核工业西南物理研究院
Southwestern Institute of Physics
中核集团 CNNC

托卡马克磁流体不稳定性理论和数值模拟

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2025年7月

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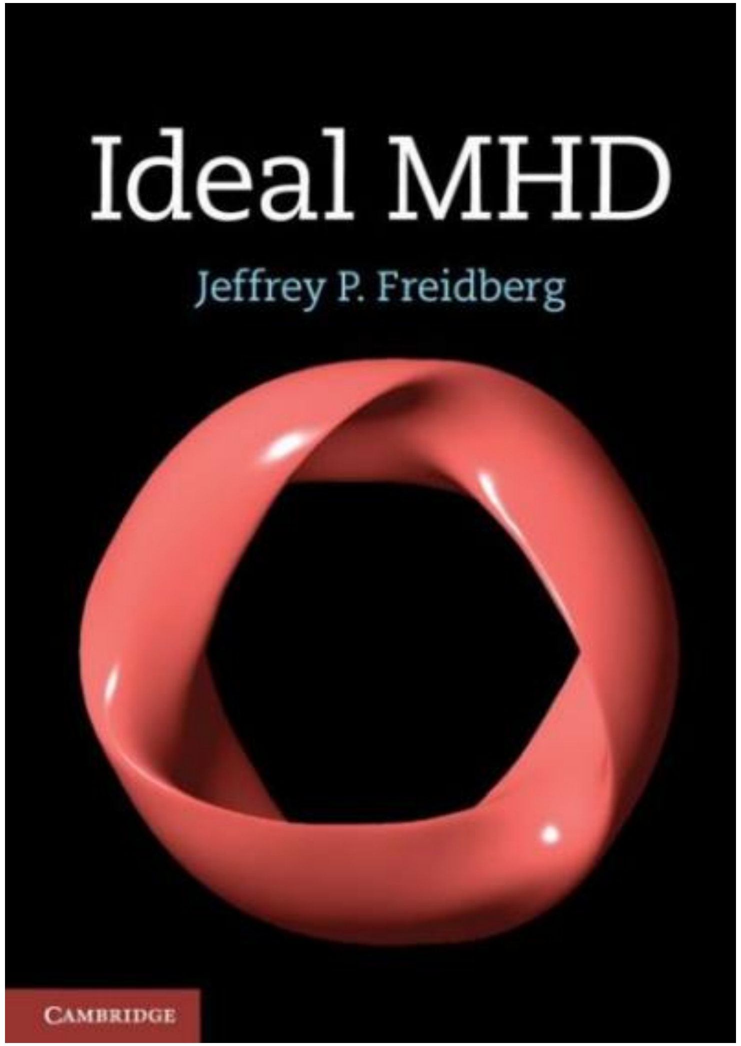
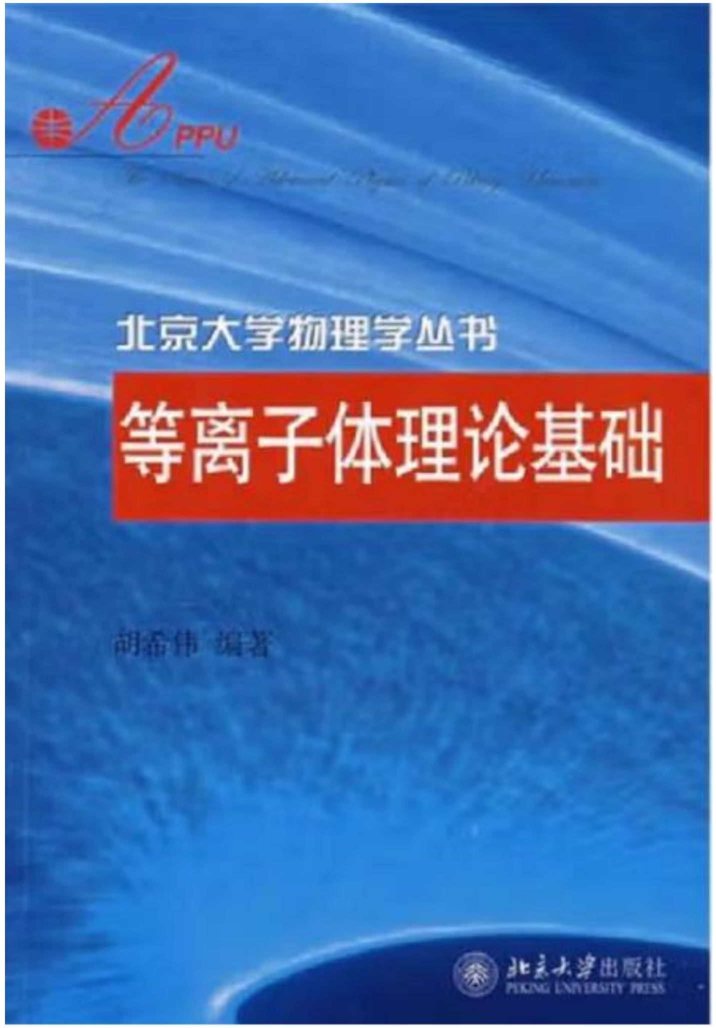
Relevant Texts



2014



2006

Outline

1. Introduction
    - Background
    - Closure of MHD equations
    - Several basic concepts
    - Revisit of energy principle
2. Physics and control of Resistive Wall Mode (RWM)
3. Physics and control of Edge Localized Mode(ELM)
4. Summary and outlook

Challenges of plasma physics for fusion reactor

Fusion Gain:

Challenge #0: Fusion technology

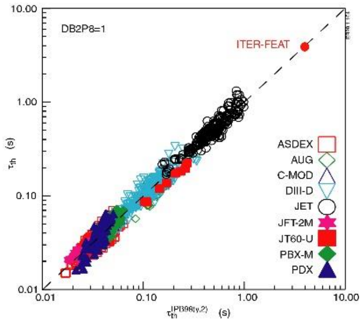
$$\boldsymbol{Q} \propto nT\tau_E \propto \beta_N H B^3 a^3 / q_{95}^2$$

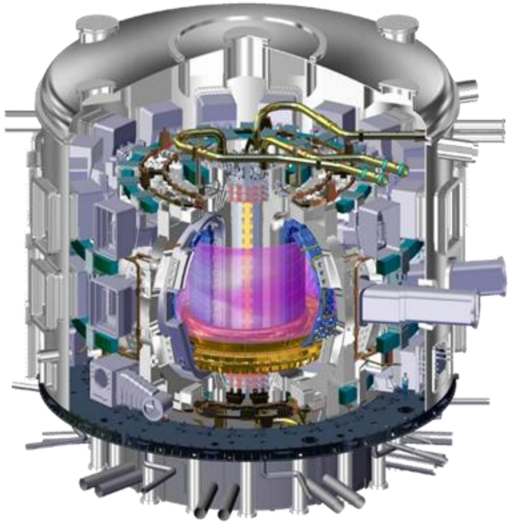
Challenge#1: MHD instabilities

Challenge#2: turbulence & transport

Scaling:

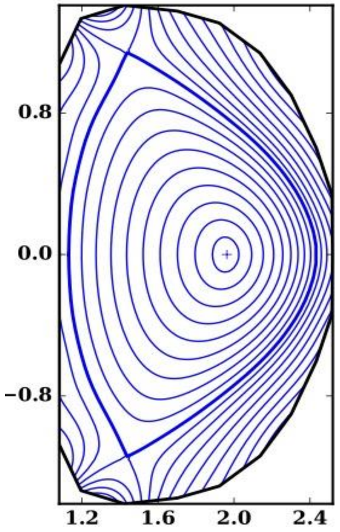
$$\tau_E = c\, I^{0.93} B^{0.15} P^{-0.69} n^{0.41} \times M^{0.19} R^{1.97} \varepsilon^{0.58} \kappa^{0.78}$$

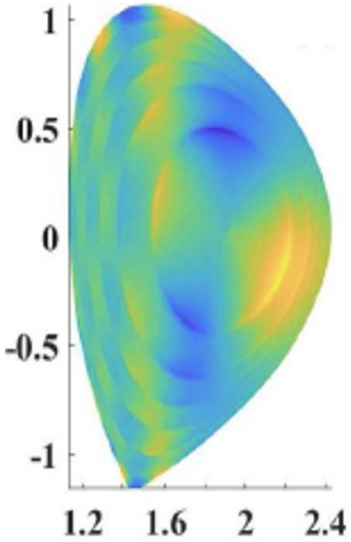


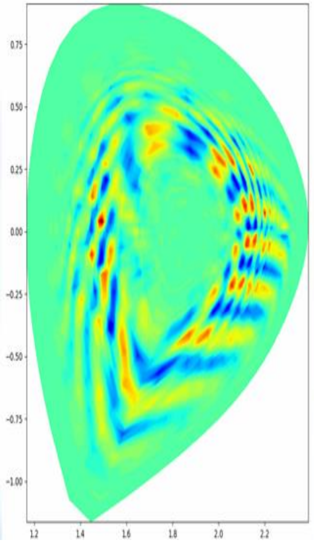


ITER： Q~10, P_{fus}=500 MW

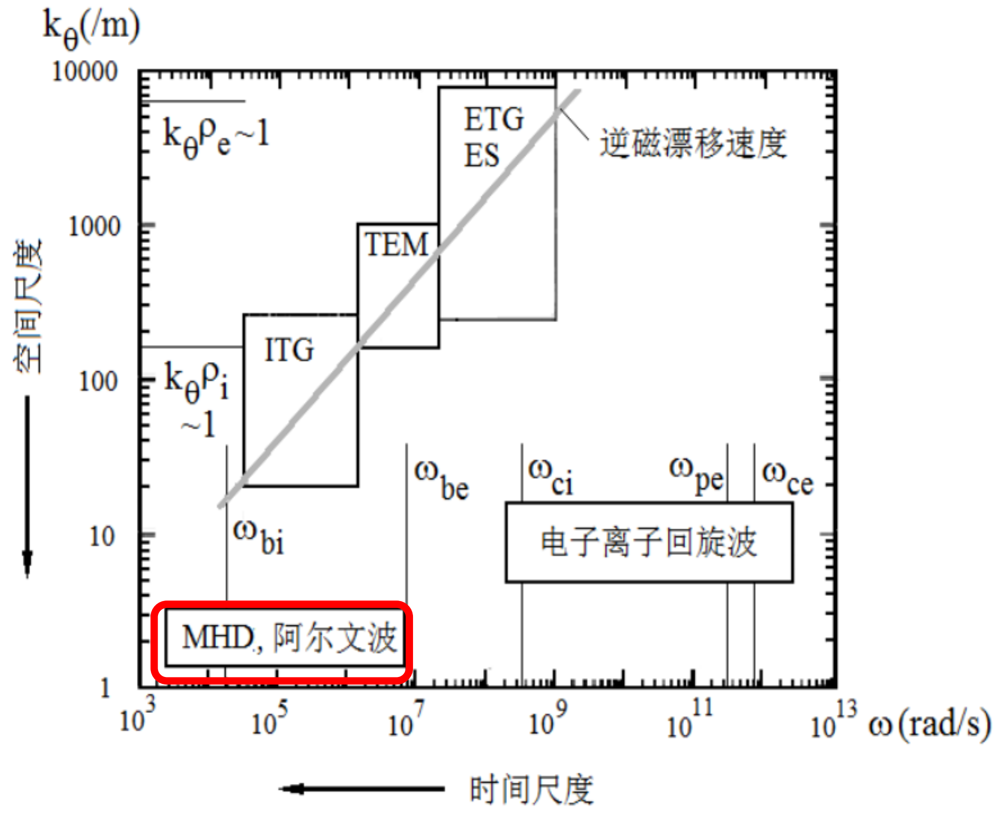
Toroidal plasmas







Scales of MHD activities



Plasma physics time scales	Formulas	Values (sec)
Electron gyro period	$\tau_{ce} = 2\pi/\omega_{ce} = 2\pi m_e/eB$	7.1×10^{-12}
Electron plasma period	$\tau_{pe} = 2\pi/\omega_{pe} = 2\pi(m_e\varepsilon_0/ne^2)^{1/2}$	1.1×10^{-11}
Ion plasma period	$\tau_{pi} = (m_i/m_e)^{1/2}\tau_{pe}$	6.7×10^{-10}
Ion gyro period	$\tau_{ci} = (m_i/m_e)\tau_{ce}$	2.8×10^{-8}
MHD time	$\tau_M = a/V_{Ti}$	1.9×10^{-6}
Electron-electron collision time	$\tau_{ee} = 6.7 \times 10^{-6}\ T^{3/2}/n$	3.0×10^{-5}
Ion-ion collision time	$\tau_{ii} = (2m_i/m_e)^{1/2}\tau_{ee}$	2.6×10^{-3}
Energy equilibration time	$\tau_{eq} \approx (m_i/2m_e)\tau_{ee}$	5.5×10^{-2}
Energy confinement time for ignition	$\tau_E = 1.7/n$	1.7
Resistive diffusion time	$\tau_D = \mu_0 a^2/\eta = 40\ a^2 T^{3/2}$	2.1×10^2

Over 14 orders

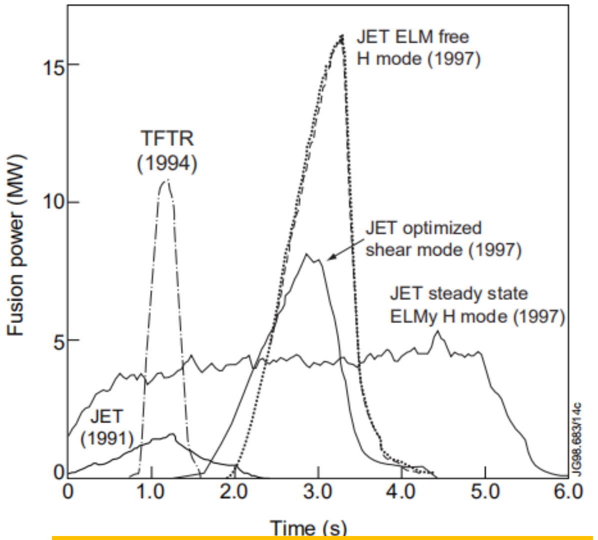
Plasma physics length scales	Formulas	Values (m)
Electron gyro radius	$r_{Le} = V_{Te}/\omega_{ce}$	3.7×10^{-5}
Debye length	$\lambda_D = V_{Te}/\omega_{pe}$	5.8×10^{-5}
Electron skin depth	$\delta_e = c/\omega_{pe}$	5.3×10^{-4}
Ion gyro radius	$r_{Li} = (m_i/m_e)^{1/2} r_{Le}$	2.2×10^{-3}
Ion skin depth	$\delta_i = (m_i/m_e)^{1/2}\delta_e$	3.2×10^{-2}
MHD length	a	1
Ion-ion mfp	$\lambda_{ii} = V_{Ti}\tau_{ii}$	1.4×10^3
Electron-electron mfp	$\lambda_{ee} = \lambda_{ii}$	1.4×10^3

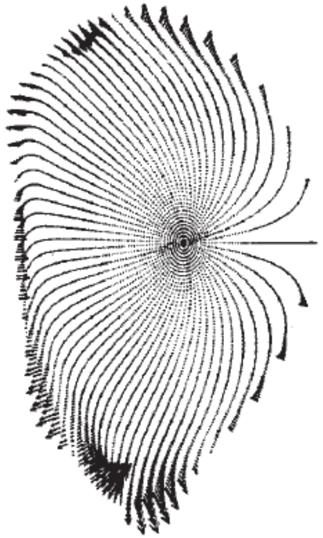
Over ~10 orders

$$a = 1\text{m},\ T_e = T_i = 3\text{keV},\ B = 5\text{T},\ n = 10^{20}\text{m}^{-3}$$

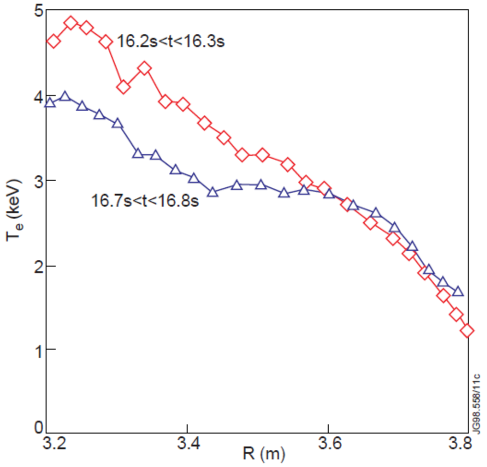
$$\ln\Lambda = 19.$$

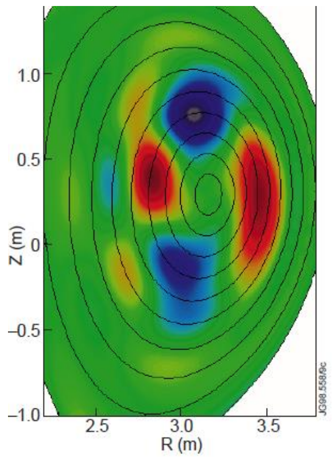
Why are MHD activities important?





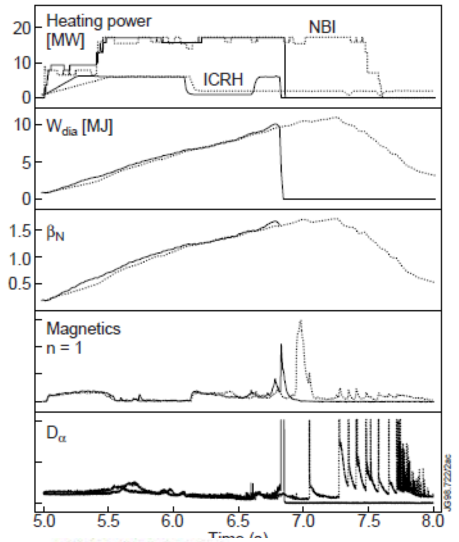
M.L. Watkins,1999, NF , **39,** (1999) , 1227
Huysmans, NF, **38,** (1998),179

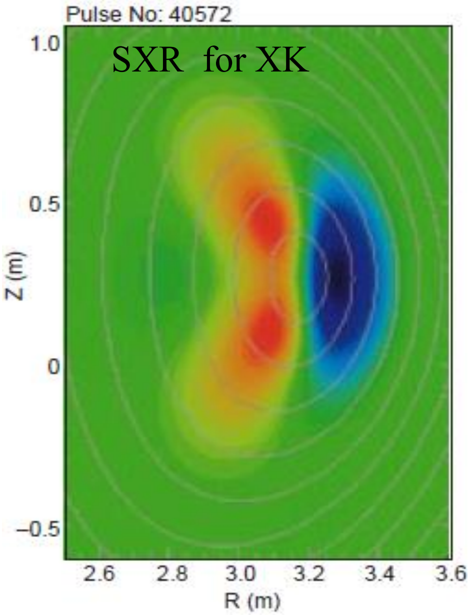




SXR measurements for NTM

Huysmans, NF, **39,** (1999),1965





Huysmans, NF, **39,** (1999), 1489

- JET DTE1 & TFTR experiments: fusion power limited by MHD activity (edge kink)
- MHD （e.g. NTM）reduces the plasma confinement (e.g. by ~13% on JET)
- MHD（e.g. XK, RWM， NTM）activities induce disruptions

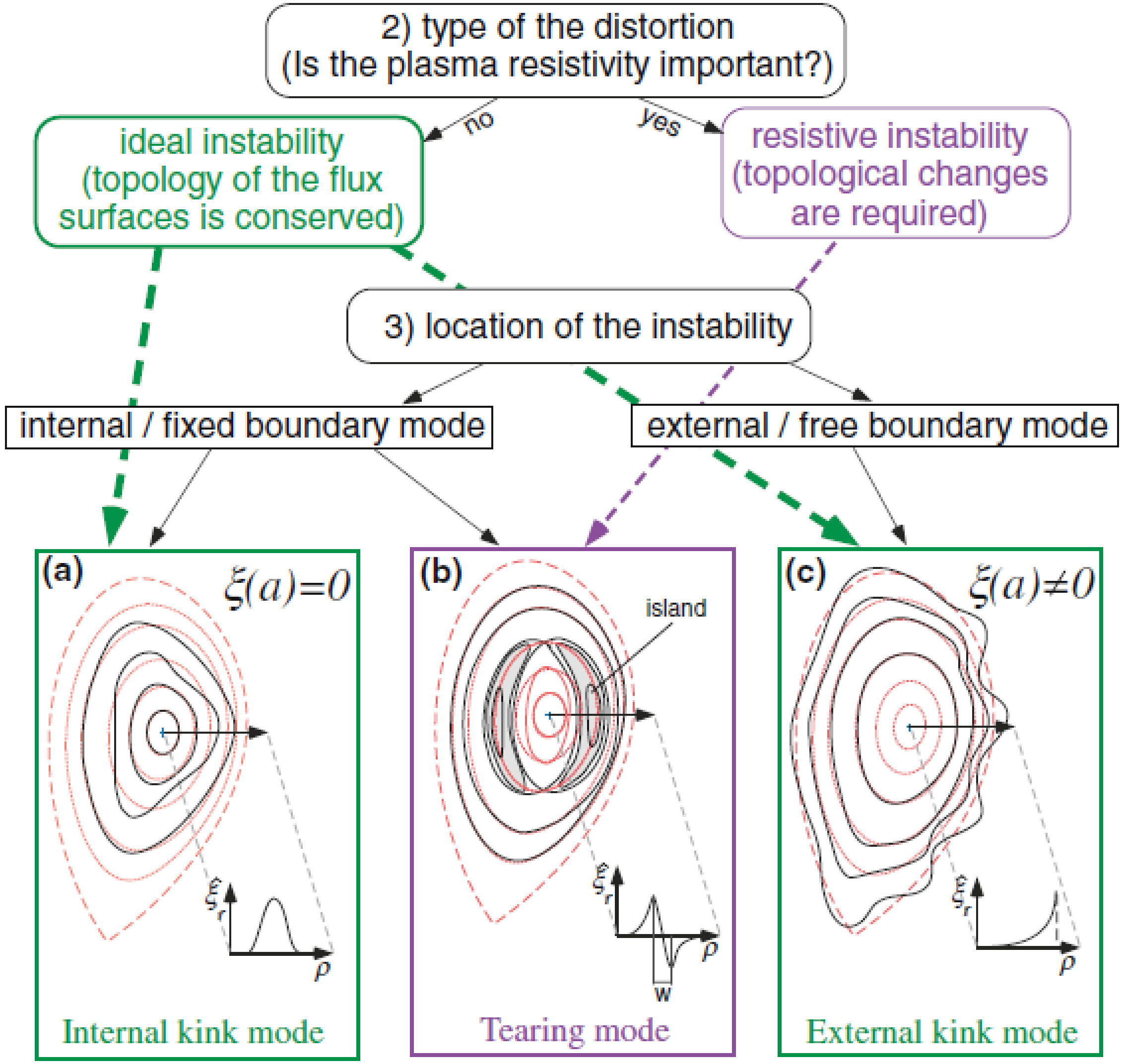
Ideal & resistivity MHD

$$\mathbf{E}+\mathbf{v}\times\mathbf{B} = \begin{cases} \eta\mathbf{j} & (resistive) \\ 0 & (ideal) \end{cases} \qquad (Ohm),$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla\times\mathbf{J} \qquad \mathbf{J} = \nabla\times\mathbf{B}}$$

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B}) + \eta\nabla^2\mathbf{B}}$$

When $\nabla\times(\mathbf{v}\times\mathbf{B}) = 0$ resistivity is important!



