta')$ **describes absorption of momentum**

$\delta J_{//,I} = \mathrm{Im}(\delta J_{//})$ **comes from the resonance between NTM and energetic ions**

- **Effective resonance can modify the evolution of NTM frequency.**

Chirping NTM

◆ **The evolution of NTM frequency**

$$G_w \frac{d}{d\hat{t}}\left[(\hat{\omega}-\hat{\omega}_0)w\right] = -\frac{6G_V}{\omega_A}\frac{\mu_a}{w}(\hat{\omega}-\hat{\omega}_0) + \Delta_{sh}$$

$$\Delta_{sh} = \frac{\pi}{2} G_h \frac{w^2}{L_{nh}R_0}\frac{\omega_A}{\omega_{dm}}\frac{n^2\beta_{\perp h}^t K_2}{K_b I_{Ep}}\frac{\hat{\omega}^{7/2}}{\hat{\omega}^{3/2}+E_c^{3/2}}\left[-\frac{\omega_{dm}}{\omega_{*h}}H_s + \frac{1}{\hat{\omega}}\left(1+K_s\frac{L_{nh}}{L_{E_c}}\right)\right]$$

- **The effects of external fields are not considered.**
- **Without TEIs, NTM frequency would reach a constant frequency, and keep unchanged.**
- **The resonance affect the dynamics of NTM frequency.**
- **Resonance condition:** $\omega\omega_d > 0 \Longrightarrow \hat{\omega} > 0.$
  - **If NTM propagates in ion diamagnetic drift direction,**
    $\omega_d < 0, \Longrightarrow \Delta_{sh} > 0,$ **increases NTM frequency**
  - **If NTM propagates in electron diamagnetic drift direction,**
    $\omega_d > 0, \Longrightarrow \Delta_{sh} < 0,$ **decreases NTM frequency**

Chirping NTM

◆ **Based on DIII-D experiment (Liu, NF2020), typical parameters:**

$$B_0 \approx 2T, R_0 = 1.7m, a = 0.61m, n_0 = 3.5\times10^{19} m^{-3},$$

$$T_i \sim T_e \sim 3.5keV, L_{E_c} \sim L_{n_i} = -a, L_{n_h} = 0.5L_{n_i}$$

◆ **Mode number and frequency of steady NTM:**

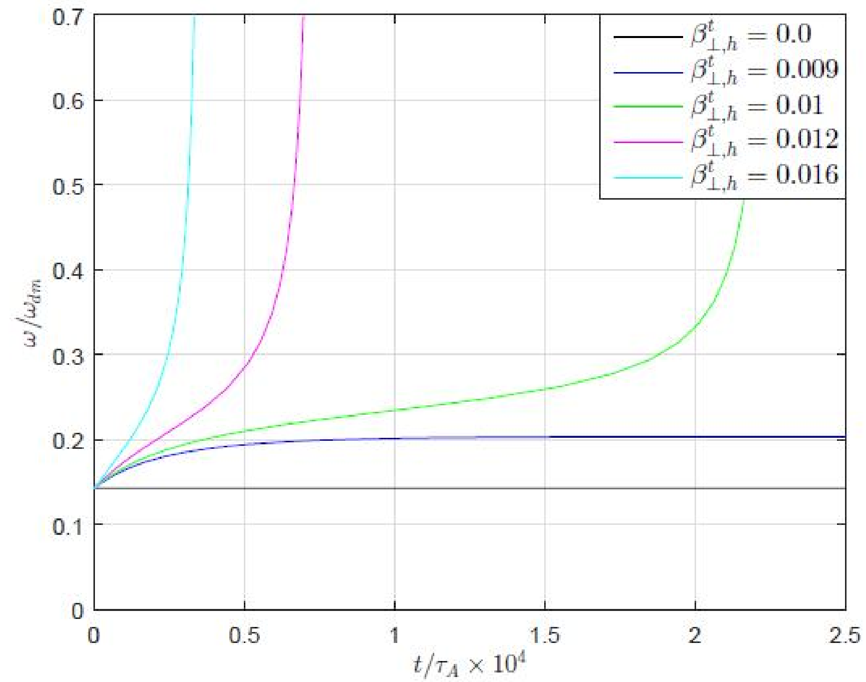
$$m/n = 3/2, \omega_0/(2\pi) = 3kHz$$

➢ **In DIII-D experiment, frequency chirps from 3kHz to 15kHz.**

◆ **Given:**

$$E_m = 65keV, \lambda B_0 = 1, anomalous\ viscosity\ \mu_a = 1m^2/s$$

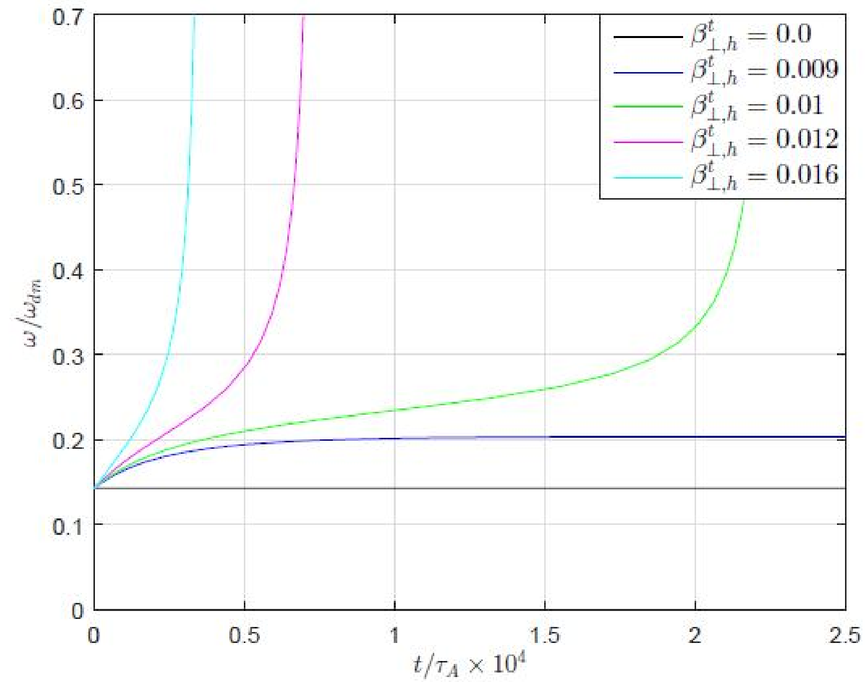
Chirping NTM_*Cai, Nucl. Fusion (2021)*



The frequency ω of NTMs against time with magnetic shear s = 0.5, where NTMs propagates in ion diamagnetic drift direction.

- **Resonant effect of TEIs on NTM frequency is significant.**
- **When energetic ion $\beta^t_{\perp,h}$ exceeds a critical value, resonant effect dominates over the viscosity damping effect, NTM frequency chirps up quickly.**
- **The chirping speed of frequency increases with the frequency owing to resonance fraction.**

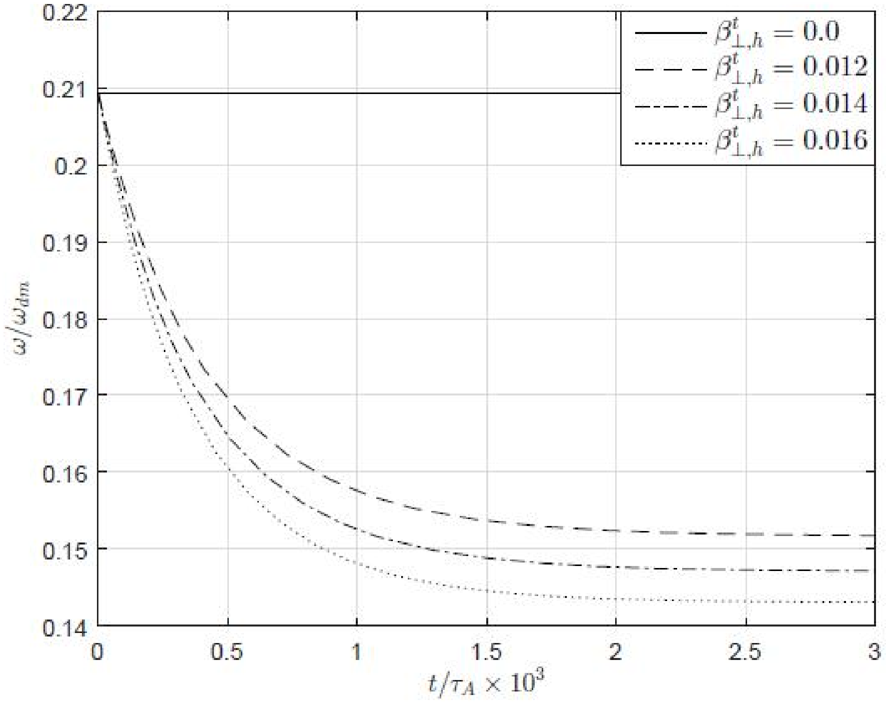
Chirping NTM_*Cai, Nucl. Fusion (2021)*



The frequency ω of NTMs against time with magnetic shear s = 0.5, where NTMs propagates in ion diamagnetic drift direction.

- **Chirping time from 3kHz to 15kHz is about 1.1ms, 2.3ms and 7.2ms for $\beta^t_{\perp,h}$=1.6%,1.2%,1.0%.**
- **Consistent with DIII-D experiment results:**
  - **Frequency chirping was observed when the island propagates in the ion diamagnetic drift direction.**
  - **The chirping time is about 1ms.**

Chirping NTM_*Cai, Nucl. Fusion (2021)*



The frequency ω of NTMs against time with magnetic shear s = -0.5, where NTMs propagates in electron diamagnetic drift direction.

- **In this case, resonance effect reduces NTM frequency.**
- **NTM frequency decreases with t, and keeps unchanged when resonance effect and restoring effect of viscosity balances.**
- **Resonance effect decreases with frequency owing to resonance fraction.**
- **Frequency chirping does not occurs when the island propagates in the electron diamagnetic drift direction.**

Summary and Discussion

- **Resonance between NTMs and TEIs provides an additional torque to alter the rotation frequency of NTMs.**
- **The rising or falling of rotation frequency depends on the rotation direction of NTMs.**
  - **If the NTMs rotation direction is the same as ion diamagnetic drift, NTM frequency chirps when energetic ion beta exceeds a critical value.**
  - **If the NTMs rotation direction is the same as electron diamagnetic drift, NTM frequency chirping does not occurs.**
- **The calculated chirping time is about a few milliseconds.**
- **The chirping time and NTM rotation direction during chirping are consistent with DIII-D results.**
- **Frequency rapid chirping of NTMs by energetic ions may impact the dynamics of island locking.**

Summary and Discussion

- **The effect of energetic ions at the island width evolution is not considered.**
- **The chirping down process is not obtained, since redistribution and loss of energetic ions is not considered.**
- **The correlation of chirping (N)TMs with n=1 fishbones is also not considered. Fishbones and (N)TMs may strongly couple together through the loss or redistribution of energetic ions. (1/1 fishbone would eject energetic ions out of 1/1 surface).**
- **Experiments uncertainty:**
  - **Is a new mode (EPM like fishbone-like) or NTM/TM? If it is a new mode, the new mode and NTM/TM should coexist.**
  - **Whether trapped energetic ions or passing energetic ions plays an important role?**
- **Further studies are needed.**

Conclusion, Outlook and Remark

- **All simulations are based on reduced fluid model. Onset threshold physics can not be simulated. Kinetic simulation is needed to be developed.**
- **Due to the limit of diagnostic, the comparison between experimental and theoretical results is difficult.**
- **The study on the interaction between NTM and energetic particles has only begun*(Hegna, 1989;Takahashi, 2009;Cai, 2011,2012,2016…)*.**
- **The interaction between NTM and turbulence is still on process. Simulations are based on fluid model. Most simulations are devoted to the interaction between tearing mode and turbulence. Most of experiments are focused on the modulation of turbulence by island.**
- **Interaction between Alfven eigenmodes and island, like BAE and tearing mode*(Buratti, 2005; Chen, 2011;Liu, 2015;Cai, 2021;Xu 2021)***
- **……**
- **Detail can be found in Ref. (H.S. Cai, D. Li, National Science Review 9: nwac019, 2022)**