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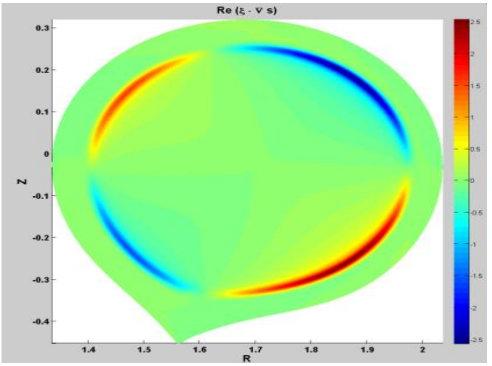
MHD instabilities in tokamak

Basic classification of MHD instabilities

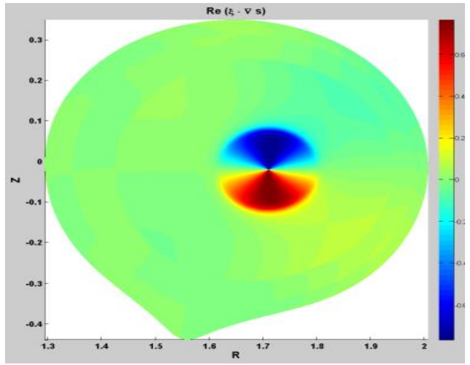
1) drive of the instability (source of the free energy)

- current driven instabilities (tearing mode, kink mode,etc)
- pressure driven instabilities (neoclassical tearing mode, kink mode at high pressure, balooning modess, etc)
- particle driven instabilities (fishbones, fast particle modes, TAEs, BAEs,etc)

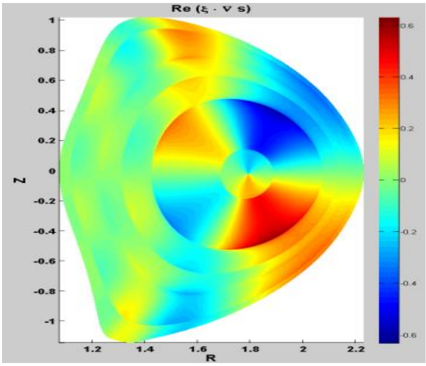
Tearing mode



Internal kink

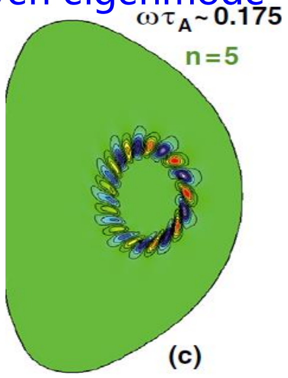


RWM（external kink)



Ballooning mode

Alfven eigenmode

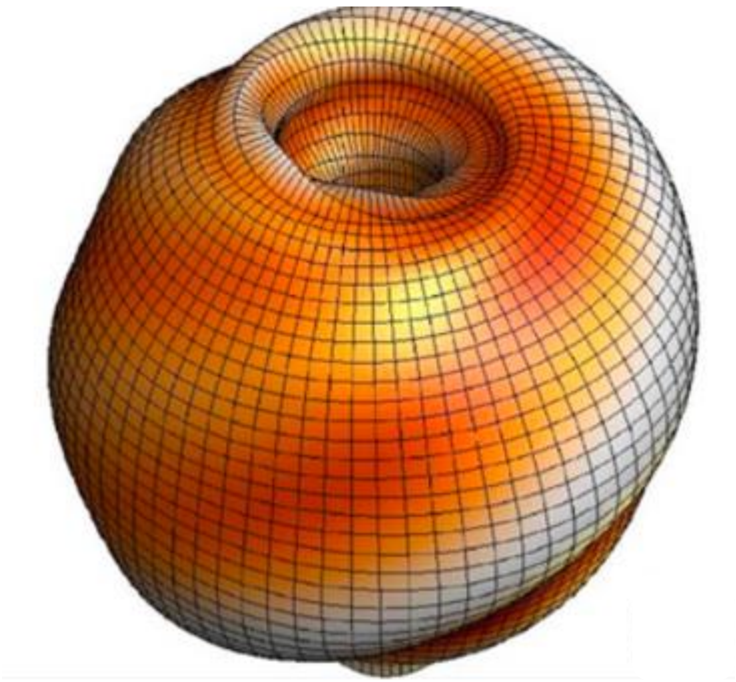


Resistive Wall Mode &Edge Localized Mode

Fusion Gain:

$$Q \propto nT\tau_E \propto \beta_N H B^3 a^3 / q_{95}^2$$

RWM ELM



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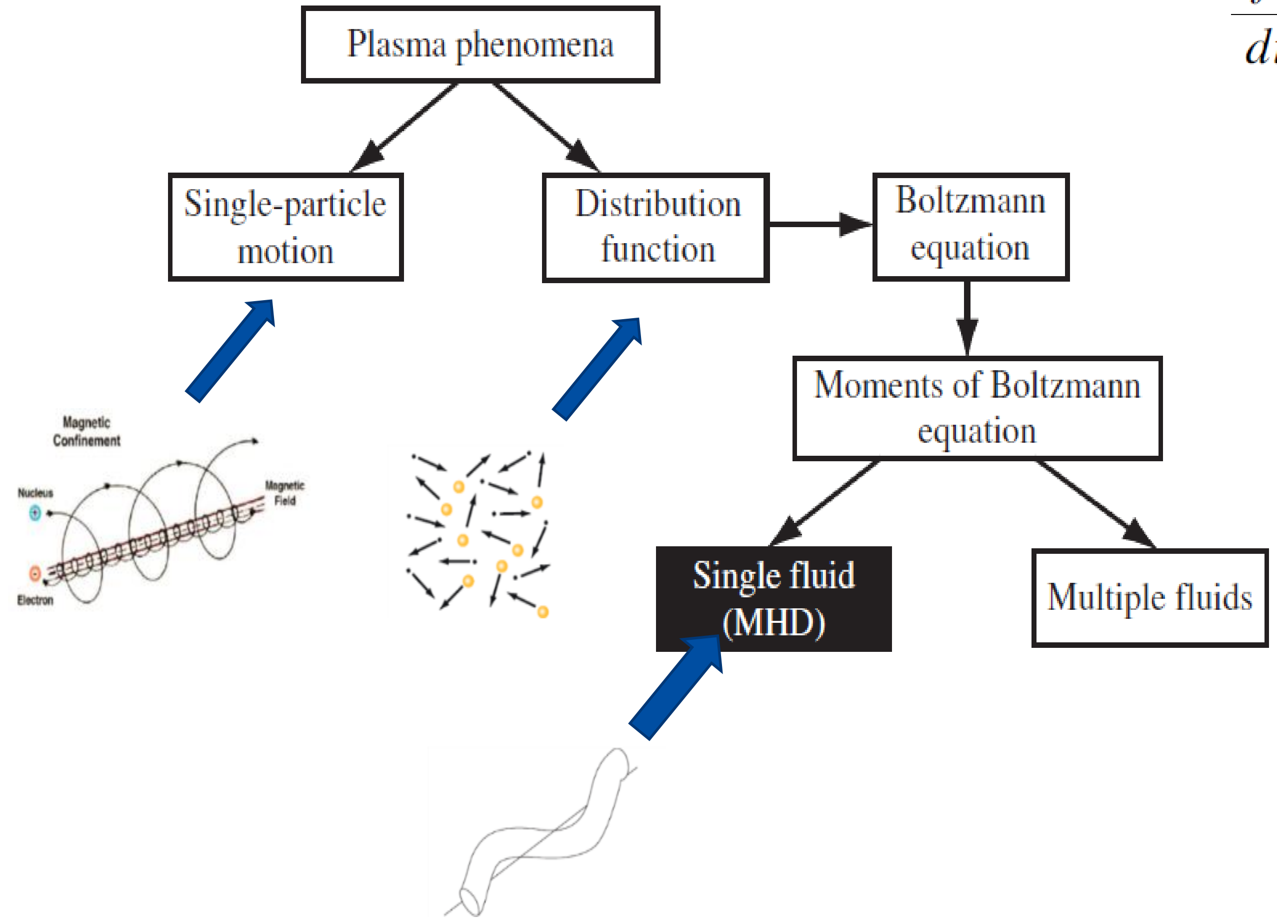
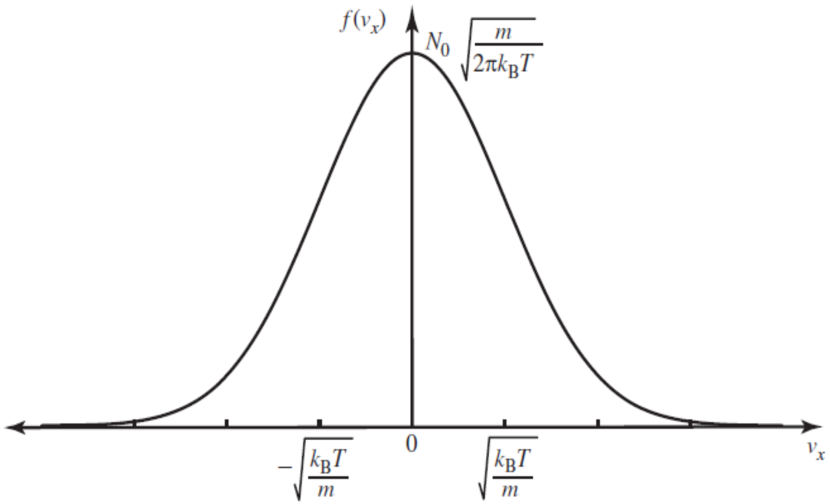
MHD model



$$\frac{df_\alpha}{dt} \equiv \frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{Z_\alpha e}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$



$$n_\alpha = \int f_\alpha d\mathbf{v}$$

Derivation of ideal MHD model

$$\frac{df_\alpha}{dt} \equiv \frac{\partial f_\alpha}{\partial t} + \mathbf{v}\cdot\nabla f_\alpha + \frac{Z_\alpha e}{m_\alpha}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\nabla_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t}\right)_c$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\cdot\mathbf{E} = \frac{\sigma}{\varepsilon_0}$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\mathbf{J} = \sum_\alpha Z_\alpha e\int \mathbf{v} f_\alpha d\mathbf{v}$$

$$\sigma = \sum Z_\alpha e\int f_\alpha d\mathbf{v}$$

$$n_\alpha = \int f_\alpha d\mathbf{v}$$

$$\mathbf{u}_\alpha = \frac{1}{n_\alpha}\int \mathbf{v} f_\alpha d\mathbf{v}$$

$$\alpha = i, e$$

$$\int g_i\left[\frac{df_\alpha}{dt} - \left(\frac{\partial f_\alpha}{\partial t}\right)_c\right] d\mathbf{v} = 0$$

for $i = 1-3$ with $g_i(\mathbf{v})$ given by

$$\begin{aligned} g_1 &= 1 && \text{(mass)} \\ g_2 &= m_\alpha \mathbf{v} && \text{(momentum)} \\ g_3 &= m_\alpha v^2/2 && \text{(energy)} \end{aligned}$$

Multi-fluid equation；

$$\left(\frac{dn_\alpha}{dt}\right)_\alpha + n_\alpha\nabla\cdot\mathbf{u}_\alpha = 0$$

$$m_\alpha n_\alpha\left(\frac{d\mathbf{u}_\alpha}{dt}\right)_\alpha - Z_\alpha e n_\alpha(\mathbf{E}+\mathbf{u}_\alpha\times\mathbf{B}) + \nabla\cdot\mathbf{P}_\alpha = \mathbf{R}_\alpha$$

$$\frac{3}{2}n_\alpha\left(\frac{dT_\alpha}{dt}\right)_\alpha + \mathbf{P}_\alpha : \nabla\mathbf{u}_\alpha + \nabla\cdot\mathbf{q}_\alpha = Q_\alpha$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{B} = \mu_0 e(n_i\mathbf{u}_i - n_e\mathbf{u}_e) + \frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}$$

$$\nabla\cdot\mathbf{E} = \frac{e}{\varepsilon_0}(n_i - n_e)$$

$$\nabla\cdot\mathbf{B} = 0$$

How to closure the set of equation?

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Closure of MHD equations

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\,\mathbf{v}) = 0$$

$$\rho \frac{d\,\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla \cdot (\mathbf{P}_e + \mathbf{P}_i)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$n_e = n_i = n$$

- More unknowns than equations.
  How to closure the set of equation?

- Ideal MHD closure

$$\mathbf{P} = \mathbf{P}_e + \mathbf{P}_i = p\,\mathbf{I}$$

$$\frac{d}{dt}\left(\frac{p}{n^\gamma}\right) = 0$$

- MHD-kinetic closure

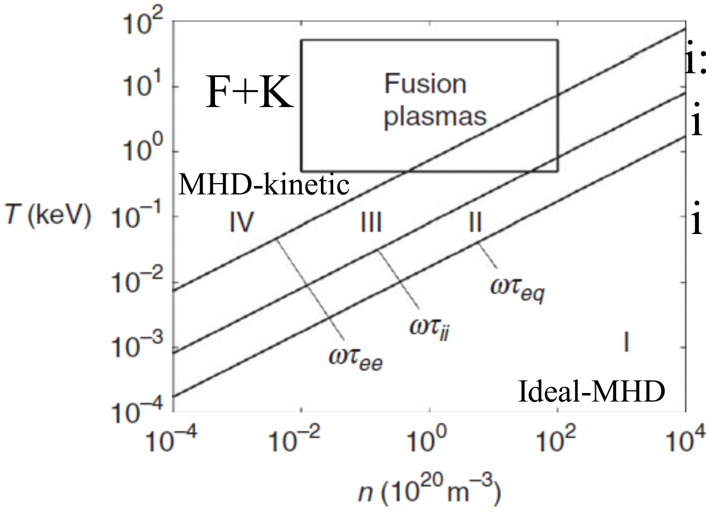
$$p_\parallel = \int v_\parallel^2 f d^3 v$$

$$p_\perp = \int v_\perp^2 f d^3 v$$

- Double adiabatic model

$$\frac{d}{dt}\left(\frac{p_\parallel B^2}{n^3}\right) = 0$$

$$\frac{d}{dt}\left(\frac{p_\perp}{nB}\right) = 0$$



- Ideal MHD: collision dominated fluid
- Kinetic MHD: collisionless kinetic
- Double adiabatic theory: collisionless fluid.

$$\omega\tau_{eq} = 3.3 \times 10^3 (T_k^2/a n_{20}) \ll 1$$

$$\omega\tau_{ii} = 1.5 \times 10^2 (T_k^2/a n_{20}) \ll 1$$

$$\omega\tau_{ee} = 1.8 (T_k^2/a n_{20}) \ll 1$$

$$\boxed{\delta W_{MHD} \le \delta W_{KIN} \le \delta W_{CGL}}$$

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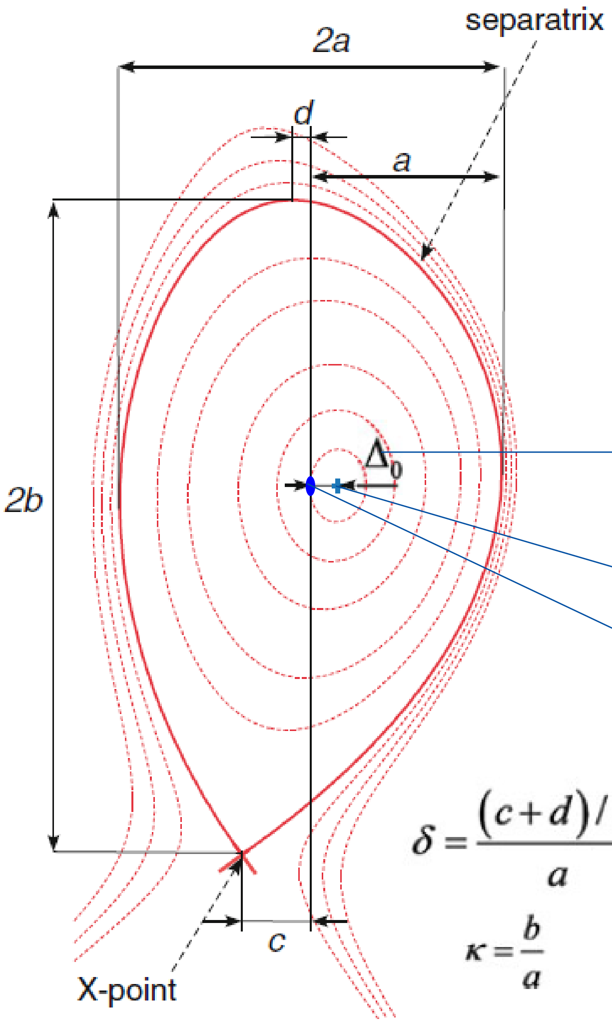
Some basic concepts about shape of tokamak plasma



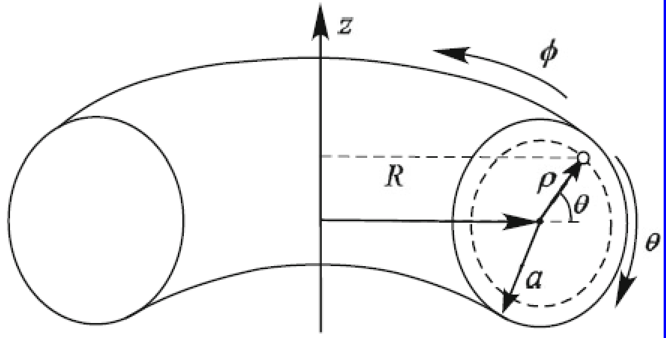
Shafranov shift

Magnetic axis

Geometry center

$\delta = \dfrac{(c+d)/2}{a}$ Tri-angularity

$\kappa = \dfrac{b}{a}$ elongation



Aspect ratio: R/a

Curvature of magnetic field

$$\boxed{\begin{aligned}\mathbf{j}\times\mathbf{B} &= \nabla P\\ \nabla\cdot\mathbf{B} &= 0\\ \mathbf{j} &= \frac{1}{\mu_0}\nabla\times\mathbf{B}\end{aligned}}$$

Substitute from Ampere's law and use vector identity →

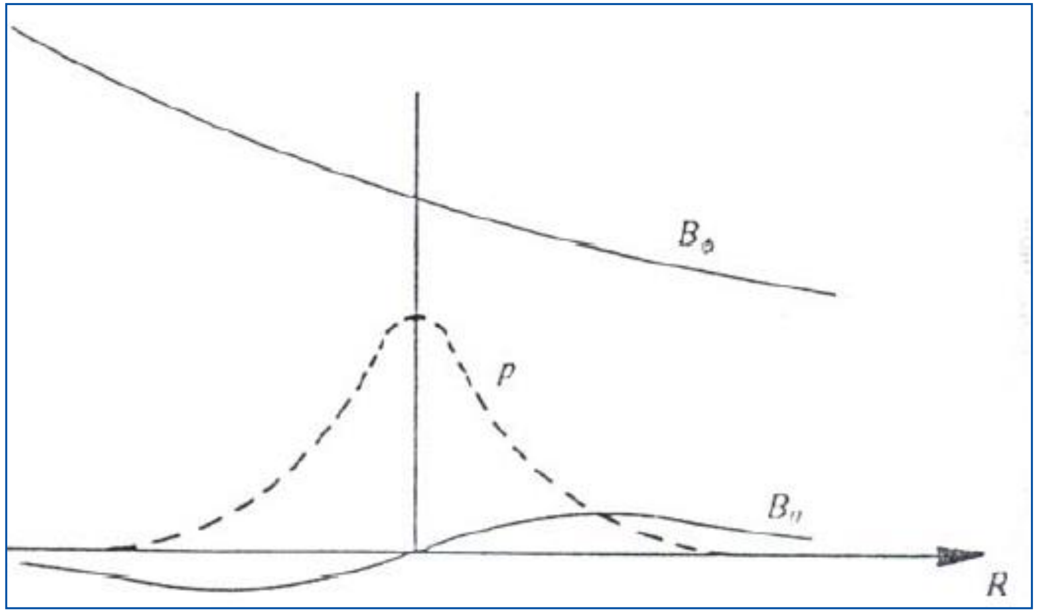
$$-\nabla P+\frac{1}{\mu_0}(\nabla\times\mathbf{B})\times\mathbf{B}=-\nabla\left(P+\frac{B^2}{2\mu_0}\right)+\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{\mu_0}=0$$

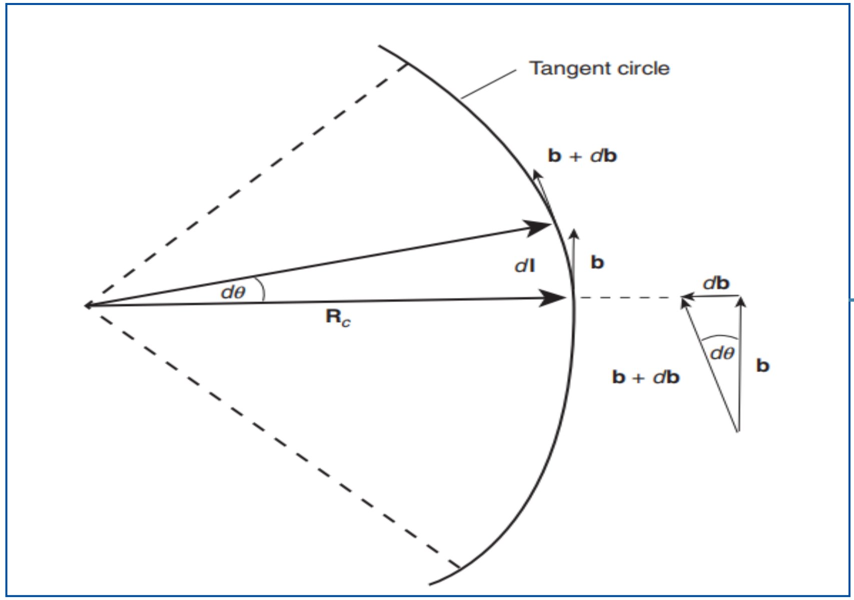
$P=$ thermal pressure

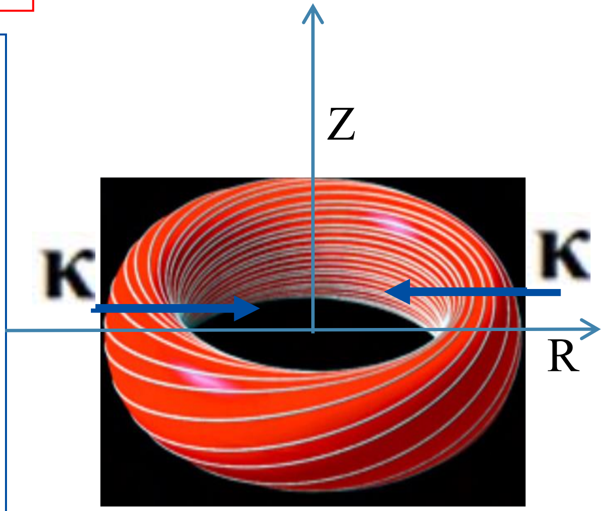
$\frac{B^2}{2\mu_0}=$ "magnetic pressure"

$$\boxed{\frac{\mathbf{B}\cdot\nabla\mathbf{B}}{\mu_o}=\frac{B^2}{\mu_0}\mathbf{b}\cdot\nabla\mathbf{b}=\frac{B^2}{\mu_0}\boldsymbol{\kappa} \qquad \boldsymbol{\kappa}=\frac{\hat{\mathbf{n}}}{R_c}=\text{curvature of field line}}$$

n"







What are normal & geodesic curvatures?

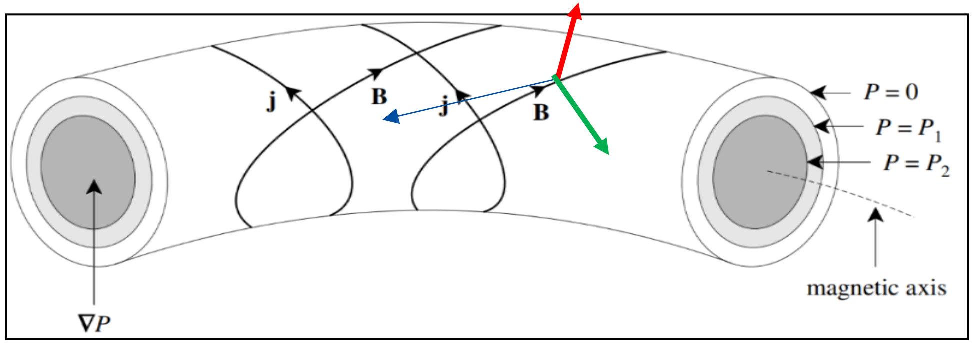
$$\mathbf{n} = \frac{\nabla\psi}{|\nabla\psi|} = \frac{\nabla\psi}{RB_p}$$

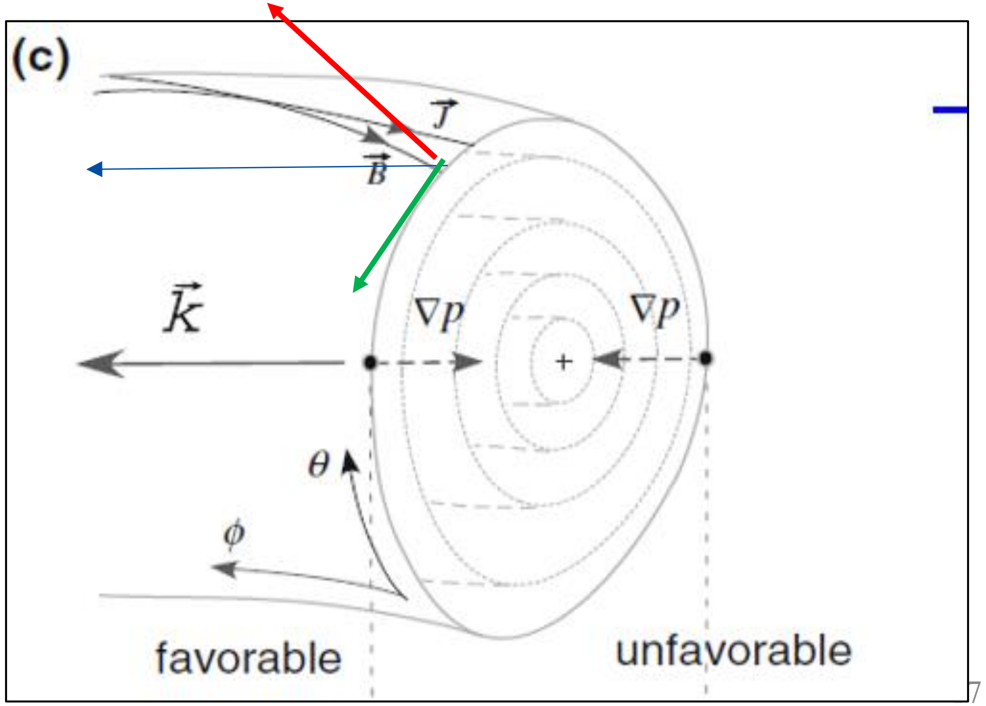
$$\mathbf{b} = \frac{B_p}{B}\mathbf{b}_p + \frac{B_\phi}{B}\mathbf{e}_\phi$$

$$\mathbf{t} = \mathbf{n}\times\mathbf{b} = \frac{B_\phi}{B}\mathbf{b}_p - \frac{B_p}{B}\mathbf{e}_\phi$$

Here, $\kappa_n = \mathbf{n}\cdot\boldsymbol{\kappa}$ is the normal curvature and $\kappa_t = \mathbf{t}\cdot\boldsymbol{\kappa}$ is the geodesic curvature (which is perpendicular to $\mathbf{B}$ but lies in the flux surface). After a short calculation these quantities can be rewritten in terms of the equilibrium fields and flux coordinates as

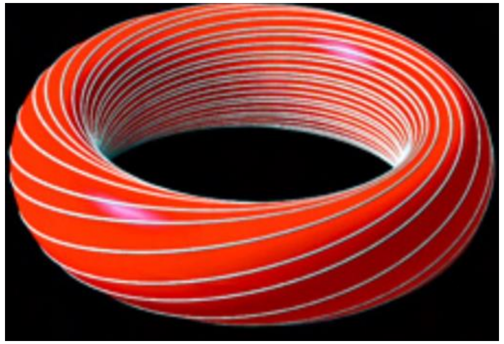
$$\begin{aligned}\kappa_n &= \frac{\mu_0 R B_p}{B^2}\frac{\partial}{\partial\psi}\left(p + \frac{B^2}{2\mu_0}\right)\\ \kappa_t &= \frac{\mu_0 F}{RB^3}\frac{\partial}{\partial l}\left(\frac{B^2}{2\mu_0}\right)\end{aligned} \qquad (12.39)$$

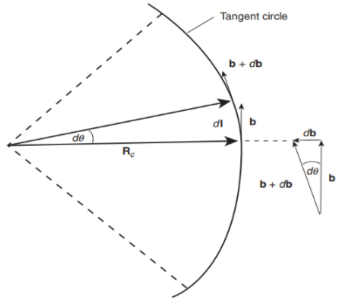




Toroidal effect is important in fusion plasma

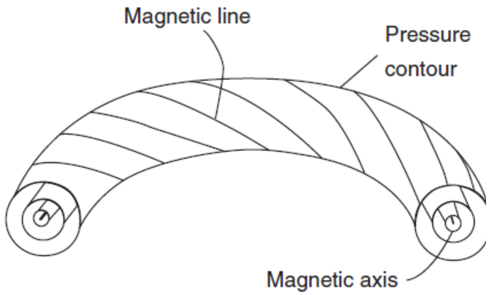
- Curvature of magnetic field (not zero after average along the poloidal direction)
  - Significant effect on MHD :TM, RWM, IK
  - GAM
  - Drive MHD: ballooning mode
  - On turbulence
  - …

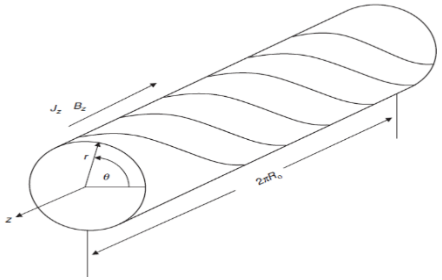




- Variation of magnetic strength along poloidal direction
  - Low field side (LFS), HFS
  - Trapped particles
  - Neoclassical effect (FOW replace of FLR)
  - Discrete (or named gap) modes: AEs
  - Coupling of different harmonics
  - On turbulence
  - …

$$B_z = B_{z0}(1 - (r/R)\cos\theta)$$





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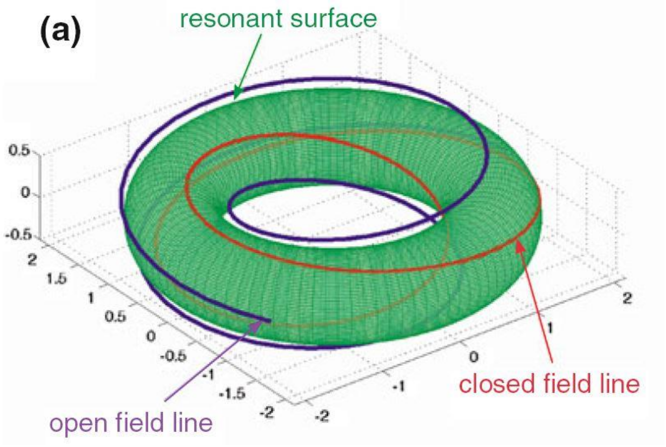
Safety Factor



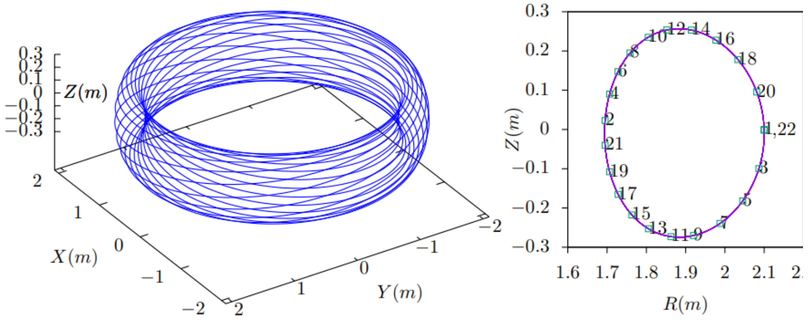


Figure 5. Left: A magnetic field line (blue) on a rational surface with $q = 2.1 = 21/10$ (magnetic field is from EAST discharge #59954@3.1s). This field line closes itself after traveling 21 toroidal loops and 10 poloidal loops. Right: The intersecting points of the magnetic field line with the $\phi = 0$ plane when it is traveling toroidally. The sequence of the intersecting points is indicated by the number labels. The 22nd intersecting point coincides with the 1st point and then the intersecting points repeat themselves.