A mechanism of neoclassical tearing modes onset by drift wave turbulence

The onset threshold of NTMs

$$\beta_\theta^{onset} = -r_s\Delta'\left(G_1\sqrt{\varepsilon_s}\frac{r_s}{sL_n}\right)^{-1}\frac{w_{pol}}{r_s}\left[\frac{\hat{w}_{seed}}{\hat{w}_{seed}^2 + w_\chi^2/w_{pol}^2} - \frac{1}{G_1\hat{w}_{seed}^3}\left(1-\frac{\sigma w_{tur}^2}{w_{pol}^2}\right)\right]^{-1}, \quad \hat{w}_{seed}=w/w_{pol}$$

- The effect of turbulence-driven current on onset threshold of NTMs depends on the ratio w_{tur}^2/w_{pol}^2.
- The effect depends on the direction of turbulence intensity gradient.
- For typical values of tokamak, $w_{tur}^2/w_{pol}^2 \sim O(1)$, namely the effect of turbulence-driven current is significant.

If $\sigma > 0$, it enhances the onset threshold of NTMs.

If $\sigma < 0$, it reduces or overcomes stabilizing effect of neoclassical polarization current, and can trigger NTMs.

The onset of NTMs_*Nucl. Fusion 59, 026009(2019)*

For the typical tokamak, like DIII-D, $R_0 = 1.7m, a = 0.61m, B = 1.6T,$

Given $T_i = T_e = 2keV, n_i = 2\times10^{19}m^{-3}, r_s\Delta' = -3, q_s = 2, s = 1, \varepsilon_s^{1/2} = 0.5, L_n = |L_I| = 0.5a,$

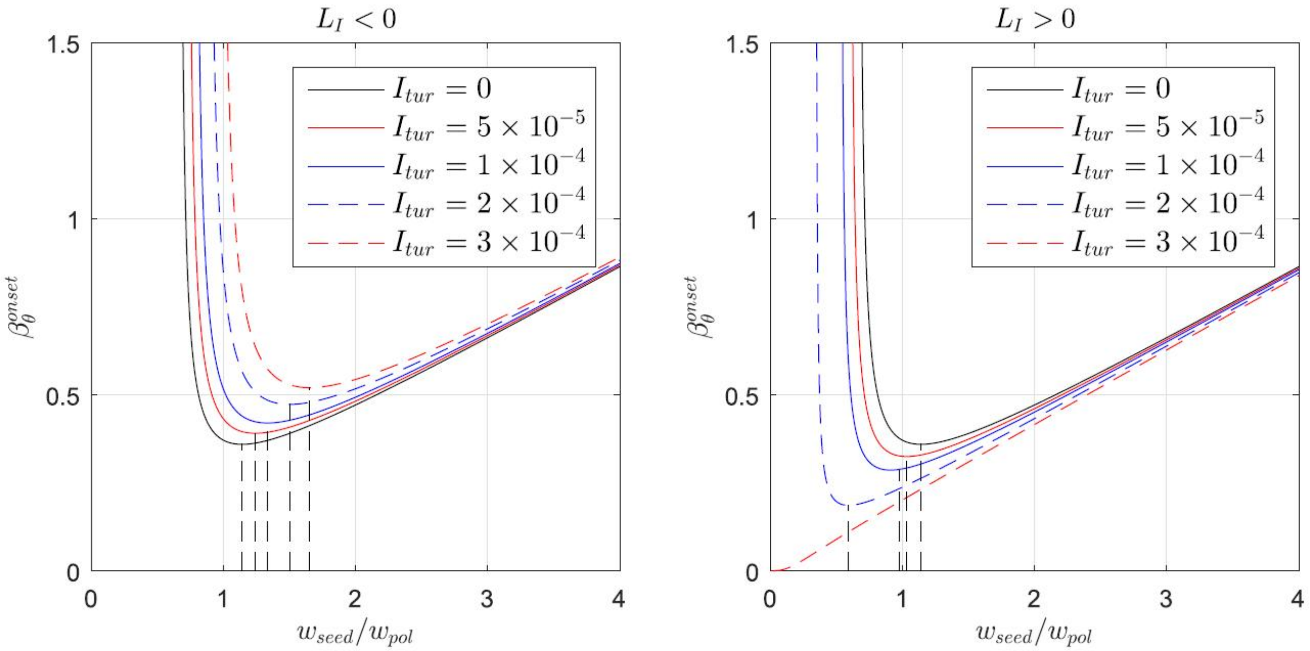


Figure 1. The dependence of $\beta_\theta^{\mathrm{onset}}$ on $w_{\mathrm{seed}}/w_{\mathrm{pol}}$ for $L_I < 0$ and $L_I > 0$, respectively.

- ➢For $L_I < 0, \beta_\theta^{onest}, w_c$ increases with I_{tur}, namely the turbulence-driven current plays a stabilizing role, and enhances the onset threshold.

The onset of NTMs_*Cai, Nucl. Fusion 59, 026009(2019)*

For the typical tokamak, like DIII-D, $R_0 = 1.7m, a = 0.61m, B = 1.6T,$

Given $T_i = T_e = 2keV, n_i = 2\times10^{19} m^{-3}, r_s\Delta' = -3, q_s = 2, s = 1, \varepsilon_s^{1/2} = 0.5, L_n = |L_I| = 0.5a,$