Rotation transform:

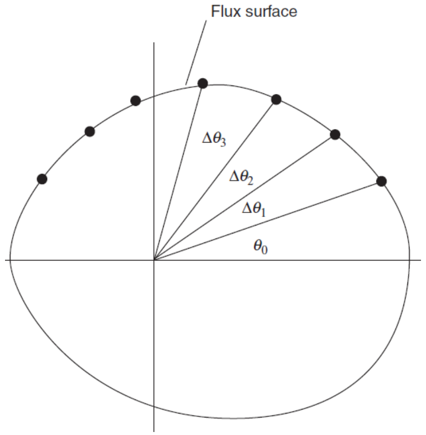
$$\iota = \lim_{N\to\infty} \frac{1}{N} \sum_{1}^{N} \Delta\theta_n$$

Safety factor:

$$q = \frac{2\pi}{\iota}$$

Magnetic shear:

$$s = \frac{r}{\iota}\frac{d\iota}{dr} = \frac{r}{q}\frac{dq}{dr}$$

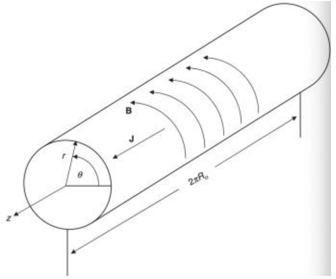


Hu's note

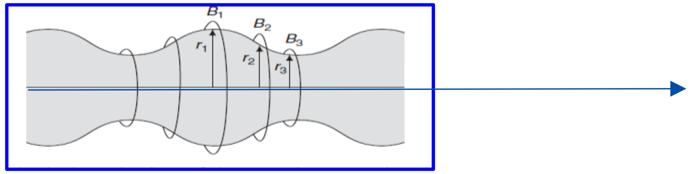
$$q_{local}(\psi, \theta) = \frac{d\phi}{d\theta}$$

$$q(\psi) = \frac{1}{2\pi}\int_0^{2\pi} q_{local}(\psi,\theta)d\theta$$

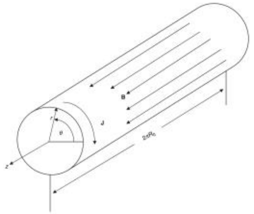
$$q \to 0$$

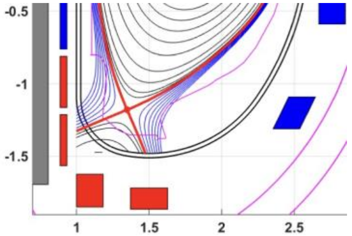


Z-pinch



$$q \to \infty$$





theta-pinch

q determines the stabilities properties

Outline

1. Introduction
   - Background
   - Closure of MHD equations
   - Several basic concepts
   - Revisit of energy principle
2. Physics and control of Resistive Wall Mode (RWM)
3. Physics and control of Edge Localized Mode(ELM)
4. Summary and outlook

Revisit of energy principle

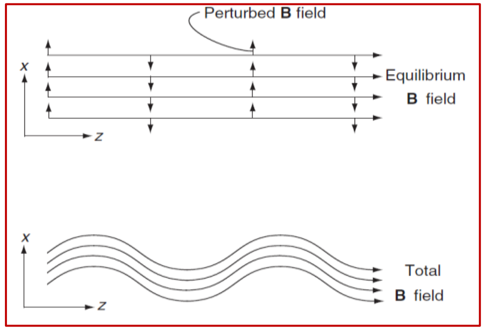
$$\omega^2 = \frac{\delta W}{\delta K} \longrightarrow -\int \xi^* \cdot \mathbf{F}(\xi)\, d\mathbf{r}$$

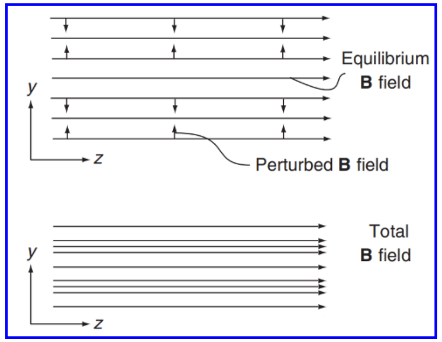
$$\delta W_F = \frac{1}{2}\int_P \mathrm{d}\boldsymbol{r}\left[\frac{|\boldsymbol{Q}_\perp|^2}{4\pi} + \frac{B^2}{4\pi}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2 + \gamma p|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}|^2 - 2(\boldsymbol{\nabla} p\cdot\boldsymbol{\xi}_\perp)(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp^*) - J_\parallel(\boldsymbol{\xi}_\perp^*\times\boldsymbol{b}\cdot\boldsymbol{Q}_\perp)\right]$$

$\omega^2 > 0$ corresponds to a pure oscillation

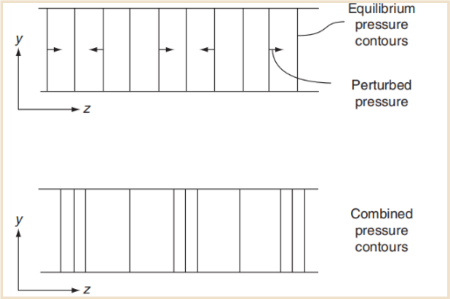
$\omega^2 < 0$ with one growing and the other decaying

$\omega = 0$ **Marginal point:** between pure growing (or decaying) mode and popagating modes





- SAW
- CAW
- sound wave



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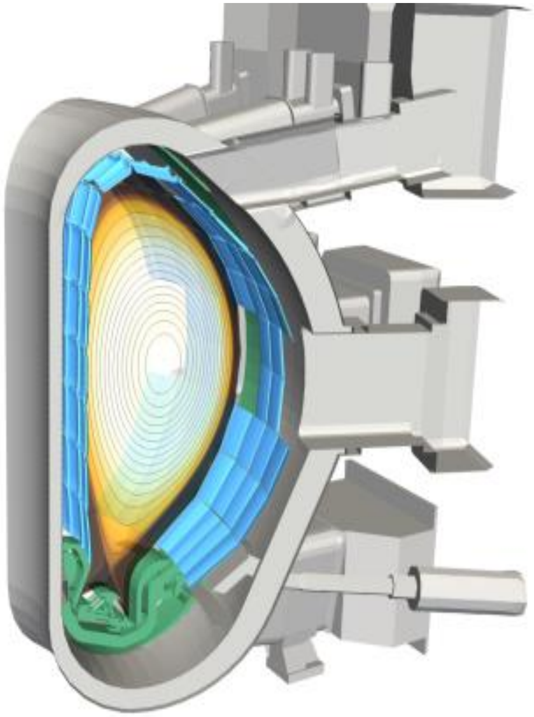
Revisit of perturbed potential energy



- fluid energy

$$\delta W_F = \frac{1}{2}\int_P \mathrm{d}\boldsymbol{r}\left[\frac{|\boldsymbol{Q}_\perp|^2}{4\pi} + \frac{B^2}{4\pi}|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2 + \gamma p|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}|^2\right.$$

$$\left. -2(\boldsymbol{\nabla} p\cdot\boldsymbol{\xi}_\perp)(\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp^*) - J_\parallel(\boldsymbol{\xi}_\perp^*\times\boldsymbol{b}\cdot\boldsymbol{Q}_\perp)\right]$$

Individual terms have the following physical interpretation:

- $|\boldsymbol{Q}_\perp|^2$ represents **magnetic field bending** energy $\Rightarrow$ **stabilizing**
- $|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\kappa}\cdot\boldsymbol{\xi}_\perp|^2$ represents **magnetic field compression** energy $\Rightarrow$ **stabilizing**
- $\gamma p|\boldsymbol{\nabla}\cdot\boldsymbol{\xi}|^2$ represents **plasma compression** energy $\Rightarrow$ **stabilizing**
- $\boldsymbol{\nabla} p$ represents **pressure gradient instability drive** $\Rightarrow$ can be **destabilizing**
- $J_\parallel$ represents **parallel current instability drive** $\Rightarrow$ can be **destabilizing**

IK/EK
RWM
BM
Infernal

TM/NTM
Peeling
RWM

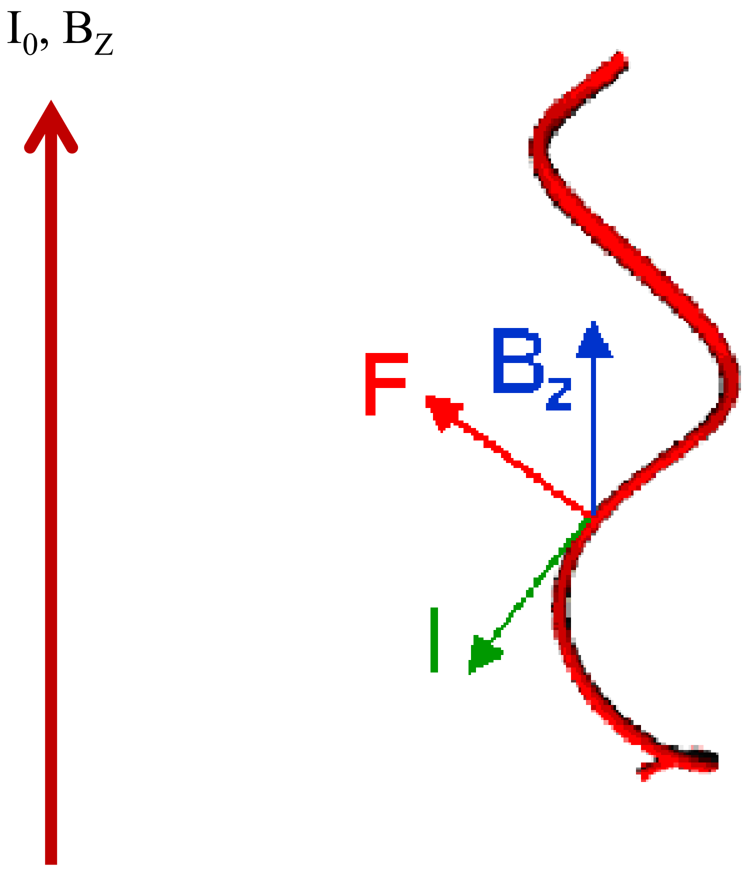
- surface energy

$$\delta W_S(\xi_\perp^*, \xi_\perp) = \frac{1}{2\mu_0}\int_{S_p} |\mathbf{n}\cdot\xi_\perp|^2 \mathbf{n}\cdot\left[\!\left[\nabla\left(\frac{B^2}{2} + \mu_0 p\right)\right]\!\right] dS$$

- vacuum energy

$$\delta W_V(\xi_\perp^*, \xi_\perp) = \frac{1}{2\mu_0}\int_V |\hat{\mathbf{B}}_1|^2 d\mathbf{r}$$

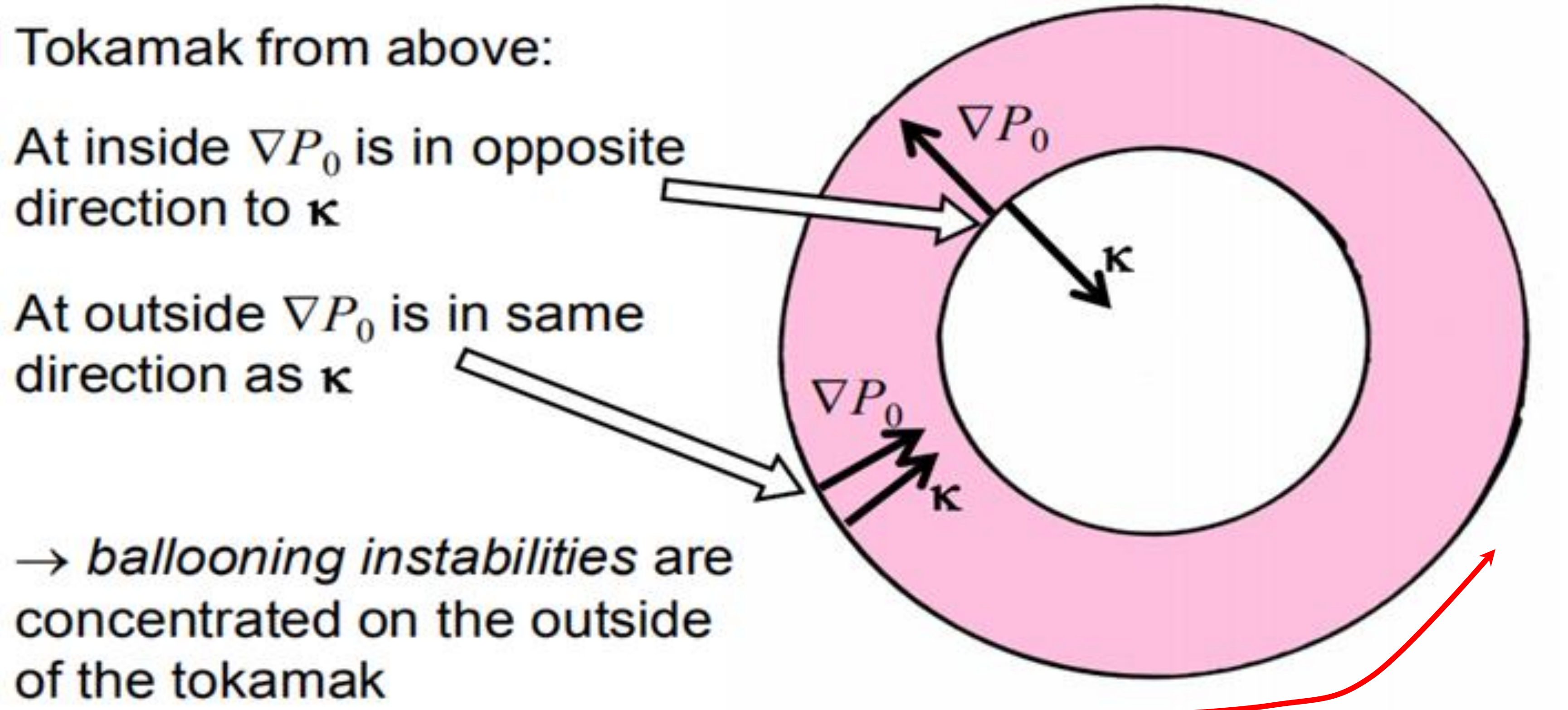
Picture of current driven instability

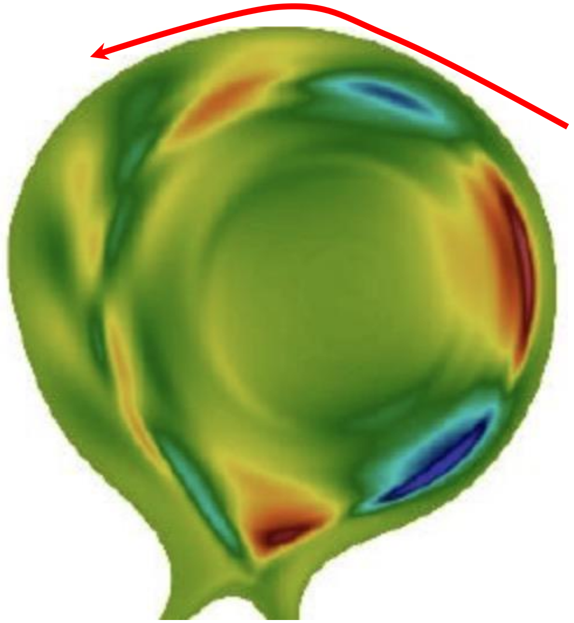


- straight wire carrying a current $\mathbf{I}_0 = I_0 \mathbf{e}_z$
- is placed in a uniform magnetic field parallel to the direction of the wire, $\mathbf{B} = B\mathbf{e}_z$
- is deformed helically: $\mathbf{x}(z) = \xi_r \cos(kz)\,\mathbf{e}_x + \xi_r \sin(kz)\,\mathbf{e}_y + z\mathbf{e}_z$
- current $\mathbf{I}$ flowing in the twisted wire: ...
- Lorentz force $\mathbf{I} \times \mathbf{B}$ acting on twisted wire: ...
- equation of motion $\rightarrow$ growth rate ...

Bad& good curvature of field line

$$-2(\nabla p \cdot \boldsymbol{\xi}_\perp)(\boldsymbol{\kappa} \cdot \boldsymbol{\xi}_\perp^*)$$





Toroidal direction

Averaged is good, to avoid interchange mode. There is a regions, in which the curvature is bad, allows the occurrence of BM.

Energy principle

Expression of δW → *Minimization* of δW → resolve E − L equation → obtain the displacment for minimum of δW → evaluate the sign of δW → stability analysis

$$\omega^2 = \frac{\delta W}{\delta \hat{K}}$$

Energy Principle:

If $\delta W(\xi^*, \xi) \geq 0$ for **all allowable** ξ, then the system is **stable**
If $\delta W(\xi^*, \xi) < 0$ for **any** ξ, then the system is **unstable**

↓

To find the minimum $\delta W(\xi^*, \xi)$ for all allowable displacements

How to minimize the perturbed energy: **Variation principle!**

Variation principle

$$\delta S = \delta \int_{x_1}^{x_2} L(g, g', x)\mathrm{d}x = 0.$$

$$\delta g\prime = \frac{d}{dx}\delta g$$

$$\delta S = \int_{x_1}^{x_2} \left(\frac{\partial L}{\partial g}\delta g + \frac{\partial L}{\partial g'}\delta g'\right)\mathrm{d}x = 0,$$

$$\delta S = \frac{\partial L}{\partial g'}\delta g(x)\Big|_{x_1}^{x_2} + \int_{x_1}^{x_2} \left(\frac{\partial L}{\partial g} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial L}{\partial g'}\right)\delta g \mathrm{d}x = 0.$$

Boundary condition: $\delta(x_1) = \delta(x_2) = 0$

$$\int_{x_1}^{x_2} \left(\frac{\partial L}{\partial g} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial L}{\partial g'}\right)\delta g \mathrm{d}x = 0.$$

Euler-Lagrange equation:

$$\frac{\partial L}{\partial g} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial L}{\partial g'} = 0,$$

Outline

1. Introduction
2. Physics and control of Resistive Wall Mode (RWM)
    - External kink
    - RWM dispersion relation
    - RWM control
    - RWM stability with kinetic damping

$$\boldsymbol{Q} \propto nT\tau_E \propto \beta_N H B^3 a^3 / q_{95}^2$$

RWM ELM

3. Physics and control of Edge Localized Mode(ELM)
4. Summary and outlook

Perturbed potential energy in whole region

$$\delta W = \underbrace{\delta W_F}_{fluid\ energy} + \underbrace{\delta W_S}_{surface\ energy} + \underbrace{\delta W_V}_{vacuum\ energy}$$

The fluid energy

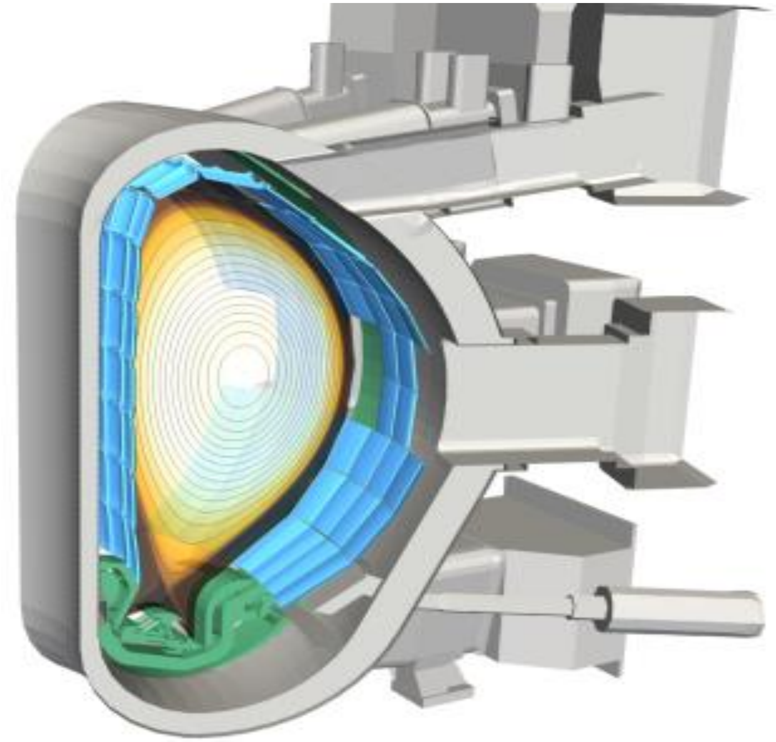
$$\boxed{\begin{aligned}\delta W_F(\xi^*,\xi) = \frac{1}{2\mu_0}\int_P \Big\{ & |\mathbf{Q}_\perp|^2 + B^2|\nabla\cdot\xi_\perp + 2\xi_\perp\cdot\boldsymbol{\kappa}|^2 + \mu_0\gamma p|\nabla\cdot\xi|^2 \\ & -\mu_0[(\xi_\perp\cdot\nabla p)(\xi_\perp^*\cdot\boldsymbol{\kappa}) + (\xi_\perp^*\cdot\nabla p)(\xi_\perp\cdot\boldsymbol{\kappa})] \\ & -(\mu_0 J_\parallel/2)[\xi_\perp^*\times\mathbf{b}\cdot\mathbf{Q}_\perp + \xi_\perp\times\mathbf{b}\cdot\mathbf{Q}_\perp^*]\Big\} d\mathbf{r}\end{aligned}}$$

The surface energy

$$\boxed{\delta W_S(\xi_\perp^*,\xi_\perp) = \frac{1}{2\mu_0}\int_{S_p} |\mathbf{n}\cdot\xi_\perp|^2 \mathbf{n}\cdot \left[\!\left[\nabla\left(\frac{B^2}{2}+\mu_0 p\right)\right]\!\right] dS}$$

The vacuum energy

$$\boxed{\delta W_V(\xi_\perp^*,\xi_\perp) = \frac{1}{2\mu_0}\int_V |\hat{\mathbf{B}}_1|^2 d\mathbf{r}}$$



δW_S can be either positive or negative， vanishes if there is no surface current.

Minimization of δW

$$\delta W_F(\xi^*,\xi) = \frac{1}{2\mu_0}\int_P \Big\{|\mathbf{Q}_\perp|^2 + B^2|\nabla\cdot\boldsymbol{\xi}_\perp + 2\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa}|^2 + \mu_0\gamma p|\nabla\cdot\boldsymbol{\xi}|^2$$
$$-\mu_0[(\boldsymbol{\xi}_\perp\cdot\nabla p)(\boldsymbol{\xi}_\perp^*\cdot\boldsymbol{\kappa}) + (\boldsymbol{\xi}_\perp^*\cdot\nabla p)(\boldsymbol{\xi}_\perp\cdot\boldsymbol{\kappa})]$$
$$-(\mu_0 J_\parallel/2)[\boldsymbol{\xi}_\perp^*\times\mathbf{b}\cdot\mathbf{Q}_\perp + \boldsymbol{\xi}_\perp\times\mathbf{b}\cdot\mathbf{Q}_\perp^*]\Big\}d\mathbf{r}$$

$$\boldsymbol{\xi} = \xi_\parallel\mathbf{b} + \boldsymbol{\xi}_\perp = \xi_\parallel\mathbf{b} + \xi_\eta\mathbf{e}_\eta + \xi\mathbf{e_r}$$

Step1: minimizing $\xi_\parallel$, $\quad \delta W(\xi,\xi_\eta,\xi_\parallel)\rightarrow\delta W(\xi,\xi_\eta)$

$$\frac{\delta W_C}{2\pi R_0} = \pi\int \gamma p\left|\nabla\cdot\xi\right|^2 r dr \quad \longrightarrow 0$$
$$\nabla\cdot\xi = 0$$
$$\xi_\parallel = i\frac{B}{F}\nabla\cdot\xi_\perp$$

Step2: minimizing ξ_η, $\quad \delta W(\xi,\xi_\eta)\rightarrow\delta W(\xi)$

$$W_\eta(r) = k_0^2B^2\left|\eta\right|^2 + 2\frac{kBB_\theta}{r}(i\eta\xi^* - i\eta^*\xi) + \frac{GB}{r}\left[i\eta\left(r\xi^*\right)' - i\eta^*(r\xi)'\right]$$
$$= \boxed{\left|ik_0B\eta + 2\frac{kB_\theta}{rk_0}\xi + \frac{G}{rk_0}(r\xi)'\right|^2} - \left|2\frac{kB_\theta}{rk_0}\xi + \frac{G}{rk_0}(r\xi)'\right|^2$$

$$\boxed{\eta = \frac{i}{rk_0^2B}\left[2kB_\theta\xi + G(r\xi)'\right]}$$

Step3: minimizing ξ

Ideal wall case

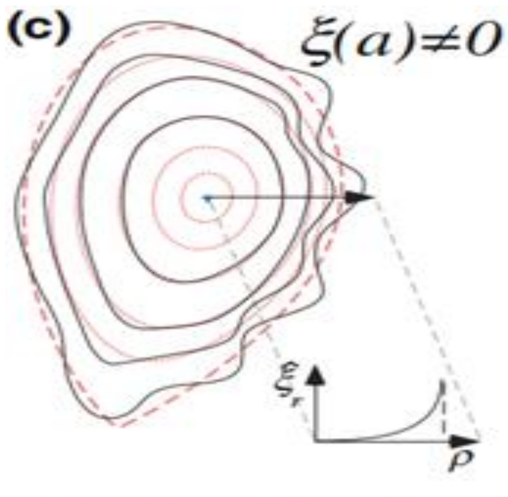
$$\delta W(\xi^*,\xi) = \delta W_F + \delta W_S + \delta W_V$$

Assume no surface current δW_S =0
Second order of fluid energy:

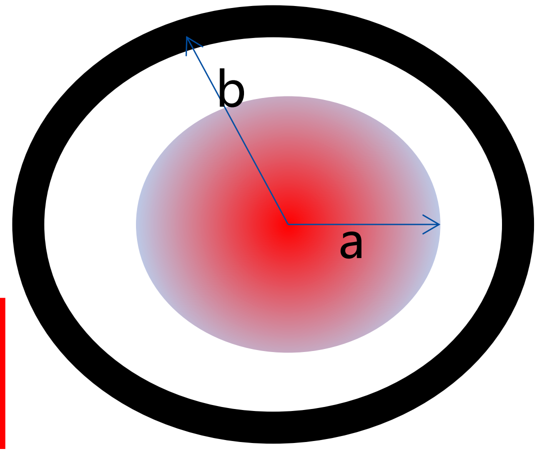
$$\delta \hat{W}_2 = \int_0^a \left(\frac{n}{m}-\frac{1}{q}\right)^2 \left[r^2\xi'^2 + (m^2-1)\,\xi^2\right] r dr + \left(\frac{n}{m}-\frac{1}{q_a}\right)\left[\left(\frac{n}{m}+\frac{1}{q}_a\right) + m\Lambda\left(\frac{n}{m}-\frac{1}{q_a}\right)\right] a^2\xi_a^2$$

$$\Lambda \simeq \frac{1+(a/b)^{2m}}{1-(a/b)^{2m}}$$:from vacuum region

Minimize $\delta \hat{W}_2$ $\rightarrow$ $\boxed{\frac{d}{dr}\left[r^3\left(\frac{n}{m}-\frac{1}{q}\right)^2 \frac{d\xi}{dr}\right] - (m^2-1)\, r\left(\frac{n}{m}-\frac{1}{q}\right)^2 \xi = 0}$



External kink mode



Ideal wall

[等离子体理论基础，胡希伟]

External kink (ideal wall)

$$\boxed{\frac{d}{dr}\left[r^3\left(\frac{n}{m}-\frac{1}{q}\right)^2\frac{d\xi}{dr}\right]-(m^2-1)\,r\left(\frac{n}{m}-\frac{1}{q}\right)^2\xi=0}$$

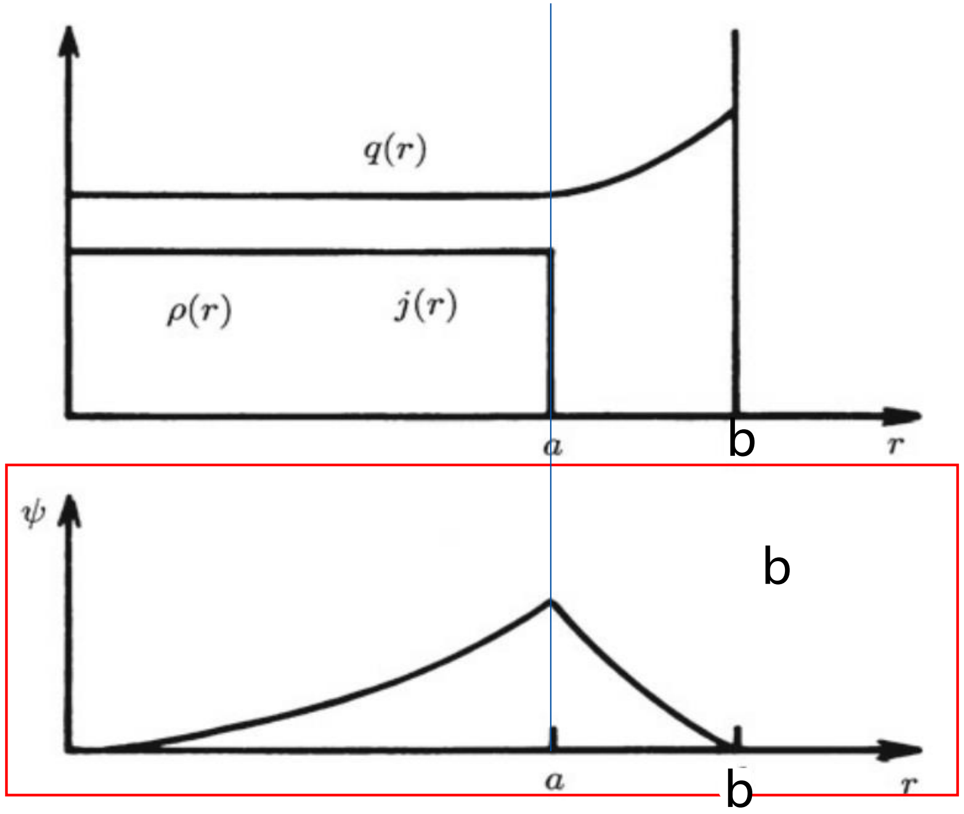
$$\frac{d}{dr}\left[r^3\frac{d\xi}{dr}\right]-(m^2-1)\,r\xi=0$$

Flat current

$$q_0=q_a=q(r)=rB_0/R_0B_\theta(r)$$

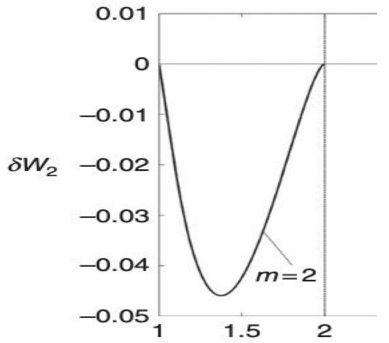
$$\boxed{\xi(r)=\xi_a\left(\frac{r}{a}\right)^{m-1}}$$

$$\frac{a\xi_a'}{\xi_a}=m-1$$



$$\boxed{\delta\hat{W}_2=\frac{2}{mq_a^2}(nq_a-m)(nq_a-m+1)}$$

Unstable condition (current driven): $m-1<nq_a<m$



P503_ideal_MHD

External kink-3 (with vacuum contributions)

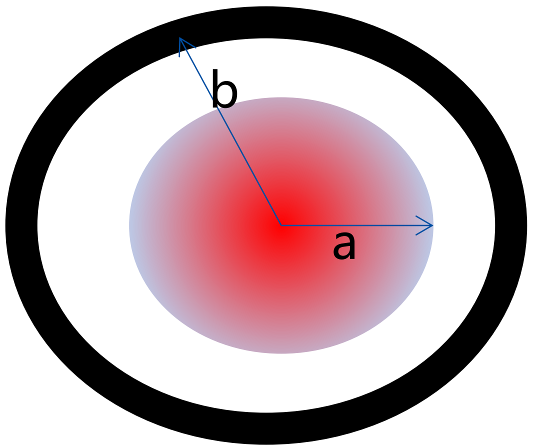
Vacuum term

$$\frac{\delta W_\infty}{2\pi^2 R_0/\mu_0} = \left(\frac{F^2}{k_0^2}\frac{r\xi'}{\xi} + \frac{FF^\dagger}{k_0^2} + \frac{r^2F^2\Lambda_\infty}{m} \right)_a \xi^2(a)$$

$$\frac{\delta W_b}{2\pi^2 R_0/\mu_0} = \left(\frac{F^2}{k_0^2}\frac{r\xi'}{\xi} + \frac{FF^\dagger}{k_0^2} + \frac{r^2F^2\Lambda_b}{m} \right)_a \xi^2(a)$$

$$\Lambda_\infty = -\frac{mK_a}{|ka|K_a'} \approx 1$$

$$\Lambda_b = \Lambda_\infty \left[\frac{1 - (K_b' I_a)/(I_b' K_a)}{1 - (K_b' I_a')/(I_b' K_a')} \right] \approx \frac{1 + (a/b)^{2m}}{1 - (a/b)^{2m}}$$



$$F^\dagger = kB_z - mB_\theta/r,$$

$$F(r) = \mathbf{k}\cdot\mathbf{B} = kB_z + mB_\theta/r$$

The approximate formulae are valid when $kb \sim ka \ll 1$. Clearly $\Lambda_b > \Lambda_\infty$. The interesting physical regime occurs when

$$\delta W_\infty < 0 < \delta W_b \qquad (11.151)$$

That is, without a wall the plasma is unstable to an external MHD mode while with a wall the mode is stabilized.

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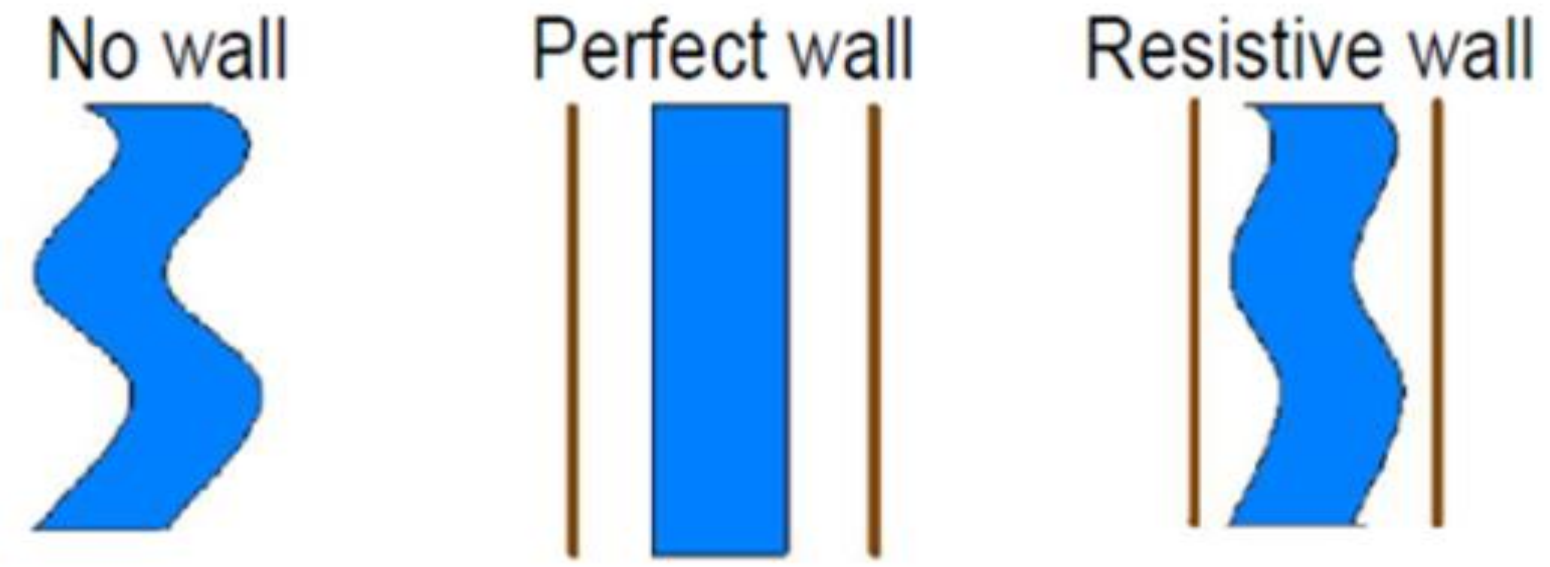
Outline

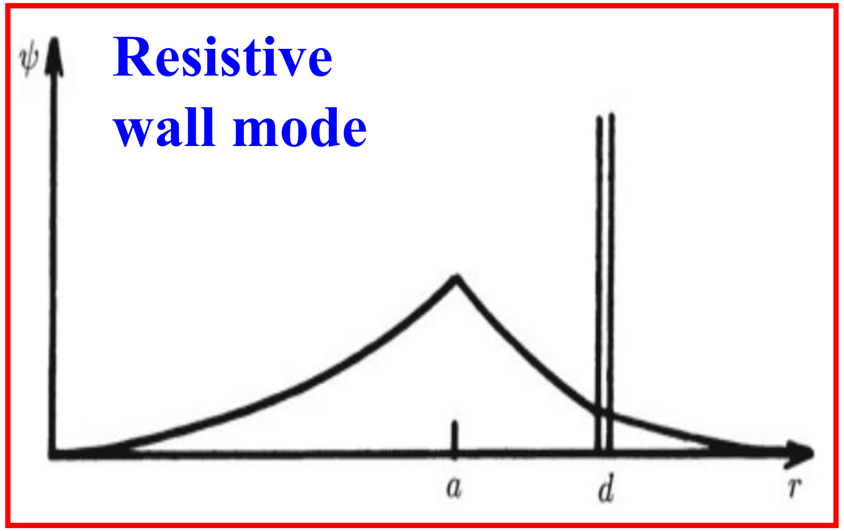
Resistive wall converts EK to RWM

$$\delta W_\infty < 0 < \delta W_b$$



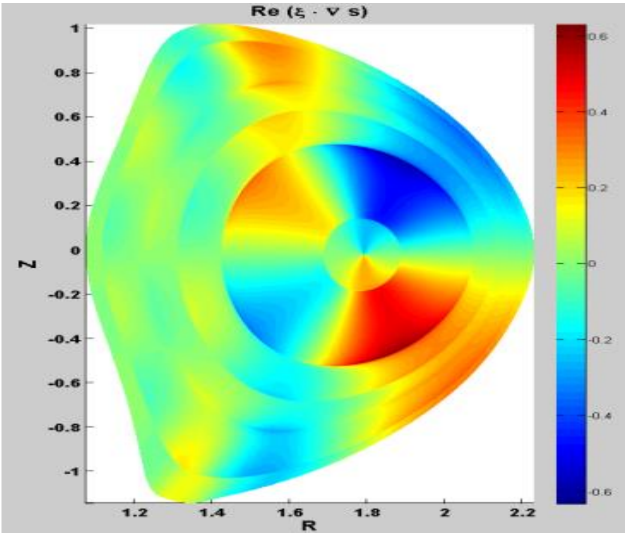
External kink ~us ➡ RWM ~ ms

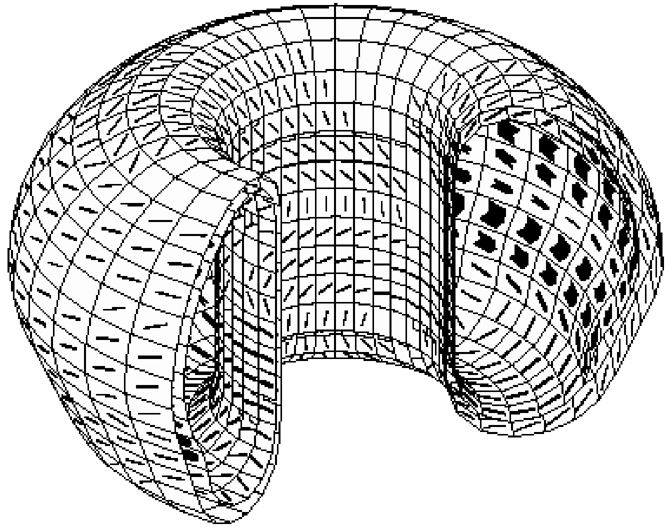
Resistive wall mode



b

RWM in DIII-D





MARS-F result

RWM:

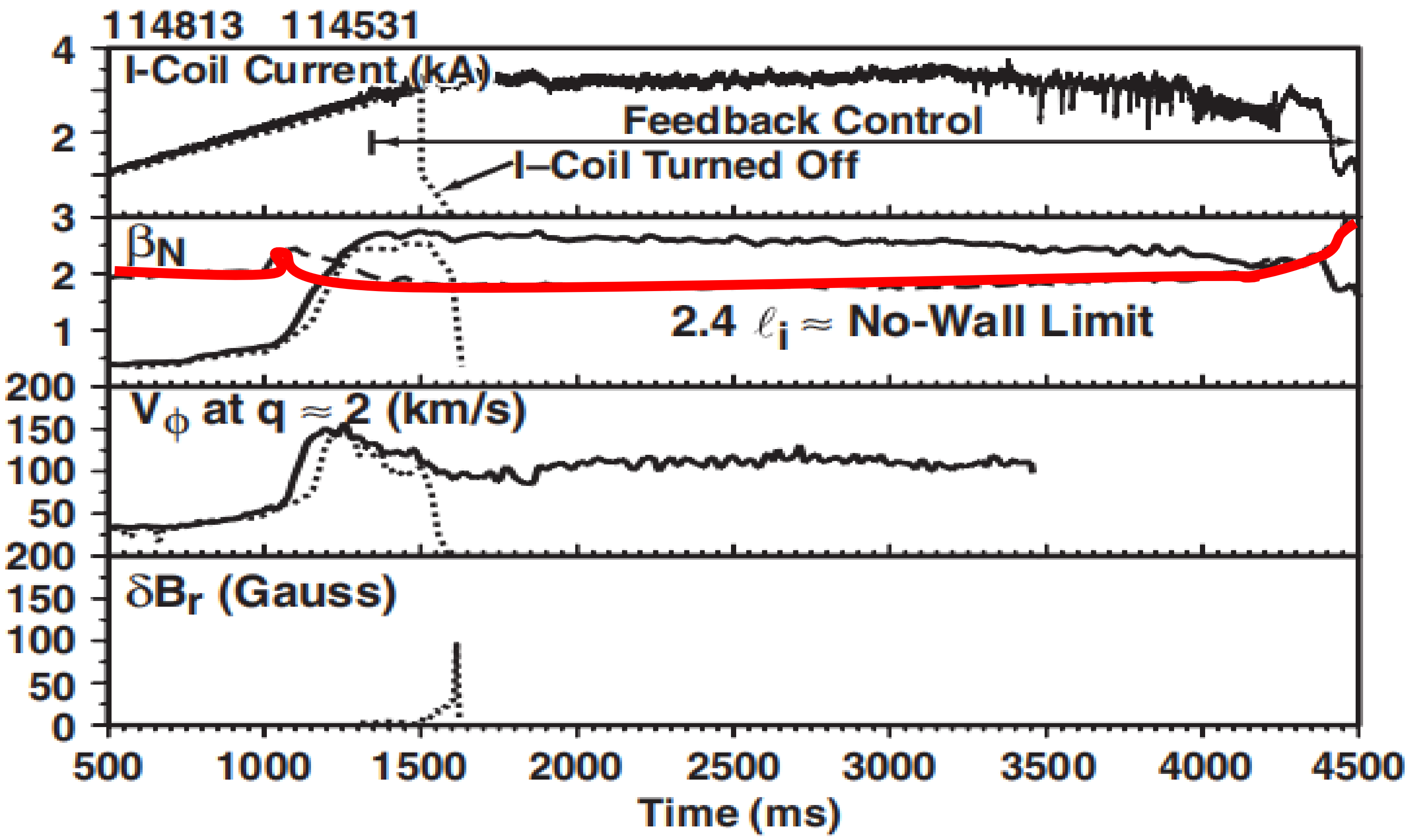
① Global MHD

② Grow time scale ~ ms (allow feedback control)

③ Real frequency < 100Hz

→Induce major disrution as beta_N > beta_N_no_wall_limit (*nT* limit)

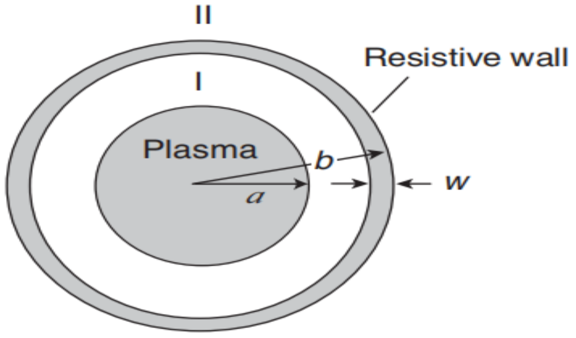
RWM induces the major disruption



[Strait 2004 Phys. Plasmas 11 2505]

RWM dispersion relation (I)

(I) $$\frac{d}{dr}\left[r^3\left(\frac{n}{m}-\frac{1}{q}\right)^2\frac{d\xi}{dr}\right]-(m^2-1)\,r\left(\frac{n}{m}-\frac{1}{q}\right)^2\xi=0$$ (inside plasma)



(II)
$$\hat{\mathbf{B}}_I=\nabla V_I$$
(because no current in vacuum region)
$$\hat{\mathbf{B}}_{II}=\nabla V_{II}$$
$$V_I=(A_0K_\rho+A_1I_\rho)\exp[\omega_i t+i(m\theta+kz)]$$ (in vacuum)
$$V_{II}=A_2K_\rho\exp[\omega_i t+i(m\theta+kz)]$$

$$\hat{B}_{wr}=\hat{B}_{wr}(x)\exp[\omega_i t+i(m\theta+kz)].$$

$$\omega_i\sim\eta/\mu_0 bw$$

$$\omega_i\,\mu_0/\eta\quad\sim\quad 1/bw$$

$$\frac{\partial^2\hat{B}_{r0}}{\partial x^2}=0$$

$$\frac{\partial^2\hat{B}_{r1}}{\partial x^2}=\frac{\mu_0\omega_i}{\eta}\hat{B}_{r0}$$

Magnetic diffusion equation for the resistive wall:

Solution of 1st order eq.

(III) $$\frac{\partial\hat{\mathbf{B}}_w}{\partial t}=\frac{\eta}{\mu_0}\nabla^2\hat{\mathbf{B}}_w$$ (across the thin wall)

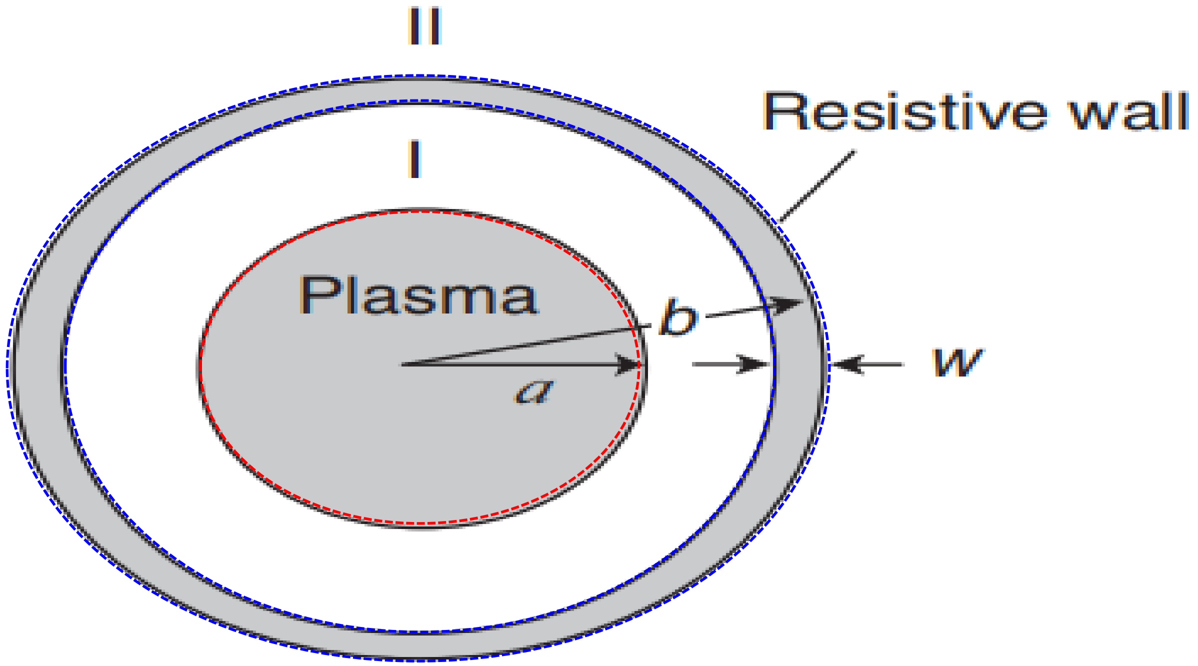
Thin wall assumption: $\delta=w/b\ll 1$ $\Rightarrow$

$$\hat{B}_{wr}\approx\hat{B}_{r0}+\hat{B}_{r1}=B_{r0}\left(1+\boxed{\frac{\mu_0\omega_i}{2\eta}x^2}\right)+\boxed{B_{r1}\left(\frac{x}{w}\right)}$$
$$\hat{B}_{r1}/\hat{B}_{r0}\sim\delta$$
(1)

RWM dispersion relation (II)

The solutions in all three regions have now been specified in terms of five unknown constants A_0, A_1, A_2, B_{r0}, B_{r1}. These constants are determined by appropriate matching conditions.

$$A_0,\quad A_1,\quad A_2,\quad B_{r0},\quad B_{r1}\quad ?$$



Match of the normal fields at two wall surfaces (A)

Left: from vacuum eq; right: from wall eq

$$\left.\frac{\partial \hat{V}_I}{\partial r}\right|_{b^-} = \left.\hat{B}_{wr}\right|_{x=0} \qquad \left.\frac{\partial \hat{V}_{II}}{\partial r}\right|_{b^+} = \left.\hat{B}_{wr}\right|_{x=w}$$

Match of the tangential fields at two wall surfaces (B)

$$\nabla\cdot\hat{\mathbf{B}}_{wr} = 0$$

$$\left.\hat{V}_I\right|_{b^-} = \left.\frac{1}{k_b^2}\frac{\partial \hat{B}_{wr}}{\partial x}\right|_{x=0} \qquad \left.\hat{V}_{II}\right|_{b^+} = \left.\frac{1}{k_b^2}\frac{\partial \hat{B}_{wr}}{\partial x}\right|_{x=w}$$

Cancel : B_{r0}, B_{r1}

1) (A) $\longrightarrow$ $$\left.\frac{\partial \hat{V}_I}{\partial r}\right|_{b^-} = \left.\frac{\partial \hat{V}_{II}}{\partial r}\right|_{b^+} \qquad \sim \delta^0$$

1) (B) $\longrightarrow$ $$\left.\hat{V}_I\right|_{b^-} = \left.\hat{V}_{II}\right|_{b^+} - \left(\frac{\mu_0\omega_i w}{\eta k_b^2}\right)\left.\frac{\partial \hat{V}_{II}}{\partial r}\right|_{b^+} \qquad \sim \delta^1$$

$$\boxed{\begin{aligned} A_0K_b' + A_1I_b' - A_2K_b' &= 0 \\ A_0K_b + A_1I_b - A_2\left(K_b - \frac{\mu_0\omega_i w|k|}{\eta k_0^2}K_b'\right) &= 0 \end{aligned}}$$

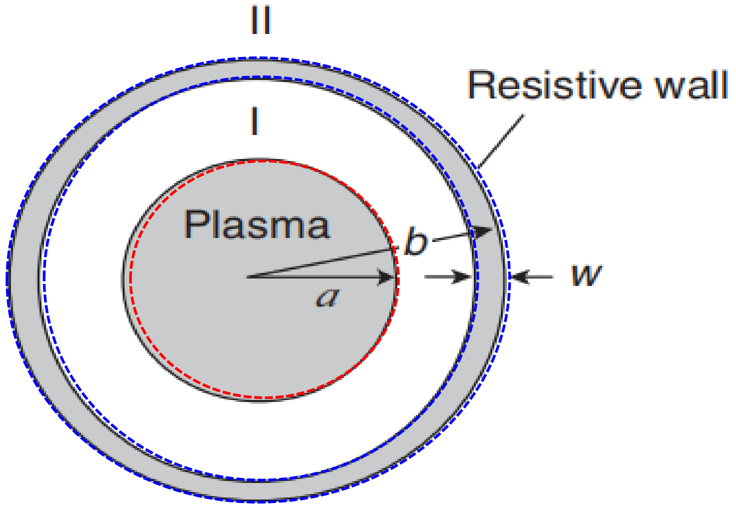
Obtain A_1, A_2 as function of A_0

RWM dispersion relation (III)

$$\Delta A_1 = -\nu K_b' A_0$$
$$\Delta A_2 = \left(I_b K_b' - K_b I_b'\right) A_0$$

with

$$\nu = \left(\mu_0 |k| w / \eta k_b^2\right) \omega_i$$
$$\Delta = I_b K_b' - K_b I_b' + \nu K_b' I_b'$$



$$\begin{aligned} V_1 &= (A_0 K_r + A_1 I_r) \\ &= A_0 (K_r + \frac{A_1}{A_0} I_r) \\ &= A_0 (K_r - \frac{\nu K_b'^2}{\Delta} I_r) \end{aligned}$$