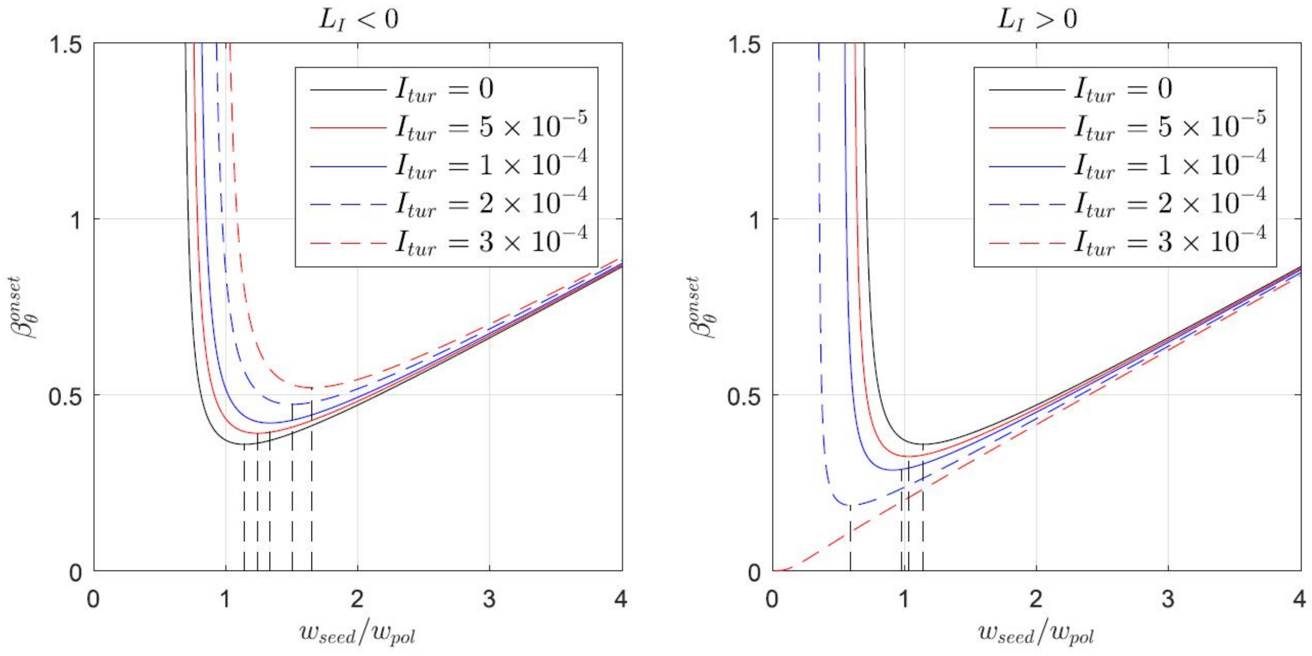
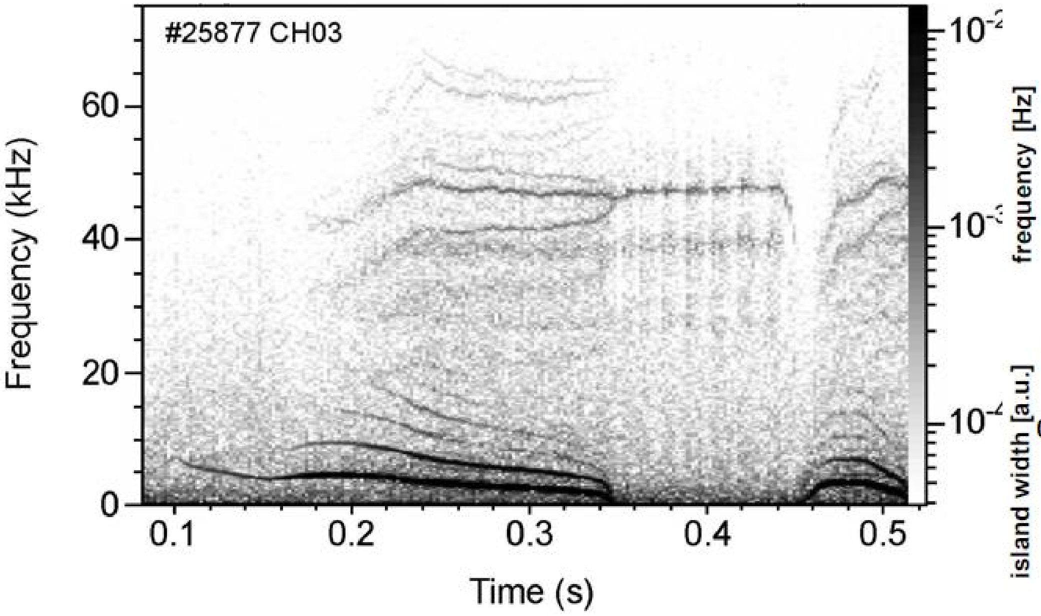


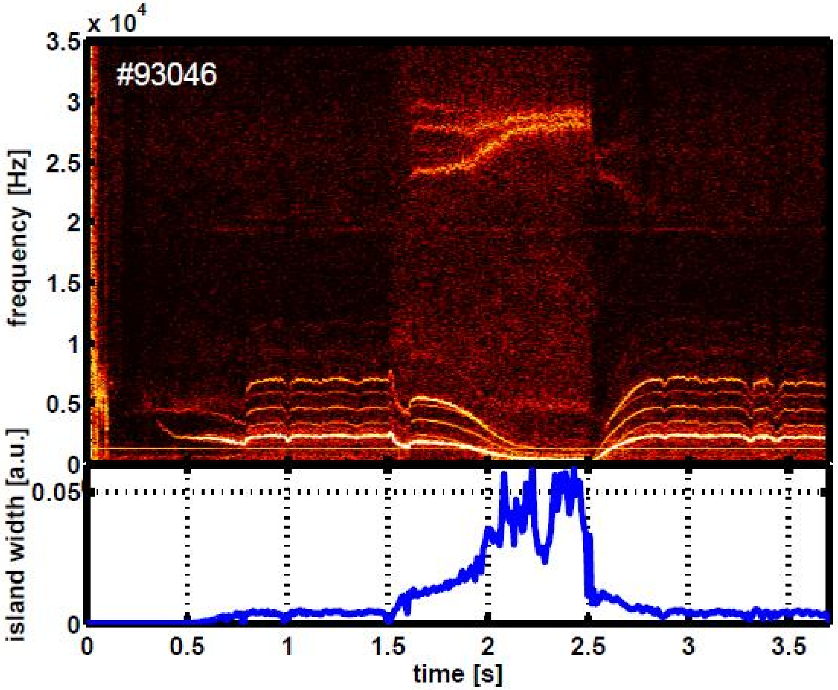
Figure 1. The dependence of β_θ^{onset} on w_{seed}/w_{pol} for $L_I < 0$ and $L_I > 0$, respectively.

- For $L_I > 0$, β_θ^{onest}, w_c decreases with I_{tur}, namely the effect is destabilizing, and cancels the stabilizing effect of neoclassical polarization current. It leads to a reduction of onset threshold and can trigger NTMs.

◆**BAEs were observed during tearing modes in FTU, TEXTOR, HL-2A, J-TEXT, EAST, TJ-II.**



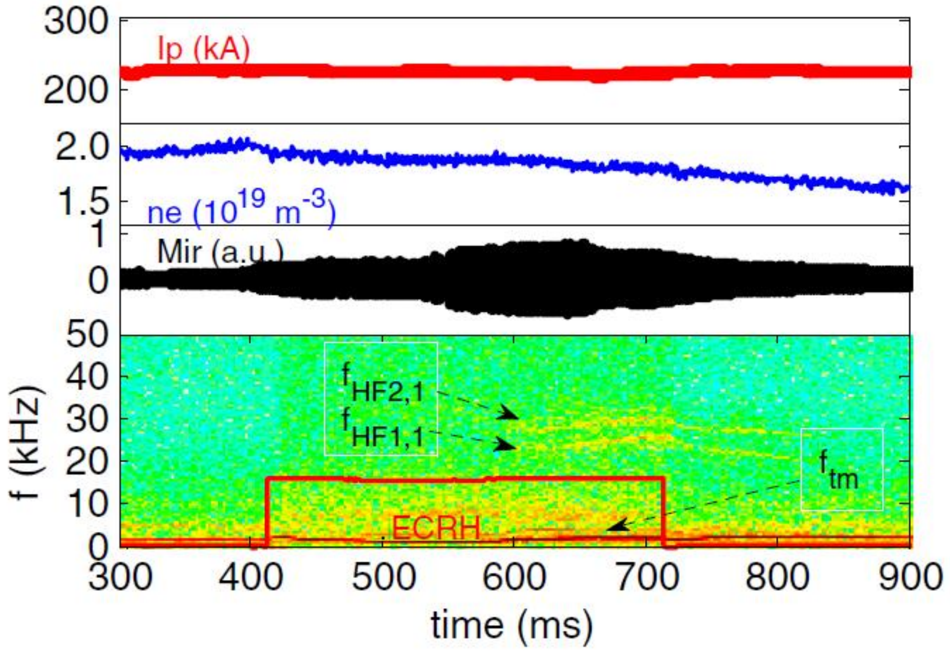
FTU, (*NF 2005*)



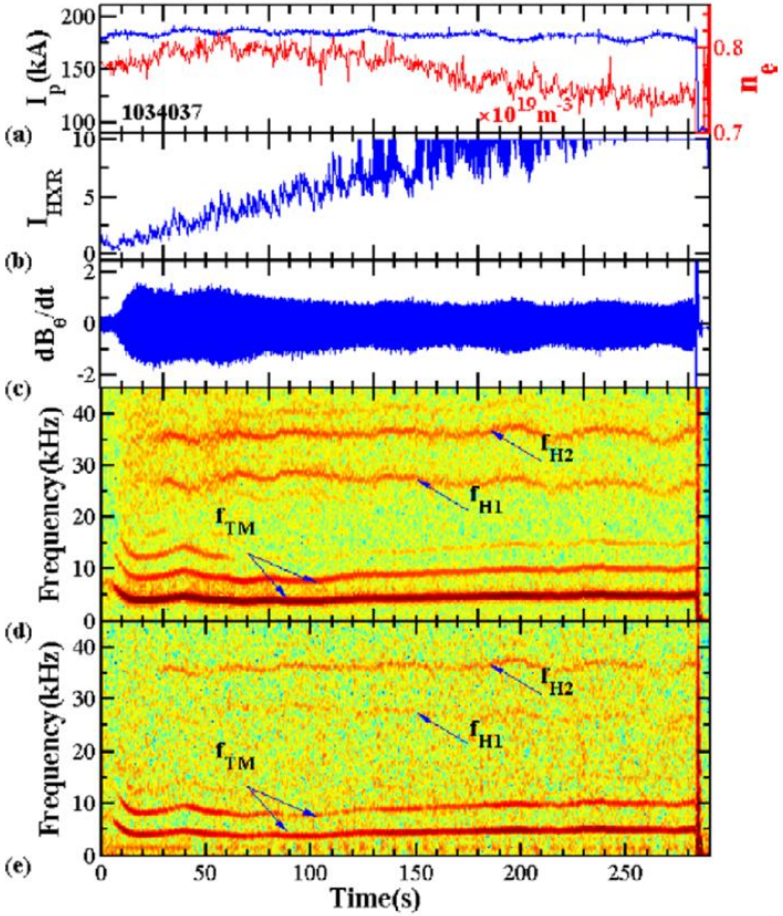
TEXTOR, (*EPS 2005*)

Excitation of BAEs by coupling between GAM and magnetic island

◆**BAEs were observed during tearing modes in FTU, TEXTOR, HL-2A, J-TEXT, EAST, TJ-II.**