Boundary condition at plasma boundary:
continuous of normal field perturbation

$$B_1 = \nabla V_1 = A_0 |k| (K_a' - \frac{\nu K_b'^2}{\Delta} I_a')$$

$$+$$

$$e_r \cdot Q|_a = iF(a)\xi(a)$$

$$\boxed{A_0 = \frac{iF(a)\xi(a)}{|k|(K_a' - \frac{\nu K_b'^2}{\Delta} I_a')}}$$

$$\mathbf{Q} = \nabla \times (\xi_\perp \times \mathbf{B})$$

[1] : Ideal MHD, J.Freidberg

RWM dispersion relation (IV)

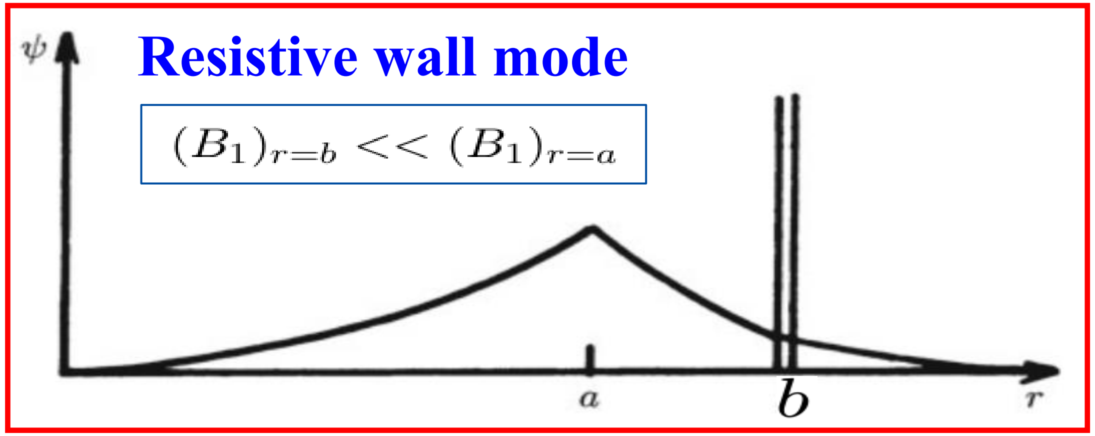
$$\delta W_v \equiv \frac{1}{2\mu}\int_v |\boldsymbol{B}_1|^2 d\boldsymbol{r}$$

$$= \frac{1}{2\mu}\int_v \nabla\cdot(V_1^*\nabla V_1)d\boldsymbol{r} = -\frac{1}{2\mu}\int_S V_1^*(\boldsymbol{n}\cdot\boldsymbol{\nabla}V_1)dS$$

$$= \frac{2\pi^2 R_0 a}{\mu}\left[\boxed{-\left(V_1^*\frac{\partial V_1}{\partial r}\right)_a} + \cancel{\left(V_1^*\frac{\partial V_1}{\partial r}\right)_b - \left(V_{II}^*\frac{\partial V_{II}}{\partial r}\right)_b}\right]$$

$$= \nu\left(\frac{\partial V_I}{\partial r}\right)^2$$

$$= -\left(\frac{F\xi}{|k|}\right)^2\frac{\Delta K_a - \nu K_b'^2 I_a}{\Delta K_a' - \nu K_b'^2 I_a'}$$



RWM dispersion relation (V)

$$\delta W_v = \frac{\left(1 + \boxed{\frac{\nu(K_b' I_b' - K_b'^2 I_a/K_a)}{I_b K_b' - K_b I_b'}}\right)}{1 - \tau_w} \frac{K_a}{K_a'}$$

$$\begin{aligned}
\boxed{1 + \frac{\nu(K_b' I_b' - K_b'^2 I_a/K_a)}{I_b K_b' - K_b I_b'}} &= 1 + \frac{\nu K_b'}{I_b K_b' - K_b I_b'}(I_b' - K_b' I_a/K_a) \\
&= 1 + \frac{\nu K_b'(I_b' - I_a' K_b'/K_a')}{I_b K_b' - K_b I_b'} \frac{I_b' K_a - K_b' I_a}{I_b' K_a' - K_b' I_a'} \frac{K_a'}{K_a} \\
&= 1 - \frac{\nu K_b'(I_b' - I_a' K_b'/K_a')}{I_b' K_b - K_b' I_b} \frac{I_b' K_a - K_b' I_a}{I_b' K_a' - K_b' I_a'} \frac{K_a'}{K_a} \\
&= 1 - \tau_w \frac{\wedge_b}{\wedge_\infty}
\end{aligned}$$

$$\boxed{\delta W_v = \frac{\wedge_\infty - \tau_w \wedge_b}{1 - \tau_w}}$$

$$\wedge_\infty = -\frac{m K_a}{|ka| K_a'}$$

$$\wedge_b = \wedge_\infty \left[\frac{1 - K_b' I_a / I_b' K_a}{1 - K_b' I_a' / I_b' K_a'}\right]$$

RWM dispersion relation (VI)

$$\omega^2 = \frac{\delta W}{\delta \hat{K}}$$

$$\delta K + \delta W = 0$$

$$\delta K \equiv -\omega^2 \delta \hat{K}$$

$$\wedge_\infty \equiv \delta W_\infty \text{ and } \wedge_b \equiv \delta W_b$$

$$\delta K + \delta W_F + \frac{\delta W_v^\infty + \tau_w \delta W_v^b}{1+\tau_w} = 0$$

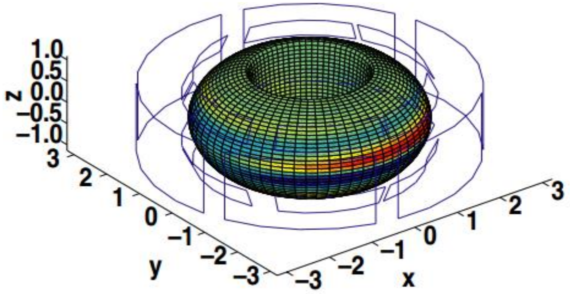
A very useful dispersion relation, valid in toroidal geometry, has been derived by several authors [Haney PF B1 1637(1989), Chu PoP 2 2236 (1995)]

$$\delta K + \delta W_A + \delta W_K + \delta W_{FB} + \delta W_D + \frac{\delta W_v^\infty + \tau_w \delta W_v^b}{1+\tau_w} = 0$$

continuum damping

$$p_\parallel^{na} = \sum_j \int M_j v_\parallel^2 f_{j,na}^1(\xi_\perp, \gamma)\, dV$$

$$p_\perp^{na} = j \frac{1}{2} M_j v_\perp^2 f_{j,na}^1(\xi_\perp, \gamma)\, dV$$

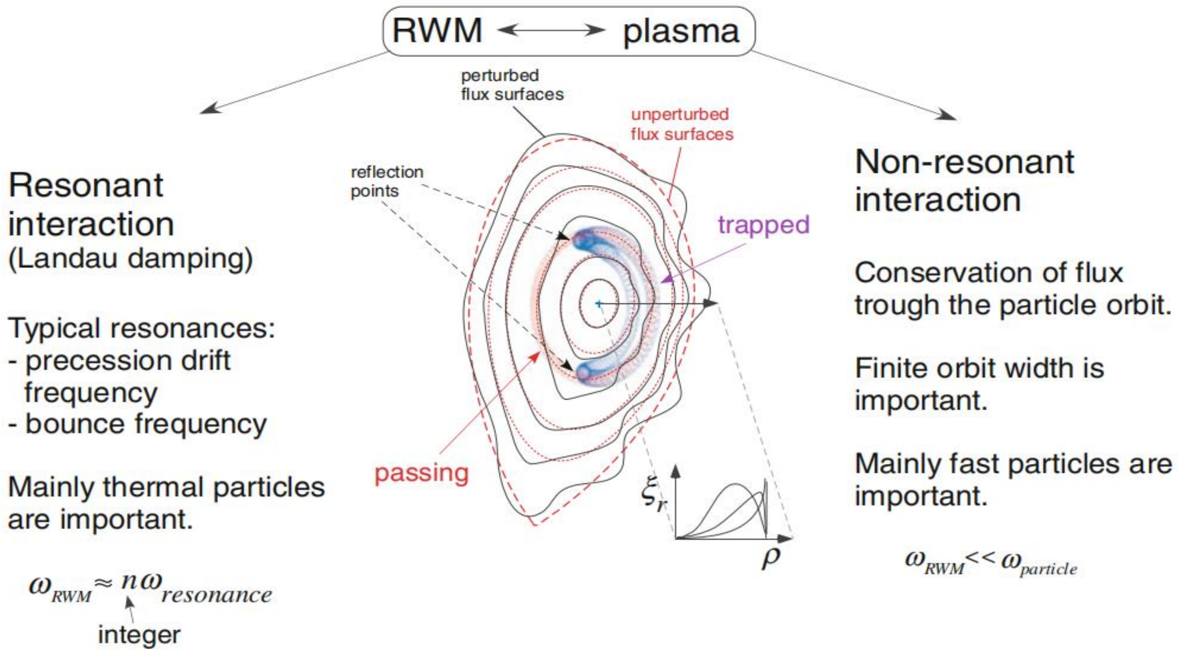


$$\vec{\Pi} = -3\eta_0 \left(\mathbf{bb} - \tfrac{1}{3}\mathbf{I}\right)\left[\mathbf{b}\cdot\nabla(\mathbf{V}\cdot\mathbf{b}) - \mathbf{V}\cdot(\mathbf{b}\cdot\nabla\mathbf{b})\right]$$

Outline

How to control RWM

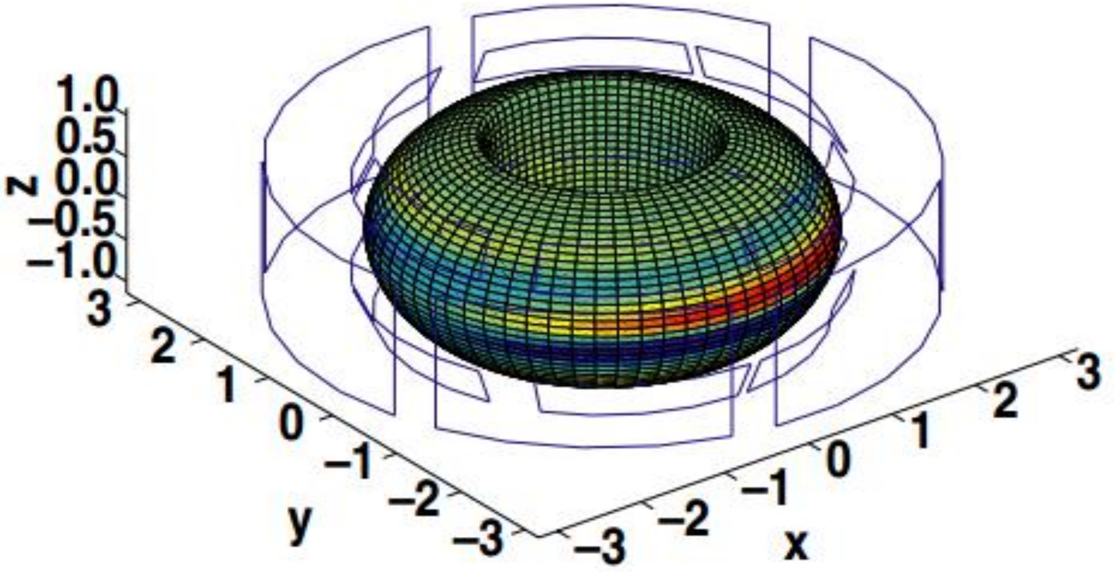
1. Passive stabilization



❑Understanding damping physics of the mode

- Continuum damping [Zheng PRL 95 255003(2005), Betti PRL 74 2949(2005)]
- Parallel damping [Chu PoP 2 2236(1995),Bondeson PRL 72 2709(1994)]
- Kinetic damping [Hu PRL 93 105002(2004), Hao PRL ,107,015001,(2011),Bondeson PoP 3 3013(1996), Liu NF 45 1131(2005)]
- Damping from plasma inertial and/or dissipation layers [Finn PoP 2 3782(1995), Gimblett PoP 7 258(2000), Fitzpatrick NF 36 11(1996)]

2. Active control



❑Choice of sensor signals (pick-up coils) (**y**) [Liu NF 47 648 (2007),Liu PRL 84, 907, 2000]

Damping physics for RWM

➢ Continuum damping

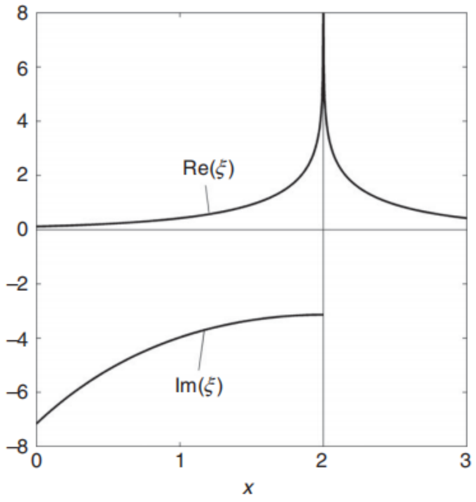
$$\frac{d^2\xi}{ds^2}+\frac{1}{s}\frac{d\xi}{ds}-\xi=0 \quad s=kx-kx_0 \text{ with } x_0=2$$

$$s\ge 0_+ \qquad \xi(s)=-\xi_0(\ln s+C)$$

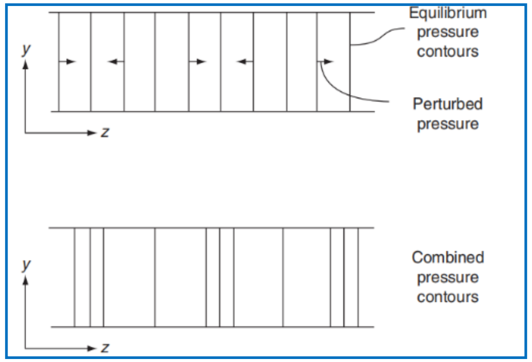
$$s\le 0_- \qquad \xi(s)=-\xi_0(\ln s+C)=-\xi_0\{\ln[-(-s)]+C\}=-\xi_0[\ln(-e^{i\pi}s)+C]$$

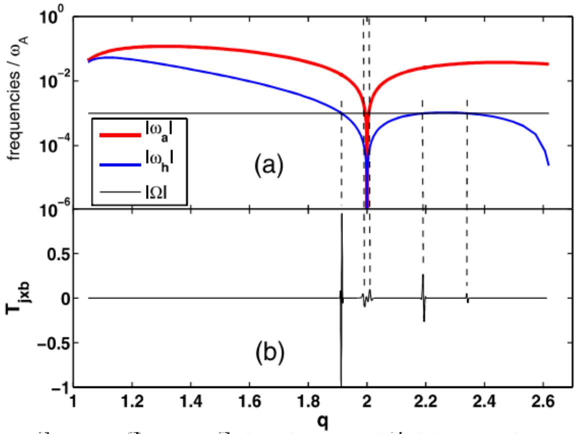
$$=-\xi_0[\ln(-s)+C+i\pi]$$

P438,Ideal_MHD



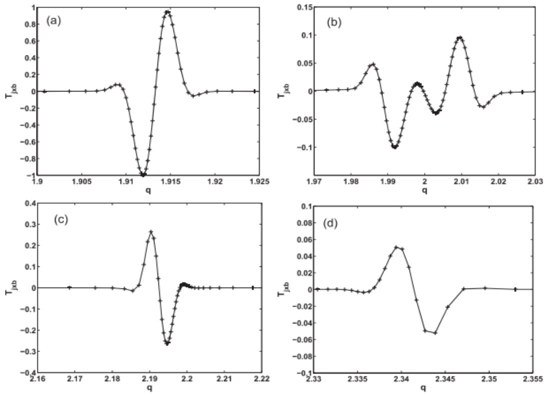
$$\boxed{[\![\xi(s)]\!]_{0_-}^{0_+}=i\pi\xi_0}$$





$$\Omega^2=\omega_a^2\equiv\omega_A^2(m/q-n)^2/(\hat{\rho}F_{PS}),$$

$$\Omega^2=\omega_h^2\equiv\omega_a^2V_s^2/(V_s^2+V_A^2/F_{PS})$$



[Liu POP,19,102507,(2012)]

- when a perturbation does not rotate together with the plasma (i.e. is rotating in the plasma frame, the continuum resonance occurs, such as for RWM.

Damping physics for RWM

➢Parallel damping

$$(\gamma + in\Omega)\xi = \mathbf{v} + (\xi \cdot \nabla\Omega)R^2\nabla\phi$$

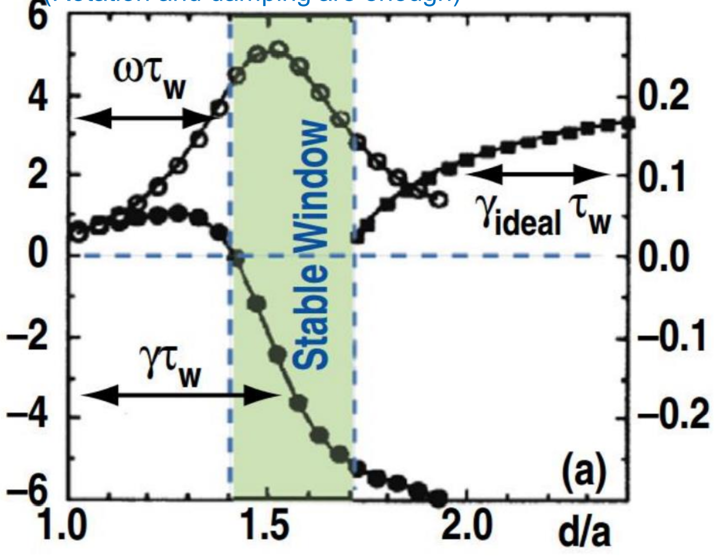
$$\rho(\gamma + in\Omega)\mathbf{v} = -\nabla p + \boldsymbol{j} \times \mathbf{B} + \mathbf{J} \times \boldsymbol{Q} + \rho[2\Omega\hat{\mathbf{Z}} \times \mathbf{v} - (\mathbf{v} \cdot \nabla\Omega)R^2\nabla\phi]$$
$$-\nabla \cdot (\rho\xi)\Omega Z \times \mathbf{V_0} - \rho\kappa_{\parallel}|k_{\parallel}|v_{th,j}[\mathrm{v} + (\xi \cdot \nabla)\mathbf{V_0}]_{\parallel}$$

$$(\gamma + in\Omega)\boldsymbol{Q} = \nabla \times (\mathbf{v} \times \boldsymbol{B}) + (\boldsymbol{Q} \cdot \nabla\Omega)R^2\nabla\phi$$

$$(\gamma + in\Omega)p = -\mathbf{v} \cdot \nabla P - \Gamma P\nabla \cdot \mathbf{v}$$

$$\mu_0\boldsymbol{j} = \nabla \times \boldsymbol{Q}$$

stable window in term of wall position for RWM
(Rotation and damping are enough)



Bondeson PRL 72 2709(1994)

➢Kinetic damping

$$(\tilde{\gamma} + in\Omega)\boldsymbol{\xi} = \mathbf{v_1} + (\boldsymbol{\xi} \cdot \nabla\Omega)R^2\nabla\phi$$

$$\rho(\tilde{\gamma} + in\Omega)\mathbf{v_1} = -\nabla \cdot \mathbf{p} + \mathbf{J_1} \times \mathbf{B_0} + \mathbf{J_0} \times \mathbf{B_1}$$
$$-\rho[2\Omega\hat{\mathbf{Z}} \times \mathbf{v_1} + (\mathbf{v_1} \cdot \nabla\Omega)R^2\nabla\phi] - \nabla \cdot \mathbf{\Pi}$$

$$(\tilde{\gamma} + in\Omega)\mathbf{B_1} = \nabla \times (\mathbf{v_1} \times \mathbf{B_0} - \eta\mathbf{J_1}) + (\mathbf{B_1} \cdot \nabla\Omega)R^2\nabla\phi$$

$$\mu_0\mathbf{J_1} = \nabla \times \mathbf{B_1}$$

$$\mathbf{p} = p\mathbf{I} + p_{\parallel}\hat{\mathbf{b}}\hat{\mathbf{b}} + \alpha_k p_{\perp}(\mathbf{I} - \hat{\mathbf{b}}\hat{\mathbf{b}})$$

$$p_{\parallel}e^{-i\omega t + in\phi} = \sum_j \int d\Gamma \frac{1}{2} M_j v_{//}^2 f_{j,na}^1$$

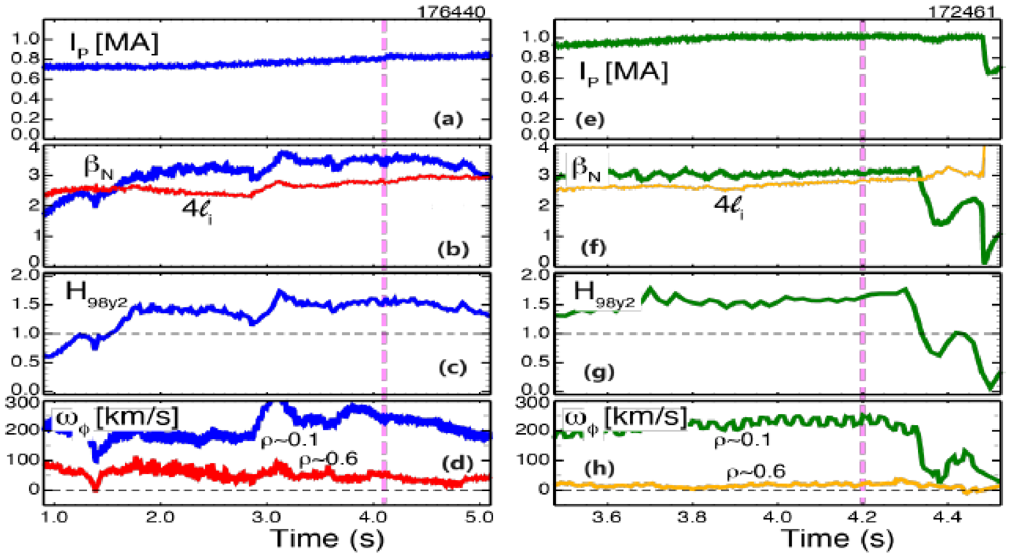
$$p_{\perp}e^{-i\omega t + in\phi} = \sum_j \int d\Gamma \frac{1}{2} M_j v_{\perp}^2 f_{j,na}^1$$

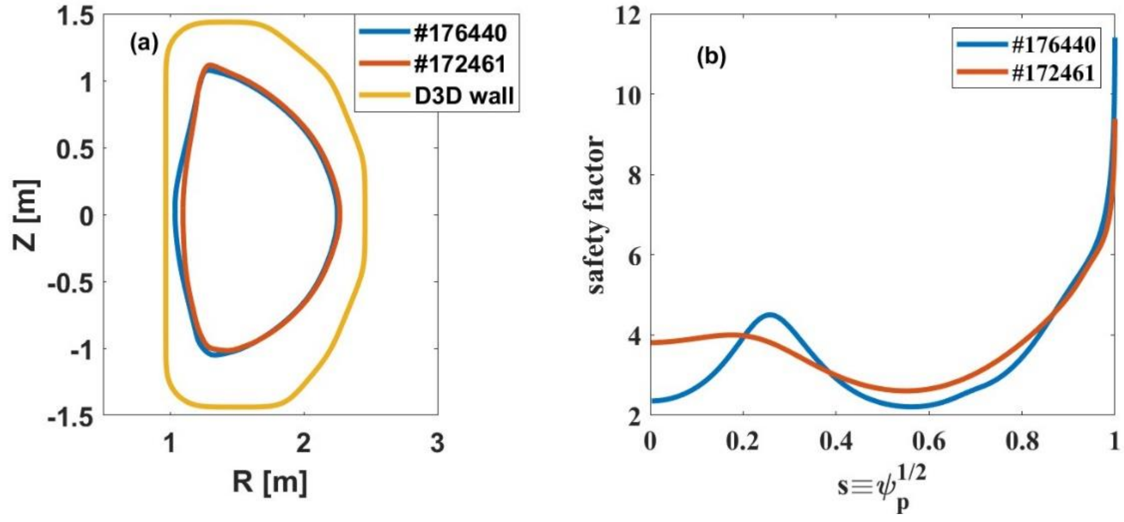
$$p = -(\mathbf{v_1} \cdot \nabla)p_0$$

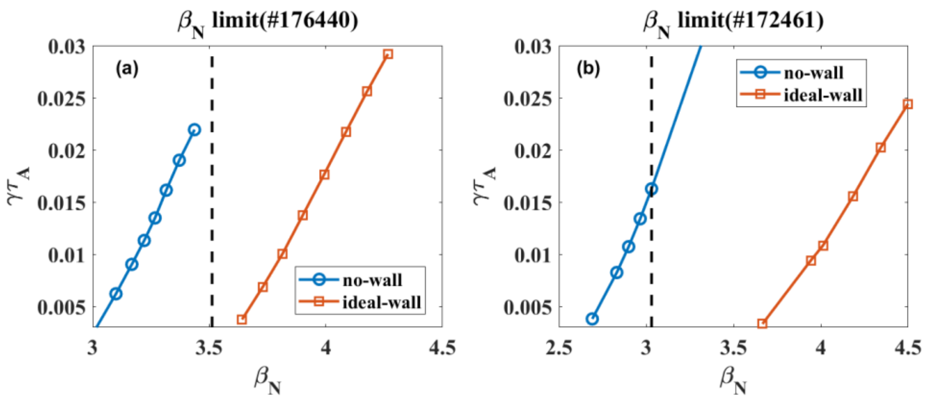
MARS-K：self-consistent treatment of kinetic damping

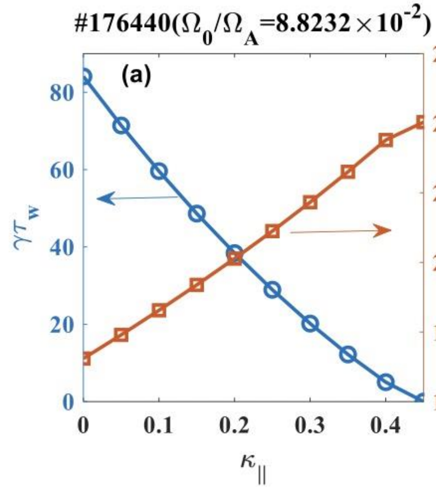
$$\lambda_{ml} = \frac{n[\omega_{*N} + (\epsilon_k - 3/2)\omega_{*T} + \omega_E] - \omega}{m < \dot{\chi} > + n < \dot{\phi} > + l\omega_b - i\nu_{\mathrm{eff}} - \omega}$$
$$= \frac{n[\omega_{*N} + (\hat{\epsilon}_k - 3/2)\omega_{*T} + \omega_E] - \omega}{n\omega_d + [\alpha(m + nq) + l]\omega_b - i\nu_{\mathrm{eff}} - \omega}$$

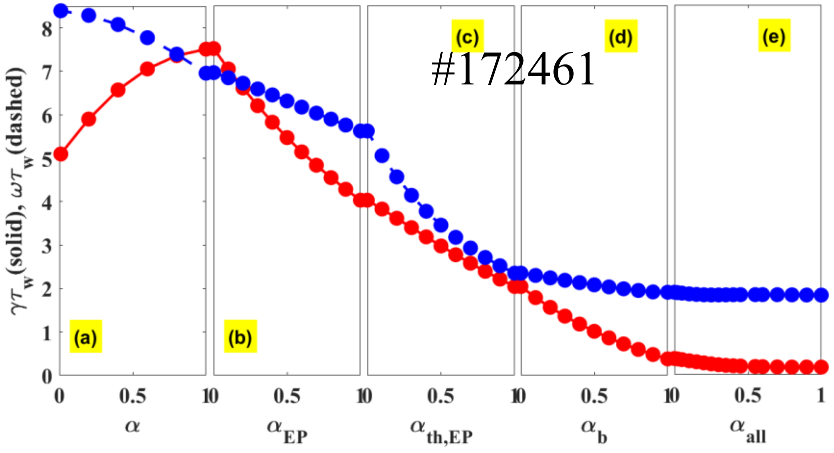
Validation study of RWM stability in DII-D high-beta-N plasmas