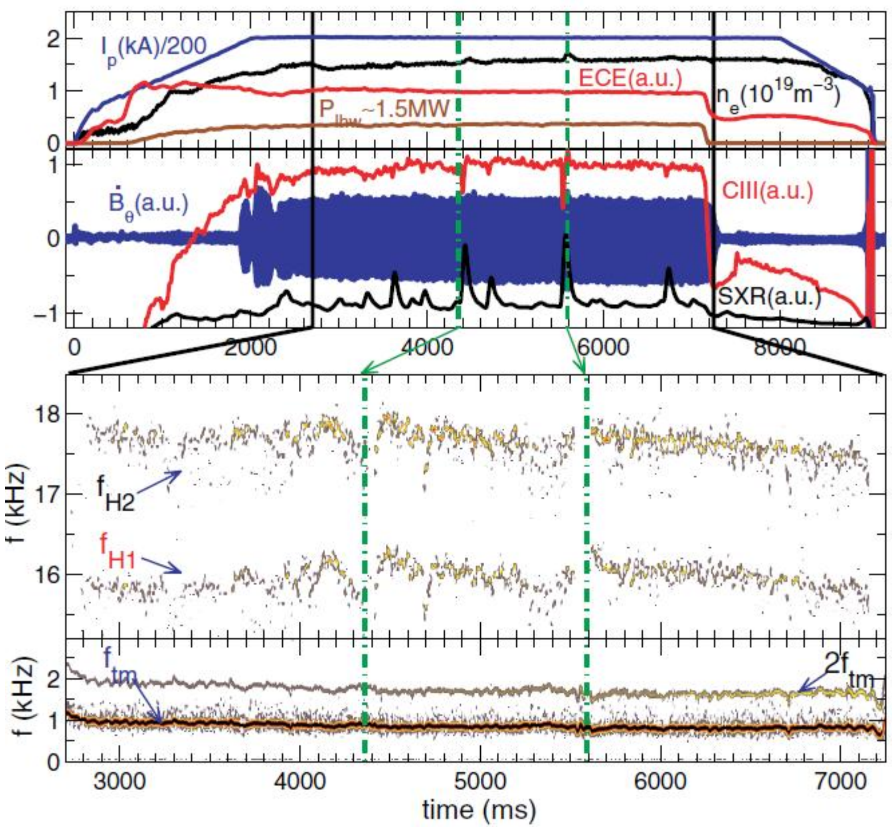


HL-2A (*JPSJ 2010*)

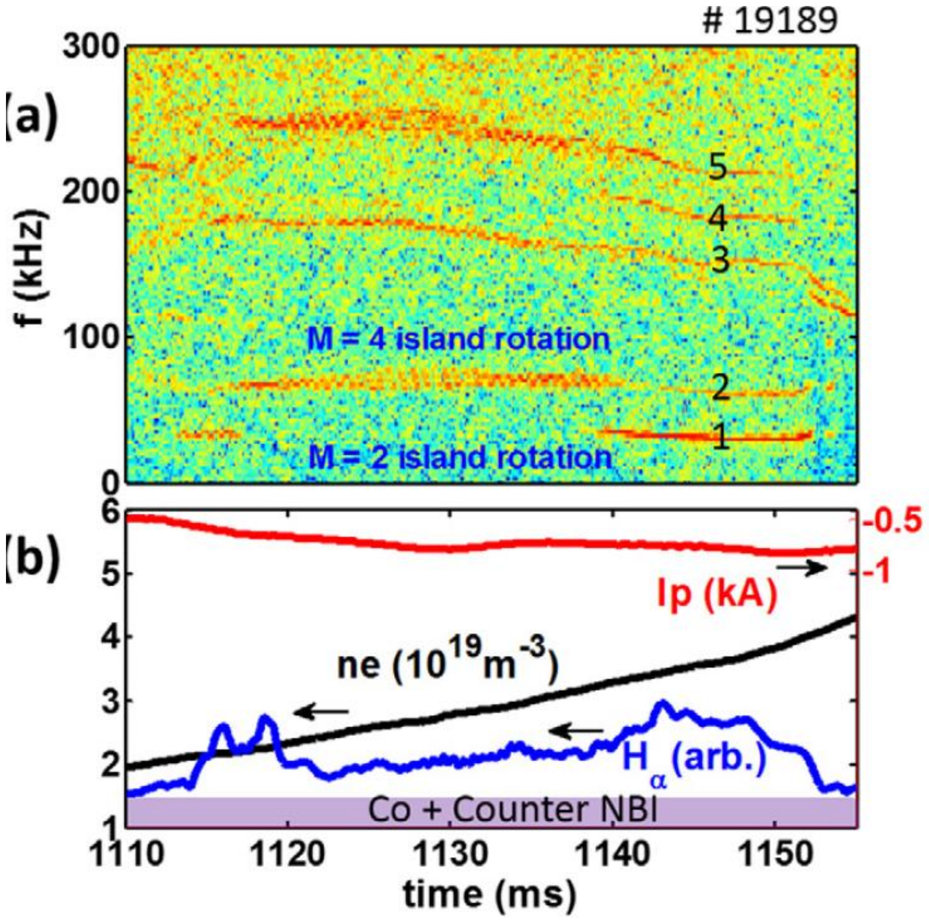


J-TEXT (*PPCF* 2015)

◆**BAEs were observed during tearing modes in FTU, TEXTOR, HL-2A, J-TEXT, EAST, TJ-II.**



EAST (*PPCF 2013*)



TJ-II (*NF* 2015)

Excitation of BAEs by coupling between GAM and magnetic island

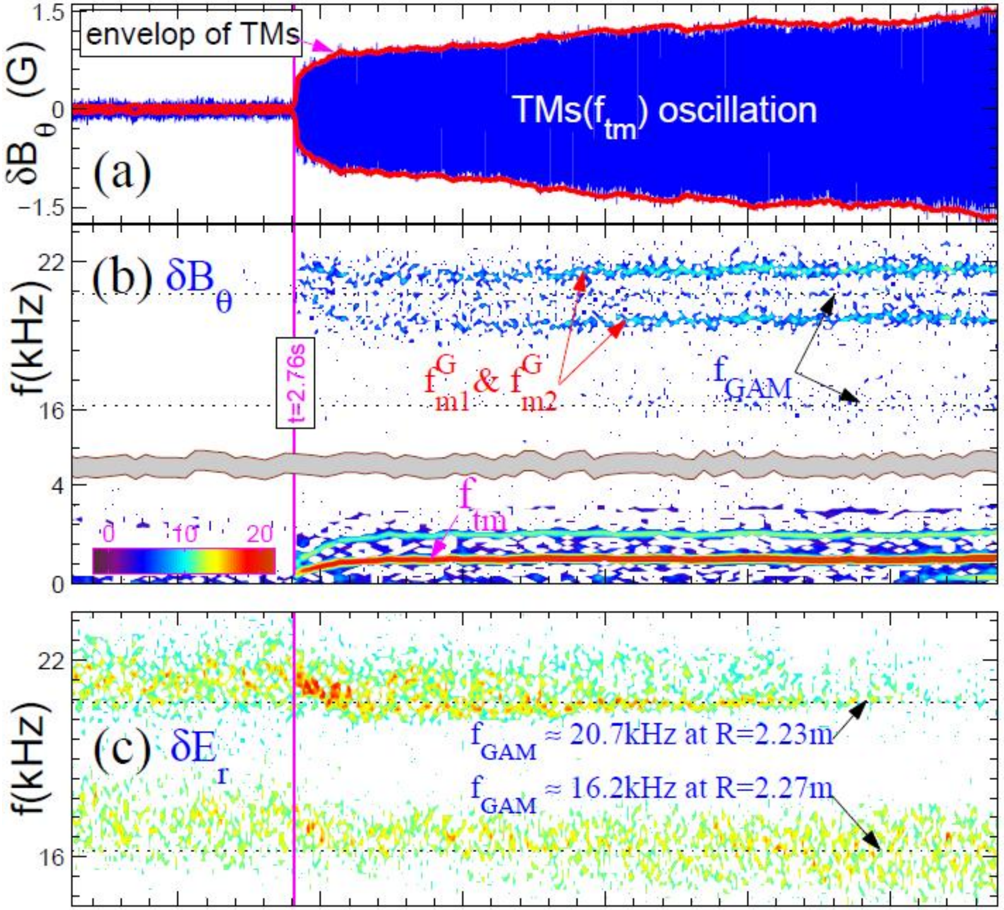
- **The common characteristics:**
  - **The excited BAEs during tearing modes are a pair waves, propagate in poloidally and toroidally with opposite direction, form a standing wave structure in the island rest frame.**
  - **The absolute values of poloidal and toroidal mode numbers are the same with tearing modes,**
  - **The frequency difference between the pair of BAEs is twice the fundamental frequency of tearing mode.**
  - **They only appear when the island width increases above a threshold.**

Excitation of BAEs by coupling between GAM and magnetic island

- **Some theories are tried to under the physics:**
  - **The BAE frequency with magnetic island has been solved (Annibaldi, *et. al.* PPCF(2007), Biancalani *et. al.*, PRL(2010), Cook *et. al.*, POP(2015)).**
  - **BAEs are excited by the plasma flow around magnetic island(Marchenko, *et. al.*, NF (2016)).**
- **The excitation physics mechanism is still unclear.**

Excitation of BAEs by coupling between GAM and magnetic island

- **Recently, experiment in EAST(#86309) showed that a GAM and magnetic island exist before the excitation of BAEs**



Xu, et. al., Nucl. Fusion 2021

- **Physics Interpretation: excitation by the coupling between GAM and magnetic island**

Excitation of BAEs by coupling between GAM and magnetic island

- A fluid model is used to describe the excitation of BAEs, for simplicity.
- GAM is assumed to be a constant pump wave, and tearing modes evolves.
- The fluctuations of fields are

$$\delta\phi = \delta\phi_B + \delta\phi_G + \delta\phi_t, \delta A_{//} = \delta A_{//,B} + \delta A_{//,t}, \delta\phi_B = \delta\phi_+ + \delta\phi_-, \delta A_{//,B} = \delta A_{//,+} + \delta A_{//,-}$$

where the subscripts of *B, G, t* denote the BAE, GAM and tearing mode, respectively. The subscripts $\pm$ are the upper and lower frequencies of the pair of BAEs, respectively.

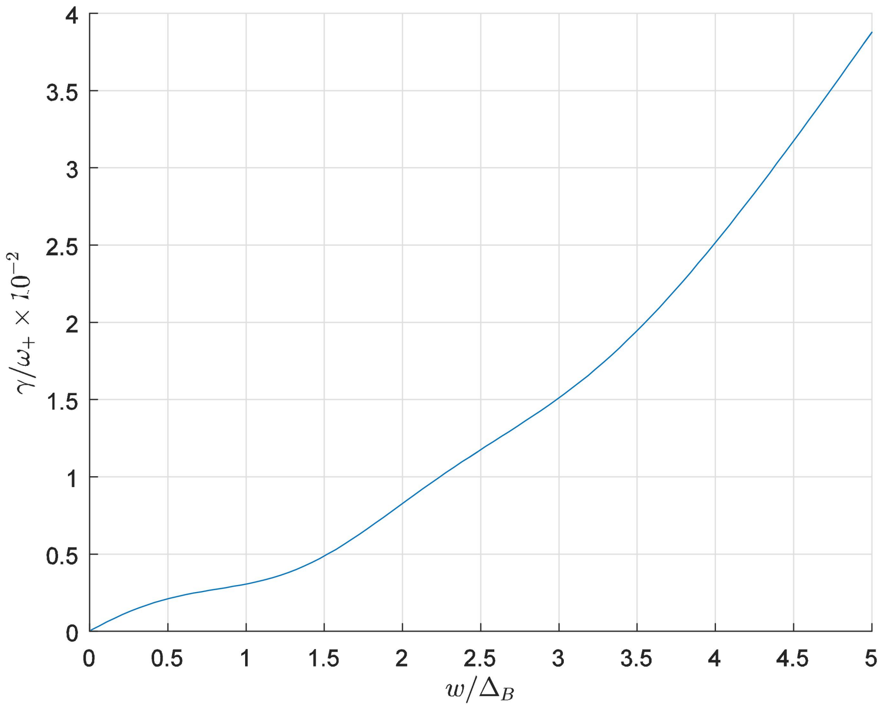
- GAM is assumed to be predominantly electrostatic $\delta\phi_G = \delta\hat{\phi}_G e^{-i\omega_G t}$
- For tearing modes, $\delta A_{//,t} = \delta\hat{A}_{//,t}(0,t)\cos\xi, \xi = m\theta - n\zeta - \omega_t t$
- For BAEs, $\delta f_\pm = \delta\hat{f}_\pm \exp\left(\pm i(m\theta - n\zeta + \varphi_0)t - i\omega_\pm t\right),$

$$\omega_\pm = \omega_G \pm \omega_t$$

Excitation of BAEs by coupling between GAM and magnetic island

- **Given the profiles of BAE and GAM, and the main parameters of the experimental results in EAST, as**

$$R_0 = 1.89m, a = 0.45m, B = 2.05T, n_0 = 1.0\times10^{19} m^{-3}, T_i = T_e = 200eV,$$
$$f_G \sim 20.7kHz, f_t \sim 1kHz, q_s = 4, r_s = 0.8a$$