Zhao, et al, Hao*, Wang*, et al, NF, 2024

Outline

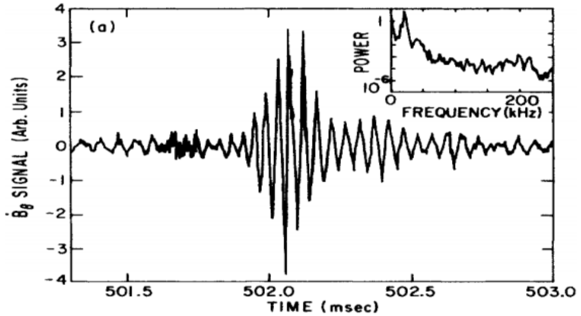
1. Introduction
2. Physics and control of Resistive Wall Mode (RWM)
   - External kink
   - RWM dispersion relation
   - RWM control
   - RWM stability with kinetic damping
3. Physics and control of Edge Localized Mode(ELM)
4. Summary and outlook

Extended energy principle

- MHD-kinetic closure

$$p_{\parallel} = \int v_{\parallel}^2 f d^3v$$

$$p_{\perp} = \int v_{\perp}^2 f d^3v$$



W. W. Heidbrink et al PRL 7,57 (1986)

$$f_{NA}^1 \longrightarrow \begin{aligned} p_{\parallel} &= \int v_{\parallel}^2 f_{NA}^1 d^3v \\ p_{\perp} &= \int v_{\perp}^2 f_{NA}^1 d^3v \end{aligned} \longrightarrow \delta W_k = -\frac{1}{2}\int d^3x \left[p_{\perp}(\nabla \cdot \boldsymbol{\xi}_{\perp}^*) - (p_{\parallel} - p_{\perp})\boldsymbol{\xi}_{\perp}^* \cdot \boldsymbol{\kappa} \right]$$

Ideal MHD

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} = -\boxed{\nabla P_1} + \mathbf{j}_0 \times \mathbf{B}_1 + \mathbf{j}_1 \times \mathbf{B}_0$$

$$\delta W = \underbrace{\delta W_F}_{fluid\ \ energy} + \underbrace{\delta W_S}_{surface\ \ energy} + \underbrace{\delta W_V}_{vacuum\ \ energy}$$

Kinetic MHD

$$\rho_0 \frac{\partial \mathbf{v}_1}{\partial t} = -\boxed{\nabla \cdot \mathbf{P}_1} + \mathbf{j}_0 \times \mathbf{B}_1 + \mathbf{j}_1 \times \mathbf{B}_0$$

$$\delta W = \underbrace{\delta W_F}_{fluid\ \ energy} + \underbrace{\delta W_S}_{surface\ \ energy} + \underbrace{\delta W_V}_{vacuum\ \ energy} + \boxed{\delta W_K}$$

Kinetic effect

Drift-kinetic equation

Drift-kinetic equation (bounce average):

$$\frac{\partial f_0}{\partial t}+\dot{\mathbf{R}}\cdot\nabla f_0+\dot{v}_{\|}\frac{\partial f_0}{\partial v_{\|}}+\dot{y}\frac{\partial f_0}{\partial y}=0$$

Linearized drift-kinetic equation:

$$\frac{df^{(1)}}{dt}+\dot{\mathbf{R}}^{(1)}\cdot\nabla F+\dot{v}_{\|}^{(1)}\frac{\partial F}{\partial v_{\|}}+y^{(1)}\frac{\partial F}{\partial y}=0$$

Solution:

$$f^1 = f^1_{NA} - \boldsymbol{\xi}\cdot\nabla f^0$$

$$\begin{aligned} f^1_{NA} &= -f^0_\epsilon \epsilon_k \sum_{m,l} X_m H_{ml} \frac{n[\omega_{*N}+(\hat{\epsilon}_k-3/2)\omega_{*T}+\omega_E]-\omega}{m<\dot{\chi}>+n<\dot{\phi}>+l\omega_b-i\nu_{\mathrm{eff}}-\omega} e^{-i\omega t+im<\dot{\chi}>t+in<\dot{\phi}>t+il\omega_d t} \\ &= -f^0_\epsilon \epsilon_k e^{-i\omega t+in\phi} \sum_{m,l} \boxed{X_m H_{ml}} \lambda_{ml} e^{-in\tilde{\phi}(t)+im<\dot{\chi}>t+il\omega_b t} \end{aligned}$$

$$\frac{1}{\epsilon_k}[M v_{\|}^2 \kappa\cdot\xi_\perp + \mu(Q_{\|}+\nabla B\cdot\xi_\perp)]$$

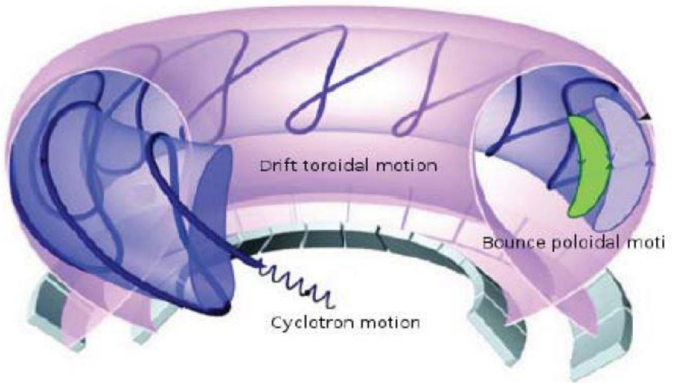
Resonance factor:

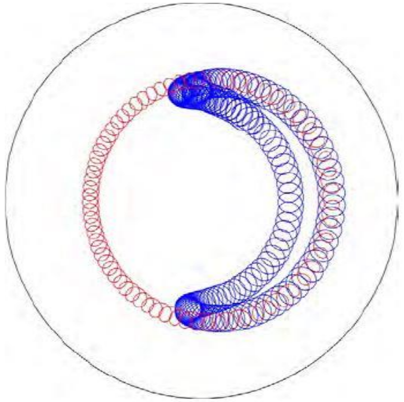
$$\boxed{\begin{aligned} \lambda_{ml} &= \frac{n[\omega_{*N}+(\epsilon_k-3/2)\omega_{*T}+\omega_E]-\omega}{m<\dot{\chi}>+n<\dot{\phi}>+l\omega_b-i\nu_{\mathrm{eff}}-\omega} \\ &= \frac{n[\omega_{*N}+(\hat{\epsilon}_k-3/2)\omega_{*T}+\omega_E]-\omega}{n\omega_d+[\alpha(m+nq)+l]\omega_b-i\nu_{\mathrm{eff}}-\omega} \end{aligned}}$$

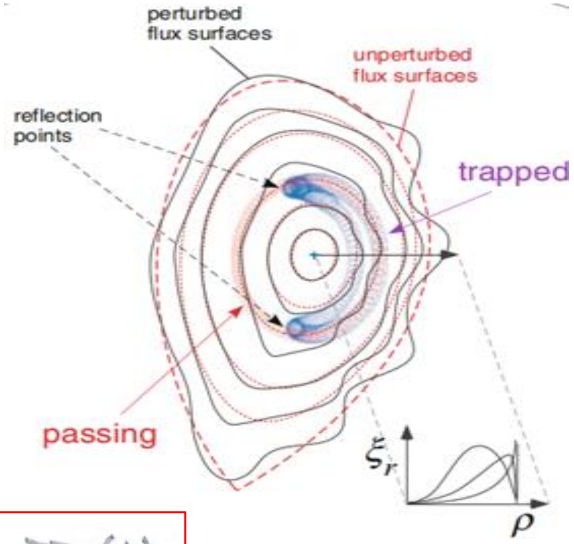
[Porcelli, Phys. Plasmas 1 (3), 1994]

Why f_{NA}^{1} is important?

1. Kinetic effect (non-adiabatic): stabilize or destabilize MHD instabilities through wave-particle resonance interaction.



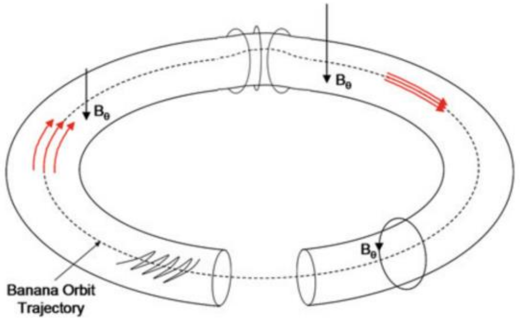




Precession drift & bounce & transit motions

$\boxed{\omega_b \gg \omega_D}$

2. Adiabatic effect: stabilize MHD



in order to conserve the poloidal flux, taking energy from the mode and so stabilising the MHD perturbation

Effect of energetic particles on RWM

Dispersion relation for resistive wall mode

$$\boxed{\gamma\tau_w = -\frac{\delta W_\infty}{\delta W_b}}$$

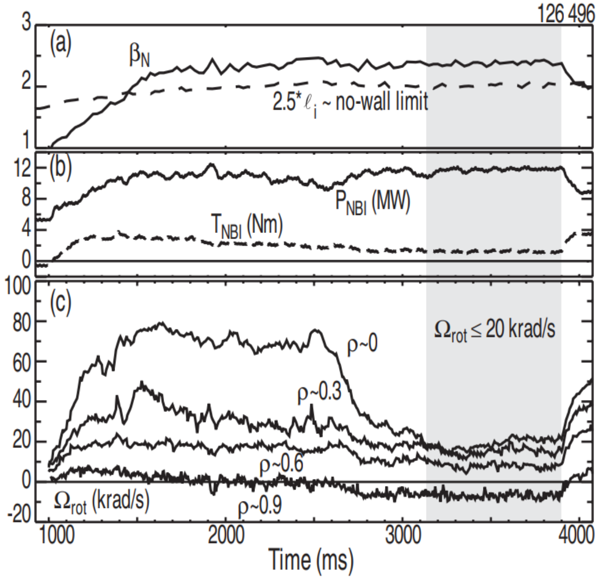
Add the kinetic effect of energetic particles:

$$\boxed{D(\omega) \equiv -i(\Omega_r + i\gamma / \omega_{ds})\omega_{ds}\tau_w^* + \frac{\delta W_f^\infty + \delta W_{K0}}{\delta W_f^b + \delta W_{K0}} = 0}$$

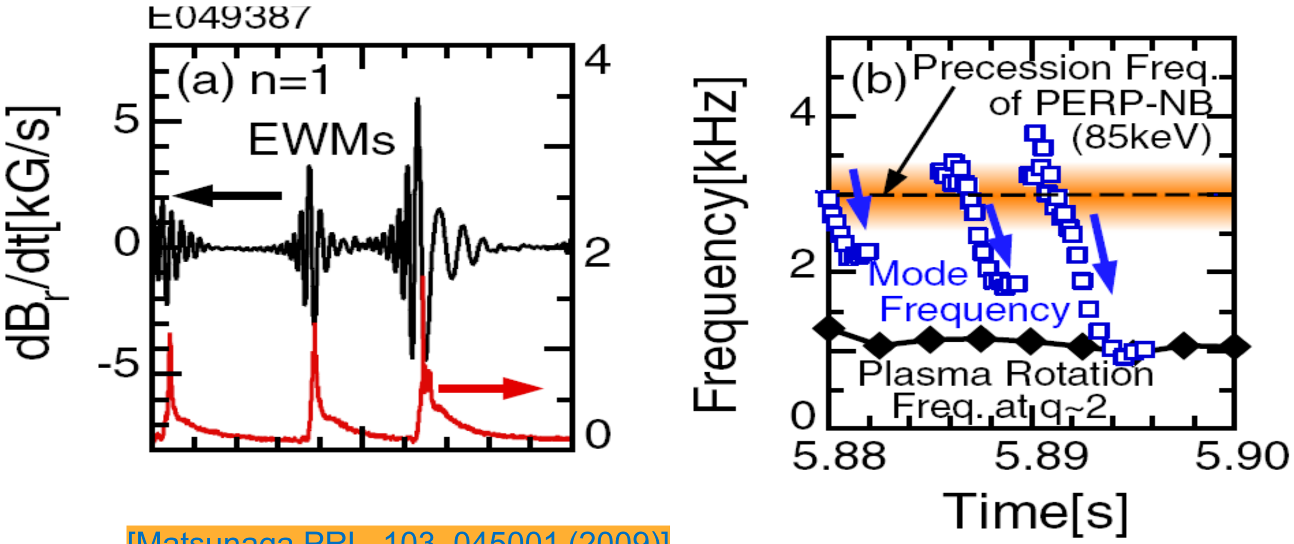
Assume $F_h = n_t \delta(\alpha_0 - \alpha) E^{-3/2}$ and global mode structure $\hat{r}^{m-1}$ and all EPs are trapped

$$\delta W_{K0} = 12\pi(1 - \frac{\alpha_0 B_0}{2})^2 \frac{\beta_h R}{Ka}\{(\hat{A} - \hat{B})\frac{2}{7}\Omega \ln(1 - \frac{1}{\Omega}) - \frac{2}{7}(\hat{A} + \frac{5}{2}\hat{B})\Omega[2(\frac{1}{5\Omega} + \frac{1}{3\Omega^2} + \frac{1}{\Omega^3}) - \frac{1}{\Omega^3}\frac{1}{\sqrt{\Omega}}\ln(\frac{1+\sqrt{\Omega}}{1-\sqrt{\Omega}})]\} + M$$

Kinetic effect of EP on RWM and FLM

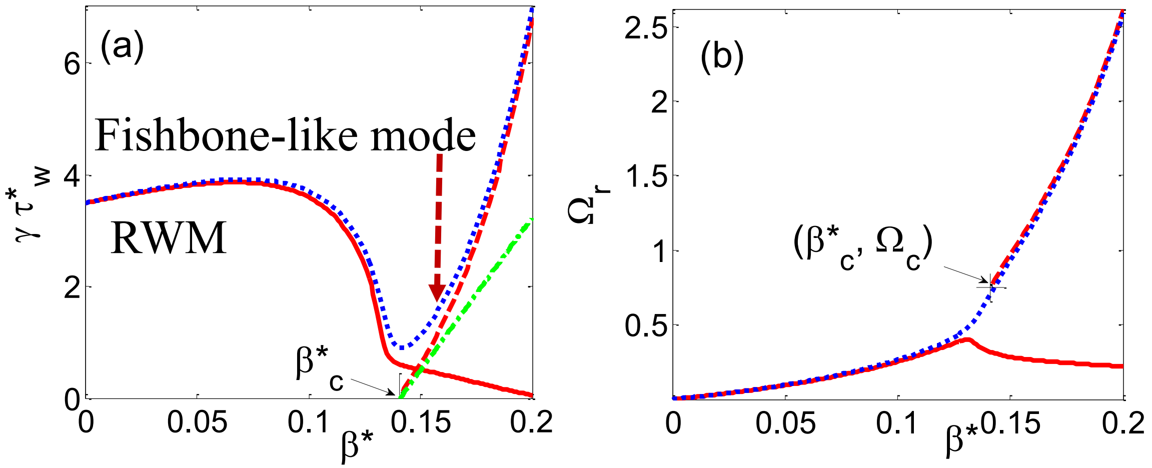


Reimerdes,,PRL. 98, 055001 (2007)



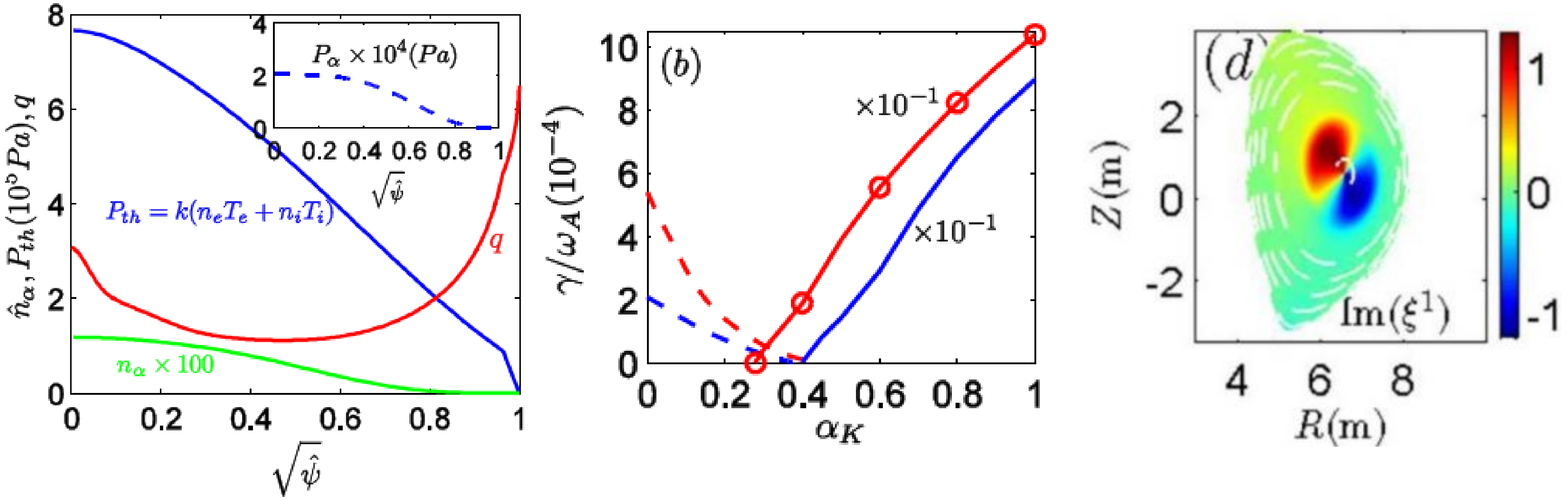
[Matsunaga,PRL, 103, 045001 (2009)]

- A complete stabilization of RWM can be achieved by the trapped EPs
- A FLM with external kink structure can be driven by trapped EPs



Hao PRL ,107,015001,(2011)

Kinetic effect of alpha particles on RWM and FLM



- ITER 10 MA scenario, RWM can be fully stablized by kinetic damping from alpha particle
- Meanwhile, alpha particle can drive fishbone-like mode with global mode structure

He,Liu, Hao, NF, 2024

核工业西南物理研究院 Southwestern Institute of Physics 中核集团 CNNC

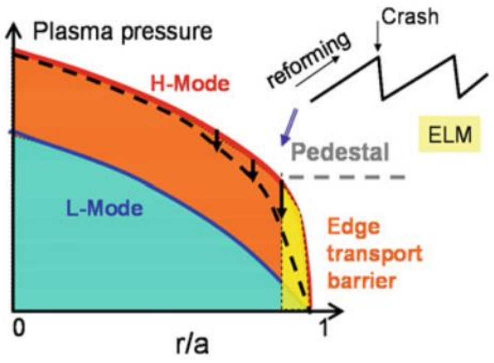
Outline

1. Introduction
2. Physics and control of Resistive Wall Mode (RWM)
3. Physics and control of Edge Localized Mode(ELM)
    - Background
    - E-L equation for Ballooning mode
    - Linear computation of RMP
    - Non-linear simulation of ELM control by RMP
4. Summary and outlook

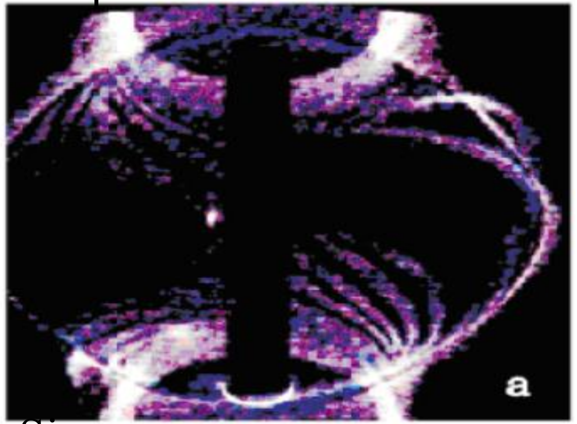
$$\boldsymbol{Q} \propto nT\tau_E \propto \beta_N H B^3 a^3 / q_{95}^2$$

RWM ELM

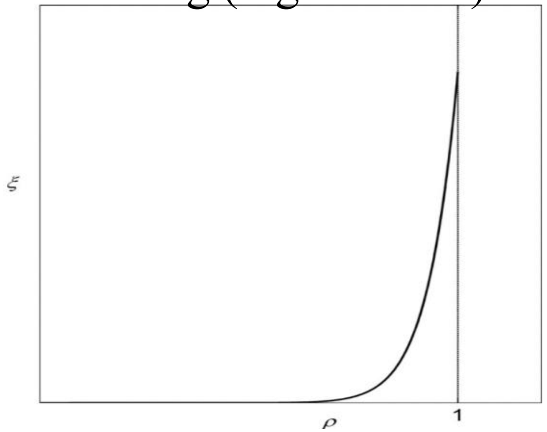
Edge localized mode

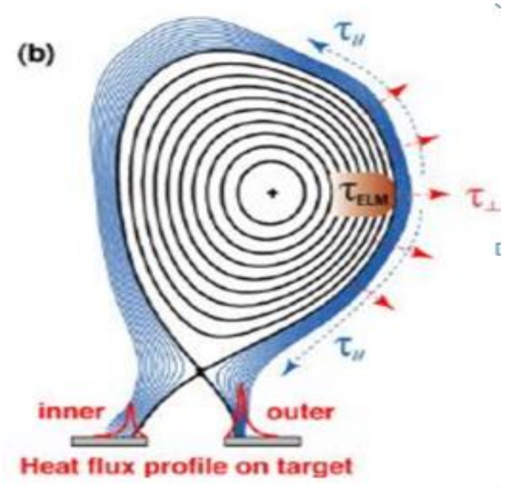


Exp.

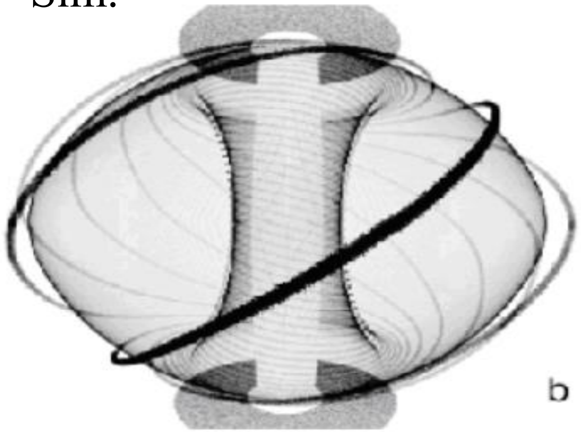


Peeling (high m kink)

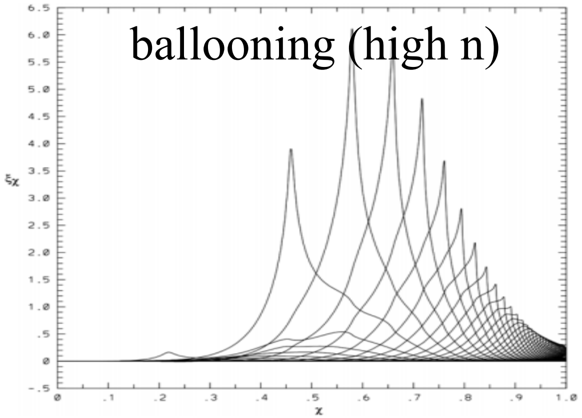




Sim.

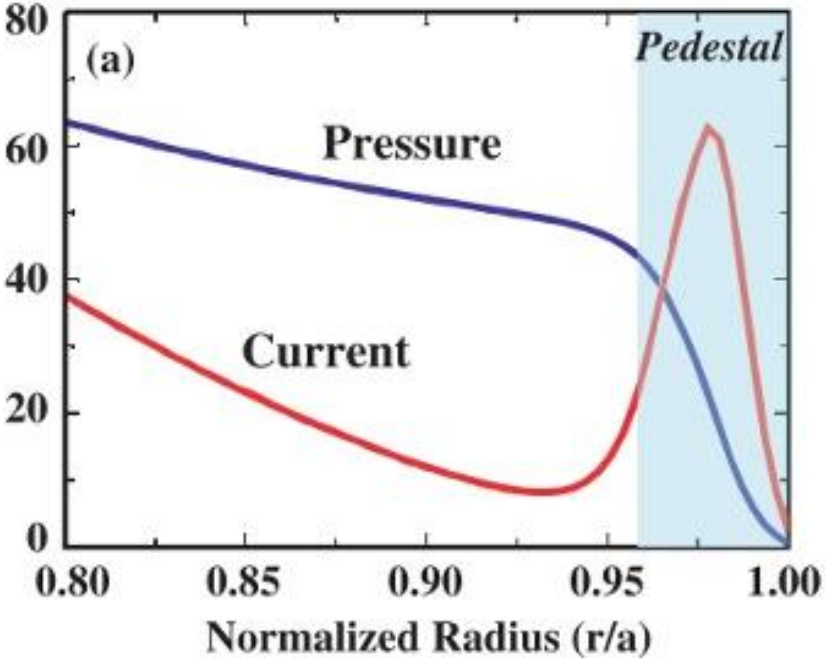


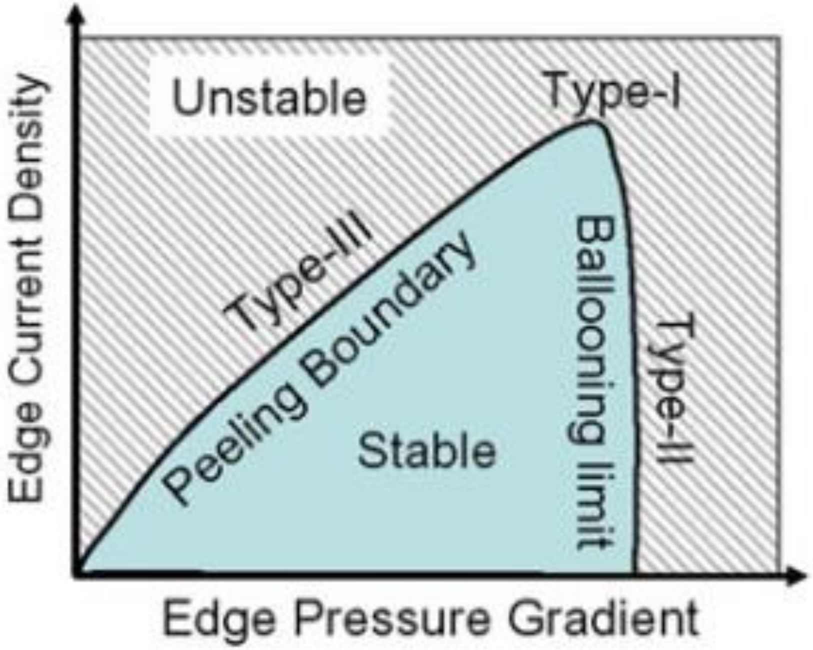
ballooning (high n)



ELM: peeling mode (high-m kink mode)+ ballooning mode

Edge localized mode

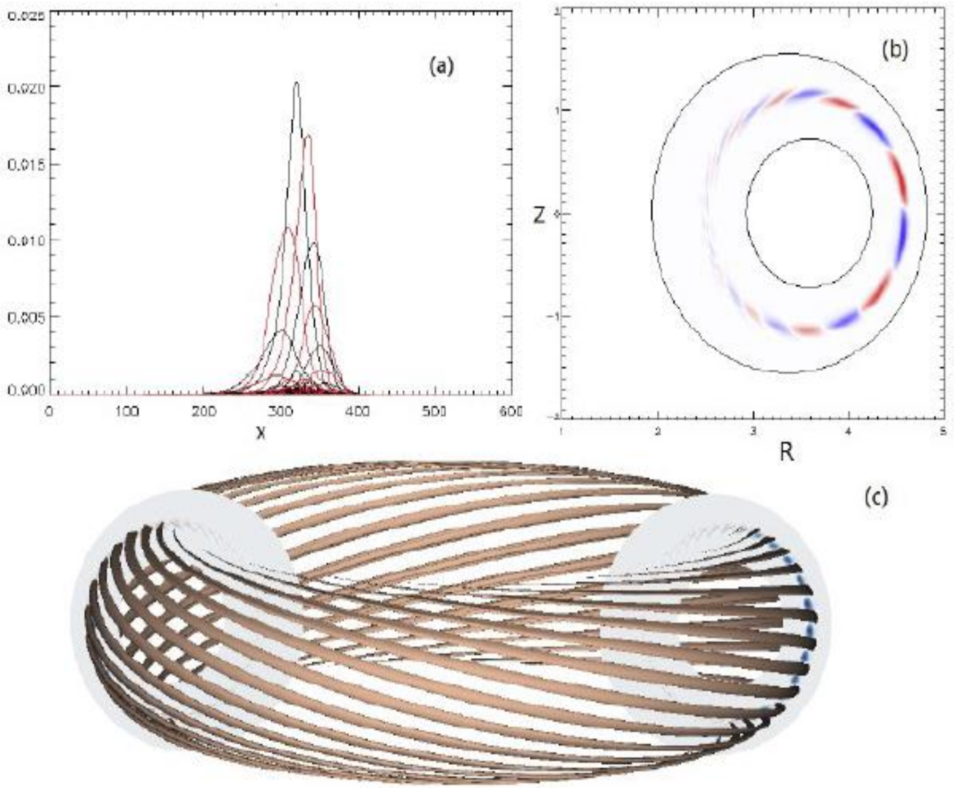




Control of Type I ELM: a challenge for reactor-level device (e.g. ITER/CFETR)

Ballooning mode

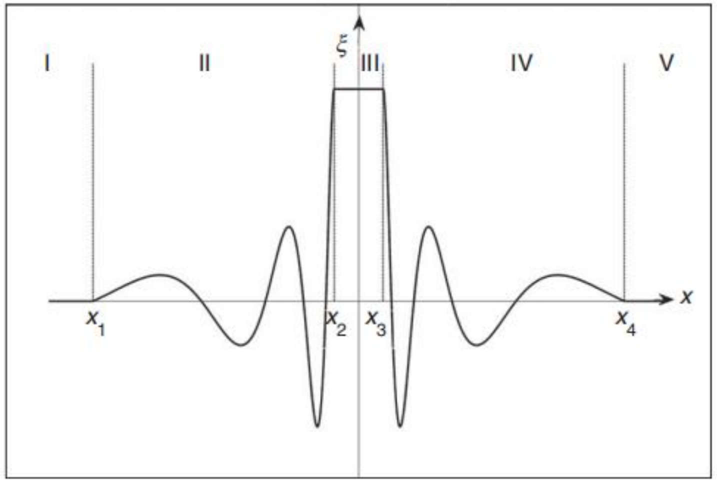
- Driven by pressure gradient
- Perp. wave number much larger than parallel one: $k_{||} \ll k_{\perp}$
- Localized in the position with large pressure gradient & unfavorable curvature region



Ballooning mode instability

Mercier criterion: for interchange mode

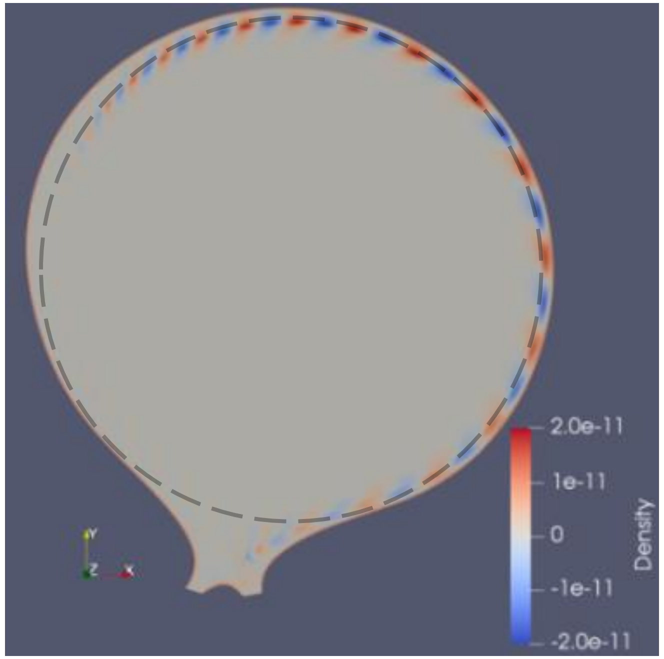
$$\left(\frac{rq'}{q}\right)^2 + \frac{8\mu_0}{B_0^2} rp'(1-q^2) > 0 \qquad \text{for stability}$$



Trial funciton for type of interchange mode

- $k_{//}$=0, minize the bending of field line
- competition between unfavorable curvature and line bending
- For tokamak, Mercier criterio is satisfied. Averaged curvature is good.

- Averaged curvature is good.
- Unfavorable curvature induces occurence of ballooning mode



Outline

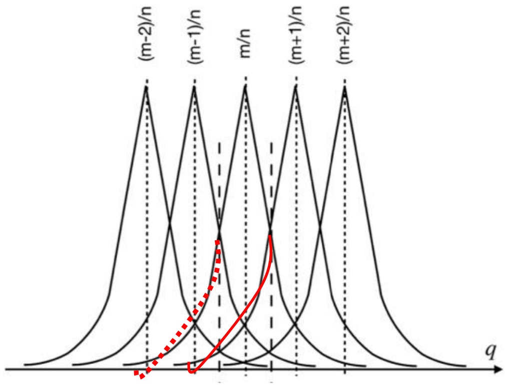
1. Introduction
2. Physics and control of Resistive Wall Mode (RWM)
3. Physics and control of Edge Localized Mode(ELM)
   - Background
   - E-L equation for Ballooning mode
   - Linear computation of RMP
   - Non-linear simulation of ELM control by RMP
4. Summary and outlook

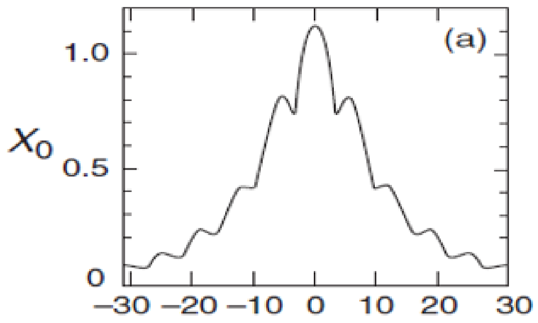
Quasi-mode

$$\xi(r,\theta,\phi) = \sum_{m,n} \bar{\xi}(r,\theta + 2m\pi, \phi + 2n\pi)$$

$\bar{\xi}(r,\theta,\phi)$, often called a "quasi-mode,"

$0 \le \theta \le 2\pi, 0 \le \phi \le 2\pi$





$\theta + 2m\pi$

$$\bar{\xi}(\mathbf{r}) = \bar{\boldsymbol{\eta}}(\mathbf{r})\, e^{iS(\mathbf{r})}$$

Key point: description of perturbation 2D (r, Θ)→1D (Θ)

vary slowly on the equilibrium scale

Rapid variation $\quad \mathbf{k} = \nabla S$

$\mathbf{B} \cdot \nabla S = 0$
(not bending field line)

$\nabla S = \mathbf{k}_\perp \gg 1/a$
(short wavelength -> high-n)

Perturbed energy based on quasi-mode concept:

$$\delta \overline{W}_F = -\frac{1}{2} \int \bar{\xi}^* \cdot \mathbf{F}(\bar{\xi}) d\mathbf{r}$$

Minimizing the perturbed energy

$$\delta \overline{W}_F = \frac{1}{2\mu_0}\int \Big[|\overline{\mathbf{Q}}_\perp|^2 + B^2 |\nabla \cdot \overline{\xi}_\perp + 2\overline{\xi}_\perp \cdot \boldsymbol{\kappa}|^2 + \mu_0 \gamma p |\nabla \cdot \overline{\xi}|^2$$

$$-2\mu_0 (\overline{\xi}_\perp \cdot \nabla p)(\overline{\xi}^*_\perp \cdot \boldsymbol{\kappa}) - \mu_0 J_{||} \overline{\xi}^*_\perp \times \mathbf{b} \cdot \overline{\mathbf{Q}}_\perp \Big] d\mathbf{r}$$

$$\boxed{\overline{\xi}(\mathbf{r}) = \overline{\boldsymbol{\eta}}(\mathbf{r})\, e^{iS(\mathbf{r})}}$$

minimization of δW , bringing us from the three dimensional vector equation F(ξ) = 0 to a one-dimensional equation along the field for one scalar function.

$$\begin{aligned} \overline{\mathbf{Q}}_\perp &= e^{iS}[\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B}) + i\nabla S \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})]_\perp \\ &= e^{iS}[\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B}) + \cancel{i(\mathbf{k}_\perp \cdot \mathbf{B})\overline{\boldsymbol{\eta}}_\perp - i(\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_\perp)\mathbf{B}}]_\perp \\ &= e^{iS}\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp \end{aligned}$$

Perp. Magnetic field perturbation induced by the slow variation part..

Substituting $\overline{\xi}$ and $\overline{\mathbf{Q}}_\perp$ into the expression for $\delta\overline{W}_F$ leads to

Minimize the compression term:

$$\nabla \cdot \overline{\xi} = 0$$

$$\delta \overline{W}_F = \frac{1}{2\mu_0}\int \Big[|\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp|^2 + B^2 |i\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_\perp + \nabla \cdot \overline{\boldsymbol{\eta}}_\perp + 2\overline{\boldsymbol{\eta}}_\perp \cdot \boldsymbol{\kappa}|^2$$

$$-2\mu_0 (\overline{\boldsymbol{\eta}}_\perp \cdot \nabla p)(\overline{\boldsymbol{\eta}}^*_\perp \cdot \boldsymbol{\kappa}) - \mu_0 J_{||} (\overline{\boldsymbol{\eta}}^*_\perp \times \mathbf{b}) \cdot \nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp \Big] d\mathbf{r}$$

Minimizing the perturbed energy

Due to $k_\perp a \gg 1$

Is the largest term and is unbalance, should set to be zero during the minimizing procedure

$$\delta \overline{W}_F = \frac{1}{2\mu_0}\int \Big[|\nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp|^2 + B^2 |\boxed{i\mathbf{k}_\perp \cdot \boldsymbol{\eta}_\perp} + \nabla \cdot \boldsymbol{\eta}_\perp + 2\boldsymbol{\eta}_\perp \cdot \boldsymbol{\kappa}|^2$$

$$-2\mu_0(\boldsymbol{\eta}_\perp \cdot \nabla p)(\boldsymbol{\eta}_\perp^* \cdot \boldsymbol{\kappa}) - \mu_0 J_\| (\boldsymbol{\eta}_\perp^* \times \mathbf{b}) \cdot \nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp \Big] d\mathbf{r}$$

$$\boldsymbol{\eta}_\perp = \boldsymbol{\eta}_{\perp 0} + \boldsymbol{\eta}_{\perp 1} + \cdots$$

$$|\boldsymbol{\eta}_{\perp 1}| / |\boldsymbol{\eta}_{\perp 0}| \sim 1/k_\perp a$$

$$\delta \overline{W}_0 = \frac{1}{2\mu_0}\int B^2 |\mathbf{k}_\perp \cdot \boldsymbol{\eta}_{\perp 0}|^2 d\mathbf{r} = 0$$

$$\Rightarrow \quad \boldsymbol{\eta}_{\perp 0} = Y\, \mathbf{b} \times \mathbf{k}_\perp$$

Minimizing the perturbed energy

$$\delta \overline{W}_F = \frac{1}{2\mu_0}\int \Big[|\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp|^2 \boxed{+ B^2 |i\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_\perp + \nabla\cdot\overline{\boldsymbol{\eta}}_\perp + 2\overline{\boldsymbol{\eta}}_\perp \cdot \boldsymbol{\kappa}|^2}$$
$$-2\mu_0 (\overline{\boldsymbol{\eta}}_\perp \cdot \nabla p)(\overline{\boldsymbol{\eta}}_\perp^* \cdot \boldsymbol{\kappa}) - \mu_0 J_\parallel (\overline{\boldsymbol{\eta}}_\perp^* \times \mathbf{b})\cdot\nabla\times(\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp \Big] d\mathbf{r}$$

$$\frac{1}{2\mu_0}\int B^2 |i\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_{\perp 1} + \nabla\cdot\overline{\boldsymbol{\eta}}_{\perp 0} + 2\overline{\boldsymbol{\eta}}_{\perp 0}\cdot\boldsymbol{\kappa}|^2 d\mathbf{r}$$

Further minimizing the perturbed energy by choose：

$$i\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_{\perp 1} = -\nabla\cdot\overline{\boldsymbol{\eta}}_{\perp 0} - 2\overline{\boldsymbol{\eta}}_{\perp 0}\cdot\boldsymbol{\kappa}.$$

Then （up to $\bar{\eta}_{\perp 1}$ order）：

$$\delta \overline{W}_F = \frac{1}{2\mu_0}\int \Big[|\nabla \times (\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp|^2 + \boxed{\cancel{B^2 |i\mathbf{k}_\perp \cdot \overline{\boldsymbol{\eta}}_\perp + \nabla\cdot\overline{\boldsymbol{\eta}}_\perp + 2\overline{\boldsymbol{\eta}}_\perp \cdot \boldsymbol{\kappa}|^2}}$$
$$-2\mu_0 (\overline{\boldsymbol{\eta}}_\perp \cdot \nabla p)(\overline{\boldsymbol{\eta}}_\perp^* \cdot \boldsymbol{\kappa}) - \mu_0 J_\parallel (\overline{\boldsymbol{\eta}}_\perp^* \times \mathbf{b})\cdot\nabla\times(\overline{\boldsymbol{\eta}}_\perp \times \mathbf{B})_\perp \Big] d\mathbf{r}$$

Minimizing the perturbed energy

$$\delta W_F = \frac{1}{2\mu_0}\int \Big[|\nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp|^2 + \cancel{B^2|i\mathbf{k}_\perp \cdot \boldsymbol{\eta}_\perp + \nabla \cdot \boldsymbol{\eta}_\perp + 2\boldsymbol{\eta}_\perp \cdot \boldsymbol{\kappa}|^2}$$

$$-2\mu_0(\boldsymbol{\eta}_\perp \cdot \nabla p)(\boldsymbol{\eta}_\perp^* \cdot \boldsymbol{\kappa}) \cancel{- \mu_0 J_\parallel (\boldsymbol{\eta}_\perp^* \times \mathbf{b}) \cdot \nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp} \Big] d\mathbf{r}$$

$$\boxed{\begin{aligned} \nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp &= \nabla \times [YB(\mathbf{b} \times \mathbf{k}_\perp) \times \mathbf{b}]_\perp \\ &= \nabla \times (YB\nabla S)_\perp \\ &= (\nabla X \times \mathbf{k}_\perp)_\perp \\ \nabla X &= \nabla_\perp X + (\mathbf{b} \cdot \nabla X)\mathbf{b} \end{aligned}}$$

$$X(\mathbf{r}) = YB.$$

$$\nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp = (\mathbf{b} \cdot \nabla X)\mathbf{b} \times \mathbf{k}_\perp$$

Kink term no contribution

$$\boxed{\begin{aligned} \delta W_2(\text{kink}) &= -\frac{1}{2}\int J_\parallel (\boldsymbol{\eta}_\perp^* \times \mathbf{b}) \cdot \nabla \times (\boldsymbol{\eta}_\perp \times \mathbf{B})_\perp d\mathbf{r} \\ &= -\frac{1}{2}\int \frac{J_\parallel}{B}[X^*(\mathbf{b} \cdot \nabla X)][\mathbf{k}_\perp \cdot (\mathbf{b} \times \mathbf{k}_\perp)] d\mathbf{r} \\ &= 0 \end{aligned}}$$

Minimizing the perturbed energy

$$\delta \overline{W}_2 = \frac{1}{2\mu_0}\int \left[k_\perp^2 |\mathbf{b}\cdot\nabla X|^2 - \frac{2\mu_0}{B^2}(\mathbf{b}\times\mathbf{k}_\perp\cdot\nabla p)(\mathbf{b}\times\mathbf{k}_\perp\cdot\boldsymbol{\kappa})|X|^2 \right] d\mathbf{r}$$

- describe a competition between the stabilizing effects of line bending and the destabilizing effects of unfavorable curvature

- transformation of the toroidal volume element to flux coordinates

$$d\mathbf{r} = R dR\, dZ d\phi = J d\psi\, dl\, d\phi$$

$$\mathbf{n} = \frac{\nabla\psi}{|\nabla\psi|} = \frac{\nabla\psi}{RB_p}$$

$$\mathbf{b} = \frac{B_p}{B}\mathbf{b}_p + \frac{B_\phi}{B}\mathbf{e}_\phi$$

$$\mathbf{t} = \mathbf{n}\times\mathbf{b} = \frac{B_\phi}{B}\mathbf{b}_p - \frac{B_p}{B}\mathbf{e}_\phi$$

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Minimizing the perturbed energy

$$\mathbf{B}\cdot\nabla S = 0 \Longrightarrow \frac{B_\phi}{R}\frac{\partial S}{\partial \phi} + B_p\frac{\partial S}{\partial l} = 0 \Longrightarrow S = n\left(-\phi + \int_{l_0}^{l}\frac{B_\phi}{RB_p}dl\right)$$

$$\mathbf{k}_\perp = k_n\mathbf{n} + k_t\mathbf{t} = \nabla S$$

$$k_n = \mathbf{n}\cdot\nabla S = (\mathbf{n}\cdot\nabla\psi)\frac{\partial S}{\partial\psi} = nRB_p\frac{\partial}{\partial\psi}\left(F\int_{l_0}^{l}\frac{dl}{R^2B_p}\right)$$

$$k_t = \mathbf{t}\cdot\nabla S = (\mathbf{t}\cdot\nabla\phi)\frac{\partial S}{\partial\phi} + (\mathbf{t}\cdot\nabla l)\frac{\partial S}{\partial l} = n\frac{B}{RB_p}$$

$$\Downarrow$$

$$\delta W_2 = \frac{1}{2\mu_0}\int\left[k_\perp^2|\mathbf{b}\cdot\nabla X|^2 - \frac{2\mu_0}{B^2}(\mathbf{b}\times\mathbf{k}_\perp\cdot\nabla p)(\mathbf{b}\times\mathbf{k}_\perp\cdot\boldsymbol{\kappa})|X|^2\right]d\mathbf{r}$$

Can be rewritten as the integration in ballooning angle space.

Minimizing the perturbed energy

$$\delta \overline{W}_2 = \frac{\pi}{\mu_0} \int_0^{\psi_a} \overline{W}(\psi, l_0)\, d\psi$$

$$\overline{W}(\psi, l_0) = \int_{-\infty}^{\infty} \left[(k_n^2 + k_t^2) \boxed{\left(\frac{B_p}{B} \frac{\partial X}{\partial l} \right)^2 - \frac{2\mu_0 R B_p}{B^2} \frac{dp}{d\psi} (k_t^2 \kappa_n - k_t k_n \kappa_t) X^2} \right] \frac{dl}{B_p}$$

$$\kappa_n = \frac{\mu_0 R B_p}{B^2} \frac{\partial}{\partial \psi} \left(p + \frac{B^2}{2\mu_0} \right)$$

$$\kappa_t = \frac{\mu_0 F}{R B^3} \frac{\partial}{\partial l} \left(\frac{B^2}{2\mu_0} \right)$$

E-L eq.

The result is the 1-D ballooning mode differential equation given by

$$\frac{\partial}{\partial l} \left[\frac{(k_n^2 + k_t^2) B_p}{B^2} \frac{\partial X}{\partial l} \right] + \frac{2\mu_0 R}{B^2} \frac{dp}{d\psi} (k_t^2 \kappa_n - k_t k_n \kappa_t) X = 0$$

E-L equation for Ballooning mode in large aspect ratio tokamak

S-alpha model

Line Bending term

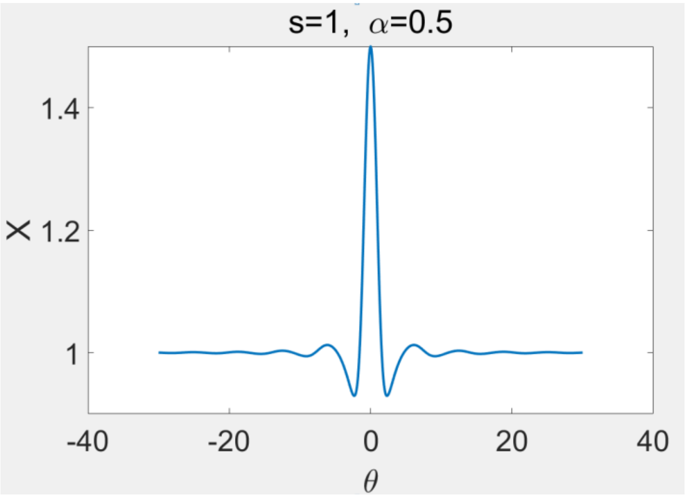
$$\frac{\partial}{\partial\theta}\left[(1+\Lambda^2)\frac{\partial X}{\partial\theta}\right]+\alpha(\Lambda\sin\theta+\cos\theta)X=0$$

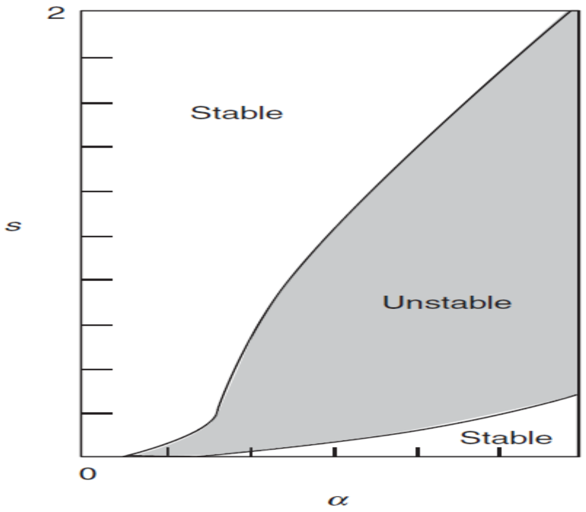
Geodesic curv. Normal curv.

$$\Lambda = s\theta - \alpha\sin\theta$$

$$s=\frac{r}{q}\frac{dq}{dr}$$

$$\alpha=-\frac{2\mu_0 r^2}{R_0 B_\theta^2}\frac{dp}{dr}=-q^2R_0\frac{d\beta}{dr}$$





$$X=1+\frac{\alpha cos\theta}{1+(s\theta-\alpha sin\theta)^2}$$

- Perdition of stability boundary of ballooning mode

  Connor et al., 1979

  P603, ideal MHD, 2014

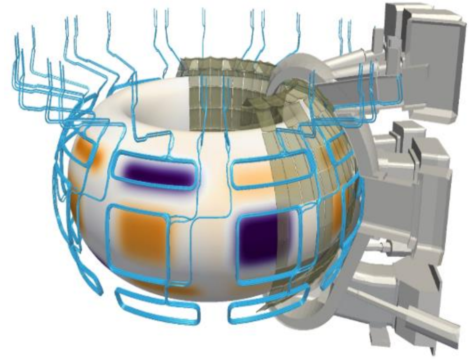
$$\boxed{\delta W_F=\frac{1}{2}\int_{-\infty}^{+\infty}d\theta\left[|\frac{dX}{d\theta}|^2+\left(\frac{(s-\alpha cos\theta)^2}{f}-\frac{\alpha cos\theta}{f}\right)|X|^2\right]=0}$$

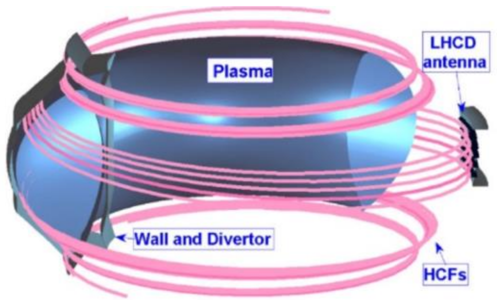
$$\boxed{\omega^2=\frac{\delta W}{\delta K}}$$

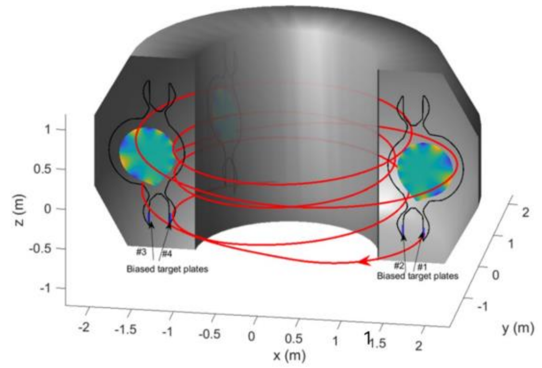
Outline

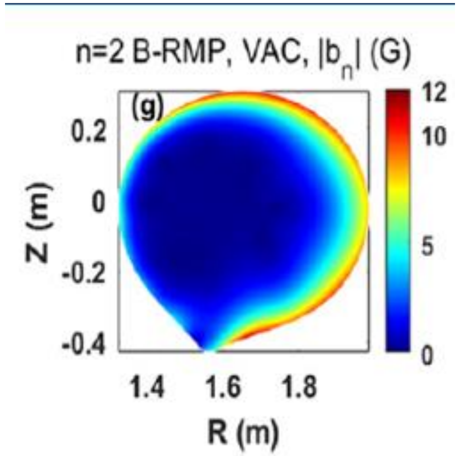
Effective control method: resonant field perturbation

- Methods of generating resonant magnetic perturbation：
  - (1) RMP Coils
  - (2) target biasing→ driven current in SOL → 3D field perturbation
  - (3) LHCD→ driven current in SOL → 3D field perturbation……