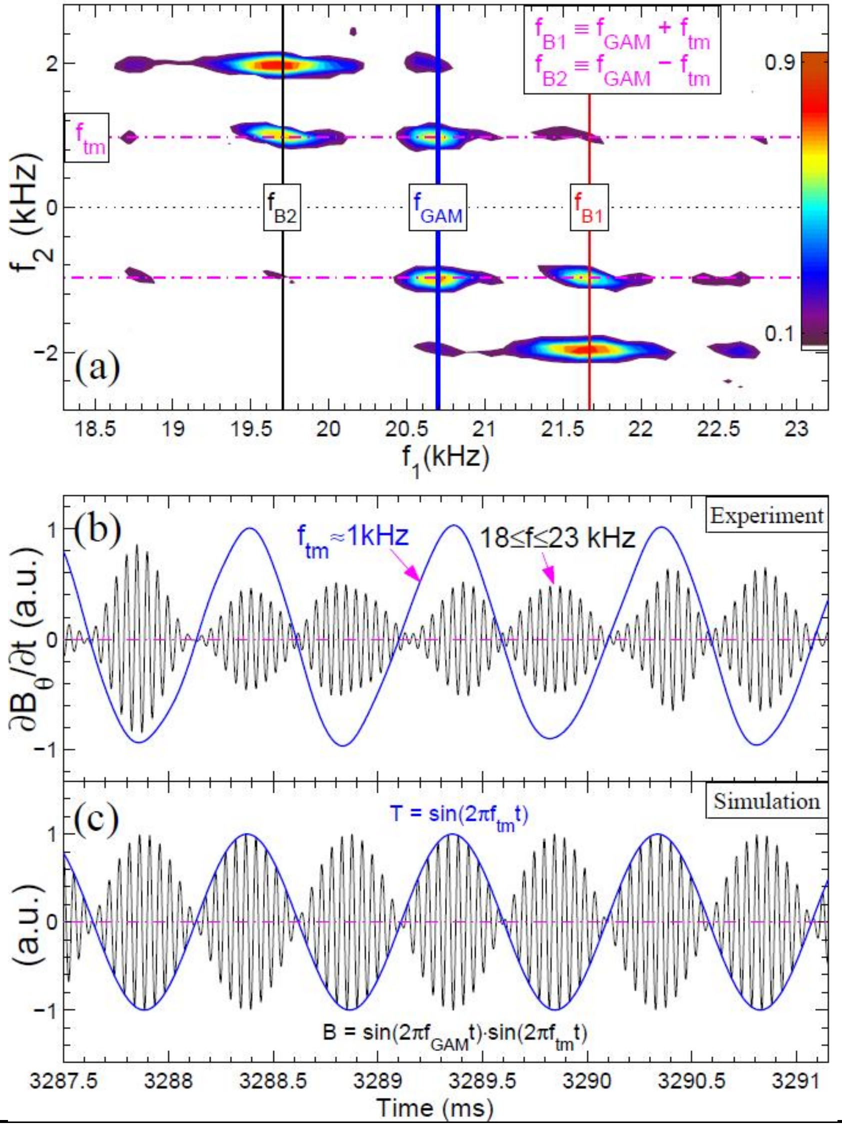


- **The growth rate without damping rate increases with island width.**
- **When the island increases larger than a threshold, growth rate dominates over damping rate, BAEs are excited.**

Cai, et. al., Nucl. Fusion 2021

Excitation of BAEs by coupling between GAM and magnetic island

◆ The phase shift $\varphi_0 = \pi/2$ is well consistent with the experimental

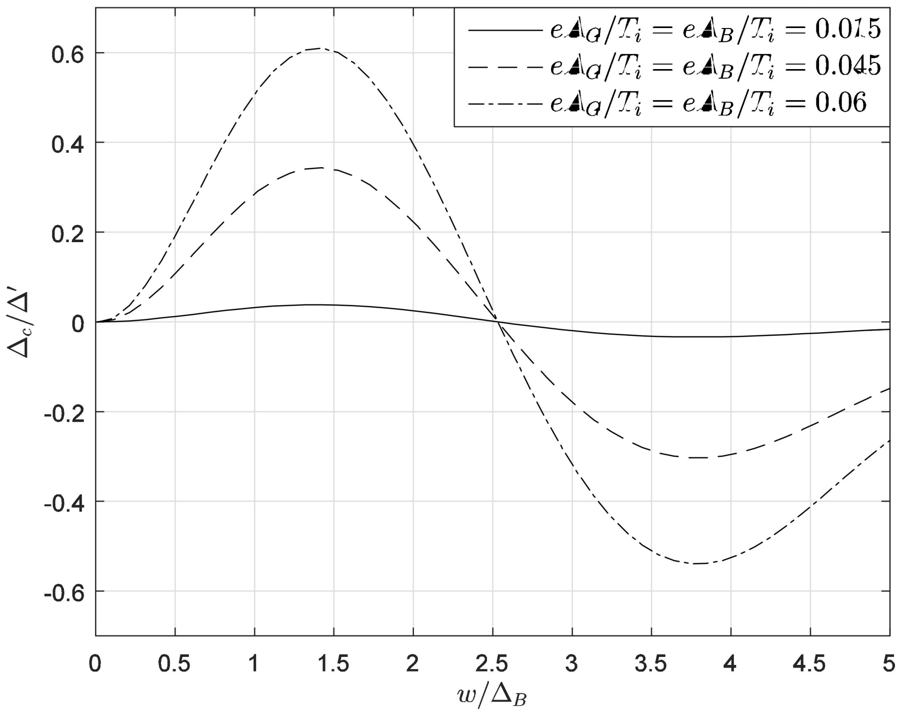


- Fig. b is plotted based on the experiment result. Fig.c is based on our model with phase shift pi/2.
- The phases are almost the same, expect the amplitude.
- This confirms our physics interpretation: **nonlinear coupling between GAM and island is responsible for the excitation of BAEs in EAST.**

Cai, et. al., Nucl. Fusion 2021

Excitation of BAEs by coupling between GAM and magnetic island

◆ **The island evolution is**



- **The nonlinear coupling effect on island is not large.**
- **For small island, the nonlinear coupling effect enhances the island evolution.**
- **For large island, the nonlinear coupling effect weakens the island evolution.**

References:

- **Wilson, *TRANSACTIONS OF FUSION SCIENCE AND TECHNOLOGY*, 2010**
- **M. Maraschek, Nucl. Fusion 2012**
- **Buttery, et. al, Plasma Phys. Control. Fusion, 2000**
- **Other references in the above references.**

Thank you for your attention!