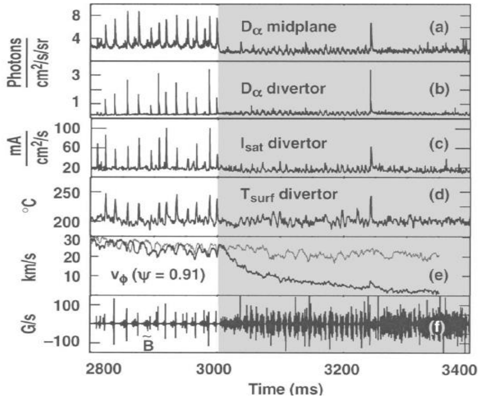




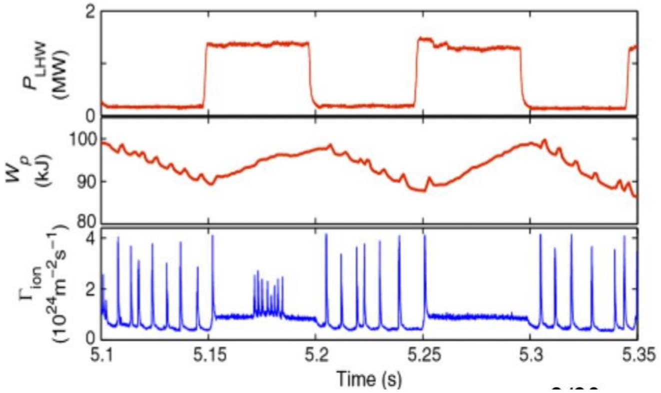




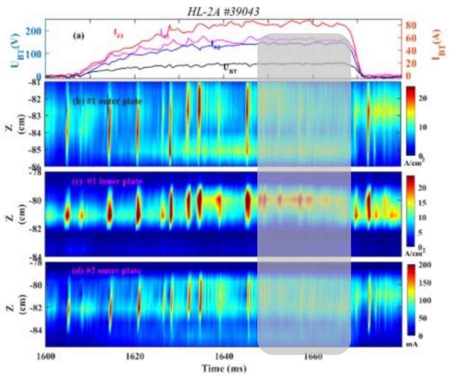
[Evans,PRL,2004]

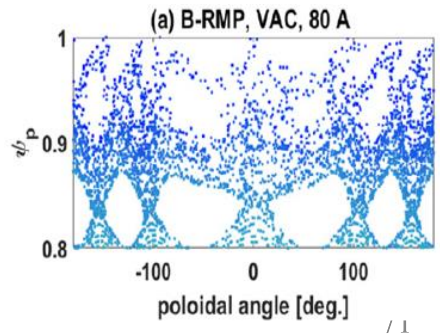


[Liang,PRL,2013]



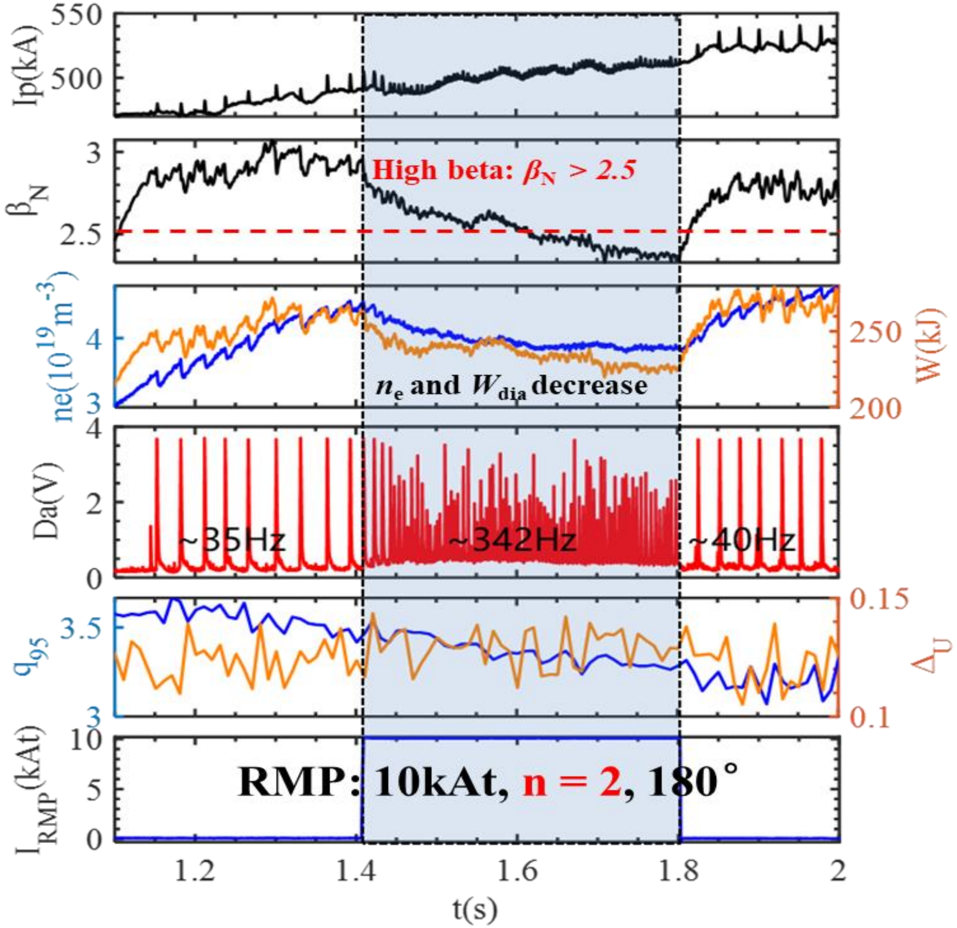
[Hao, NF-1, 2023]



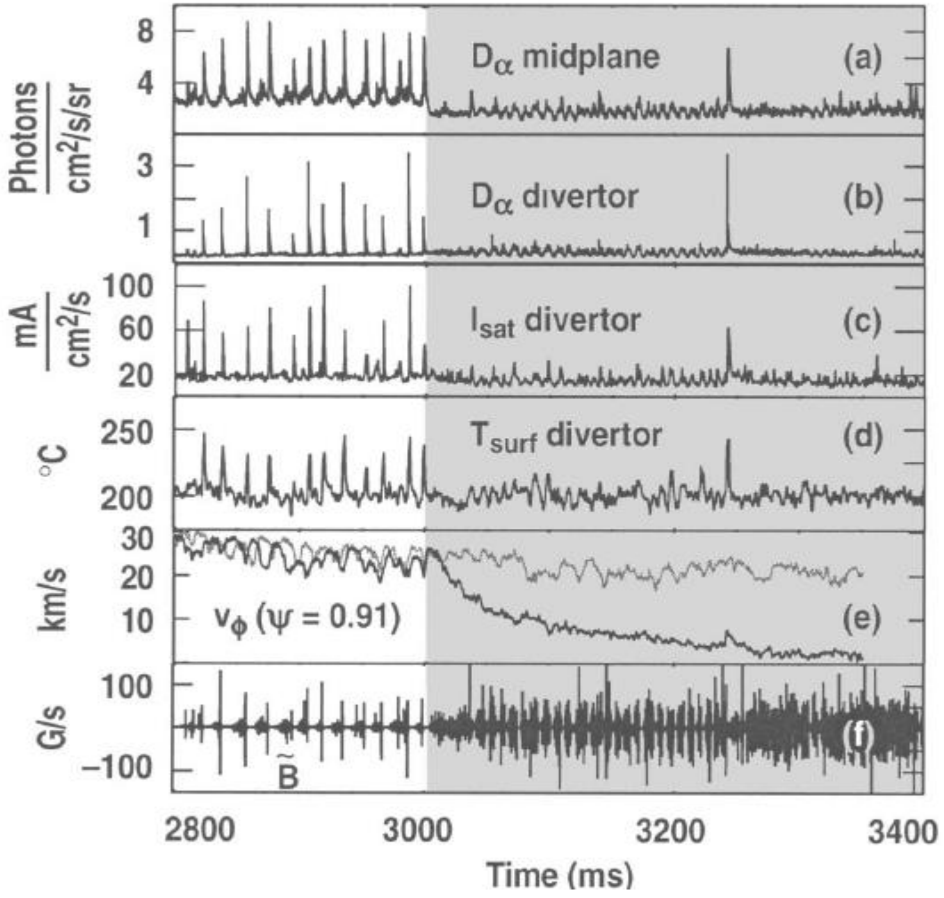


ELM mitigation & suppression

HL-3



DIII-D



Mitigation: freq. of occurence increases, while amplitude decreases

Suppression: disappear of ELM

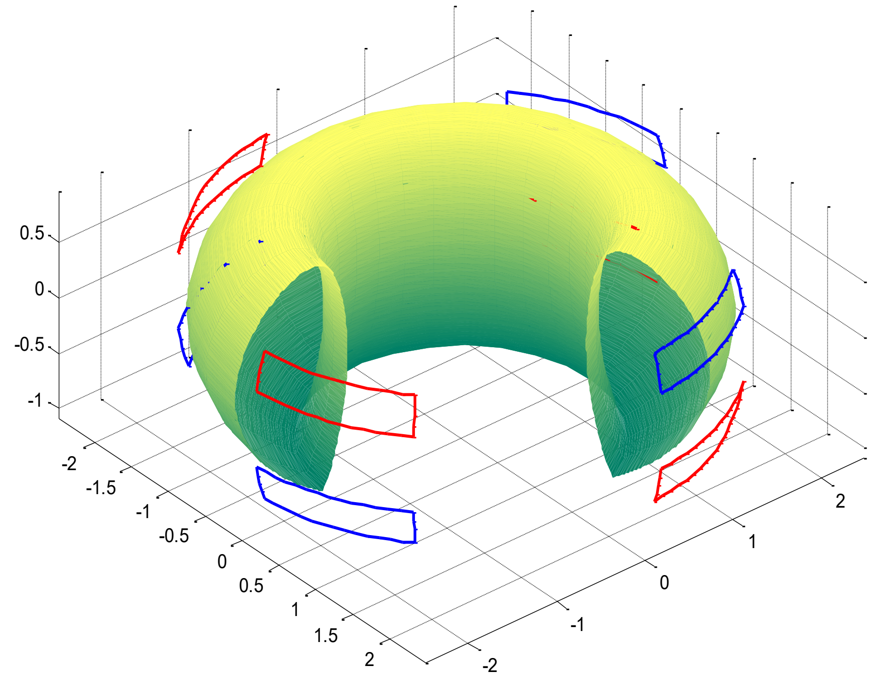
ELM mitigation & suppression

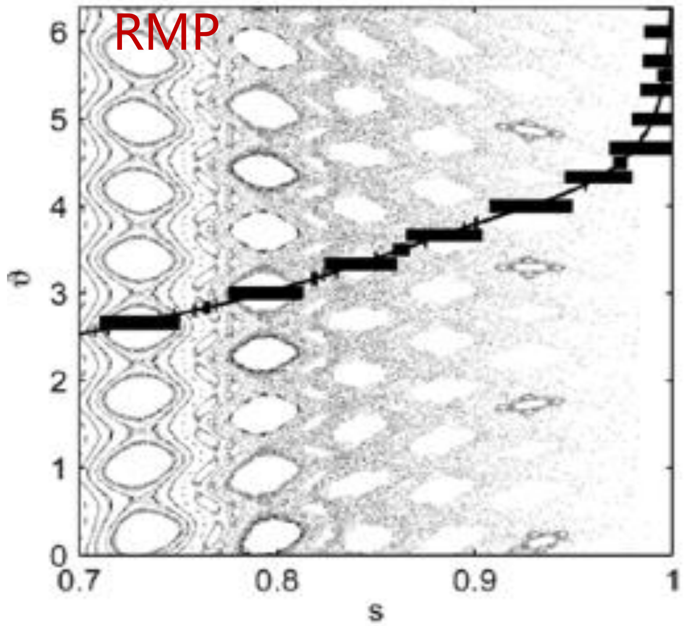
Table 2. Characteristics of Initial RMP ELM Control Observations. *Legend: Machine, ELM Suppression (S) or Mitigation (M), VIOW criterion satisfied (Y, N), toroidal n-number of perturbation, Coil configuration, Internal or External coils, q_{95} window effect (Y, N), Phasing dependence (Y, N), Collisionality (High, Moderate, Low), Density dependence (Y, N), Effects on T_e and T_i in pedestal (Y, N), Effect on edge rotation (Y, N), Density pump-out (Y, N), Confinement degradation (Y, N).*

Device	S, M	VIOW	n_{tor}	Coils	In/Ext	q_{95}	Phase	ν_e^*	n_e	T_e, T_i	Rot	Pump-out	Conf
DIII-D, high ν_e^*	S	Y	3	2×6	I	3.6	Y	H	N	N	Y	N	N
DIII-D, low ν_e^*	S	Y	3	2×6	I	3.6	Y	L	Y	Y	N	Y	Y
AUG, high ν_e^*	M	Y	2	2×4	I	5.6	N	H	Y	N	N	N	N
AUG, low ν_e^*	S	Y	2	2×8	I	3.7	Y	L	Y	Y	N	Y	Y
JET	M	Y	1–2	4	E	Y	N/A	L	Y	Y	Y	Y	Y
EAST	S	N	1–4	2×8	I	3.6 (high n), wide (low n)	Y	L-M	Y	Low n (Y) High n (Y)	Y	Y	Low n (Y) High n (N/minor)
KSTAR	S	Y	1, 2	3×4	I	3–7 (robust near 4 or 5)	Y (+90, $n = 1$)	M-H	Y	Y ($T_i \downarrow$)	Y ($V_t \downarrow$)	Y	Y
MAST	M	Y	1–6	$6 + 12$	I		Y	L-H	Y	N	Y	Y	Y

[Fenstermacher, NF,2025]

Effect of RMP fields on ELM instabilities





linear plasma response computations guide the optimizing of coil phasing

- **Parameters affecting plasma response (in descending order of importance):**
  - RMP configurations (toroidal and poloidal spectra)
  - q_{95} (and q_a)
  - Plasma flow
  - Plasma shape
  - Plasma pressure (b_N as well as pedestal pressure)

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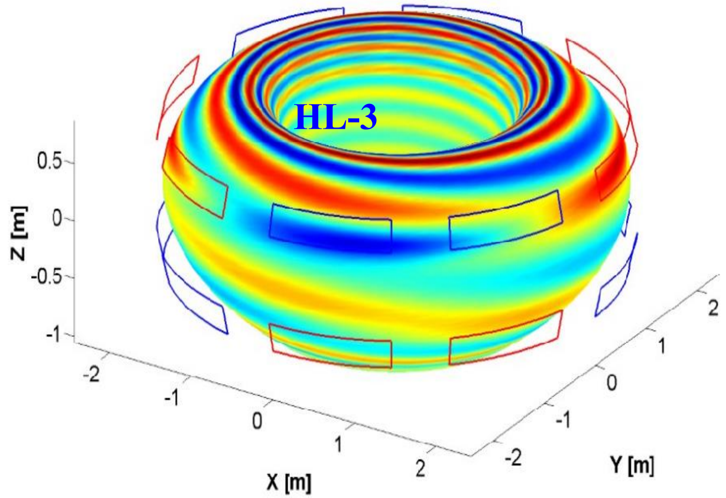
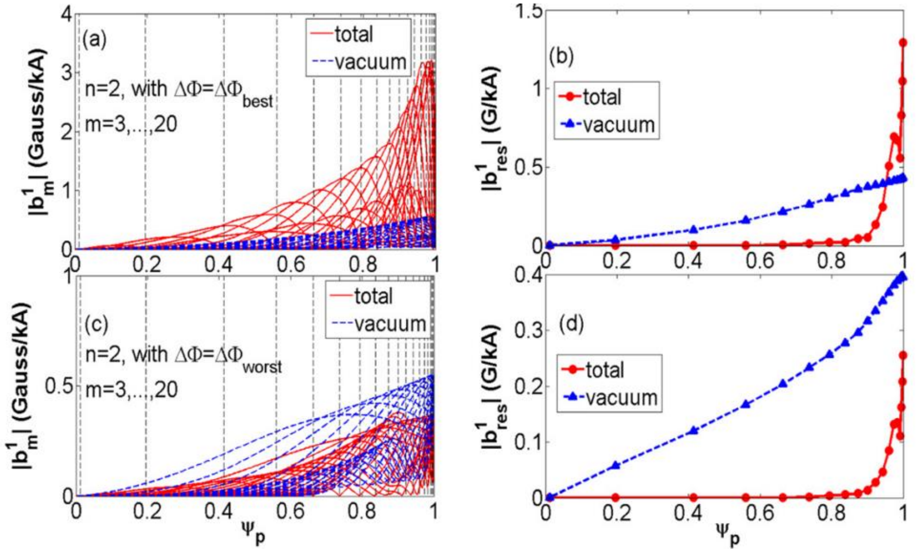
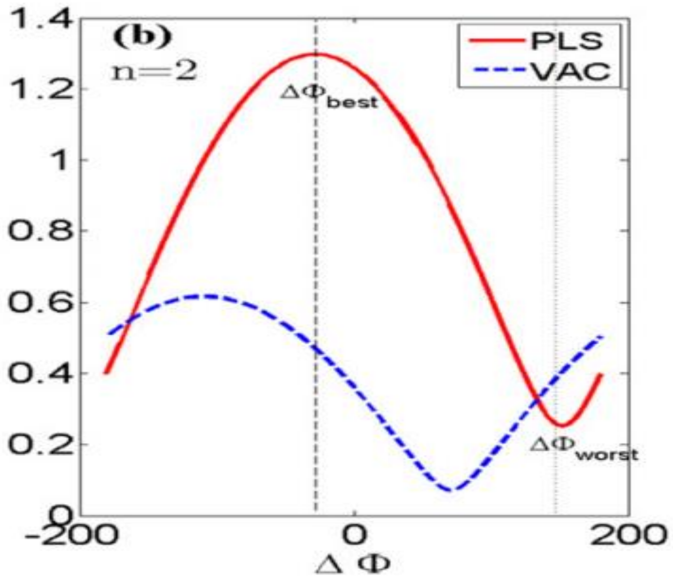
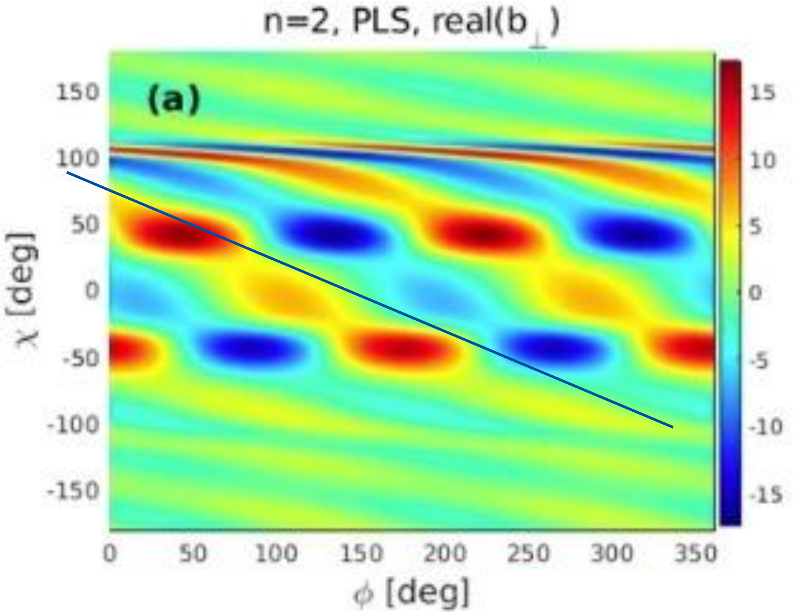
Plasma response is important for optimizing coil phasing for ELM control

- ➢ screening effect (rotation)
- ➢ amplification (pressure)
- ➢ penestration (resistive)







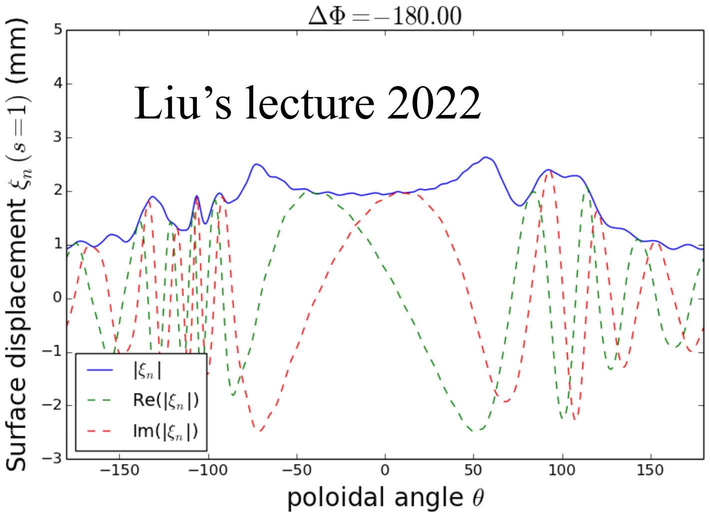


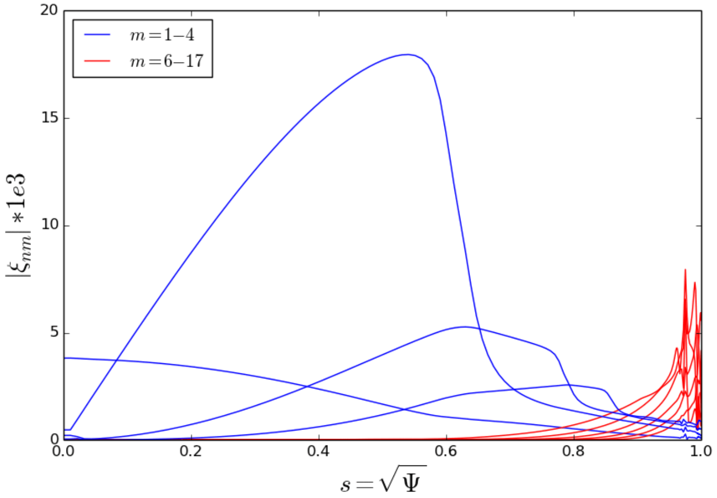
[Hao, NF-1, 2021]

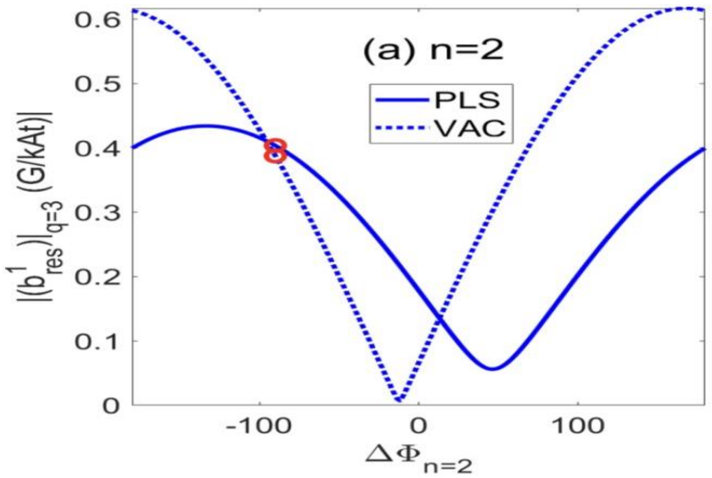
Basic RMP physics: linear plasma response: eigenmode view

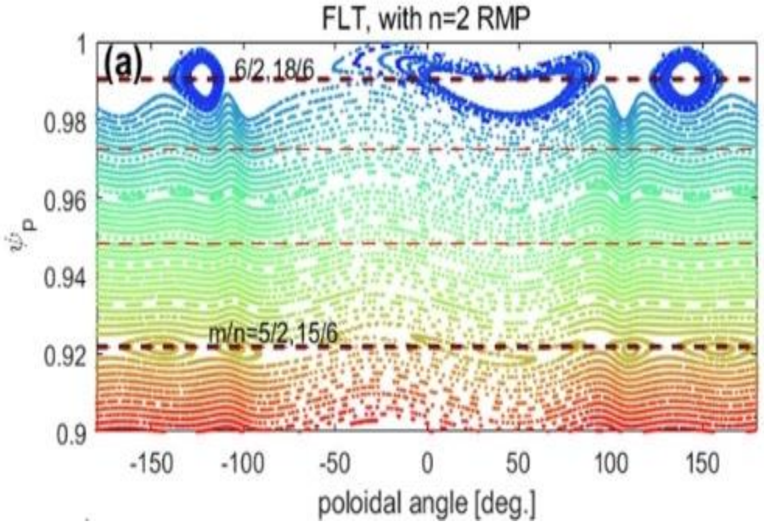
- **Core-kink vs edge-peeling response**

MARS-F computations









[Hao, NF-2, 2023]

Outline

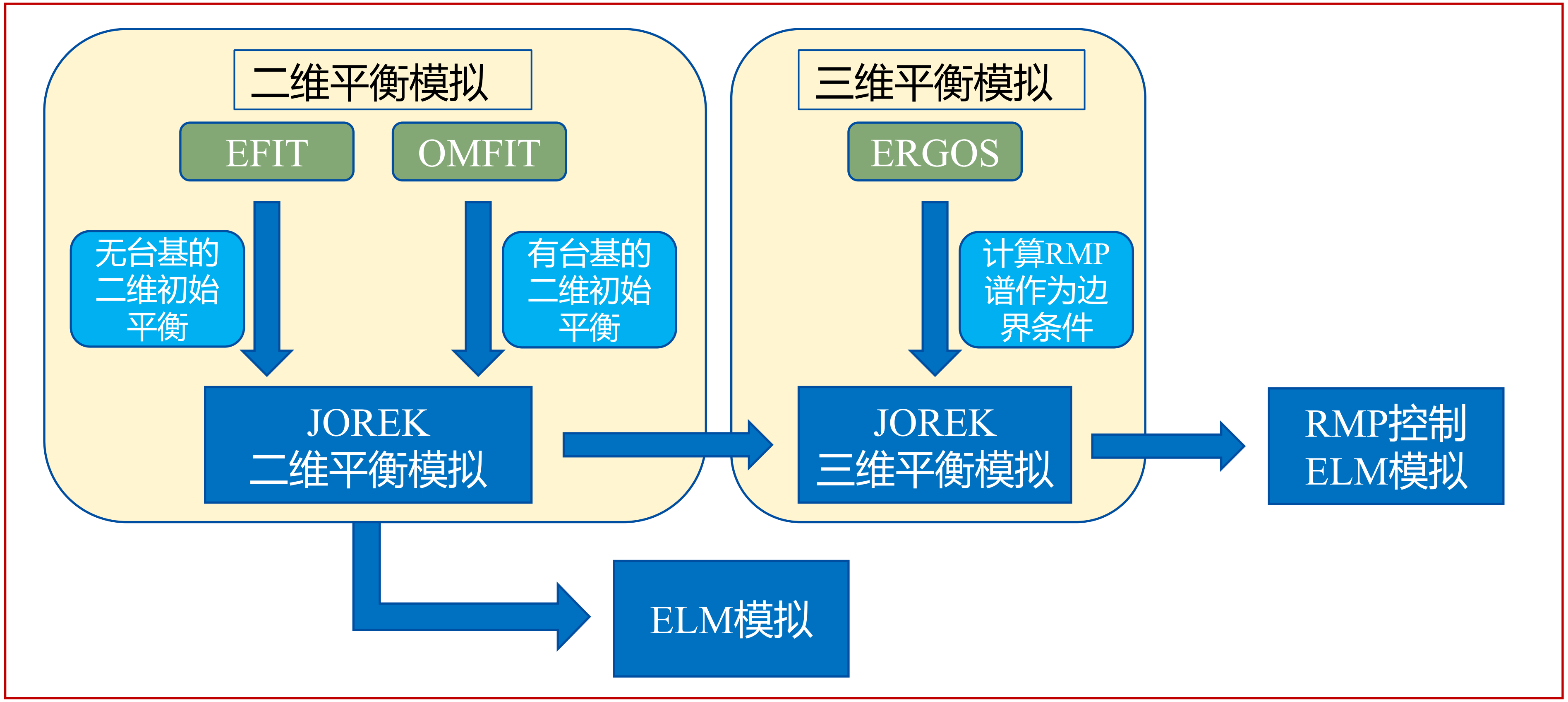
A tool for modeling ELM control by RMP：JOREK

$$\frac{\partial \mathbf{A}}{\partial t} = -\mathbf{E} - \nabla\Phi$$

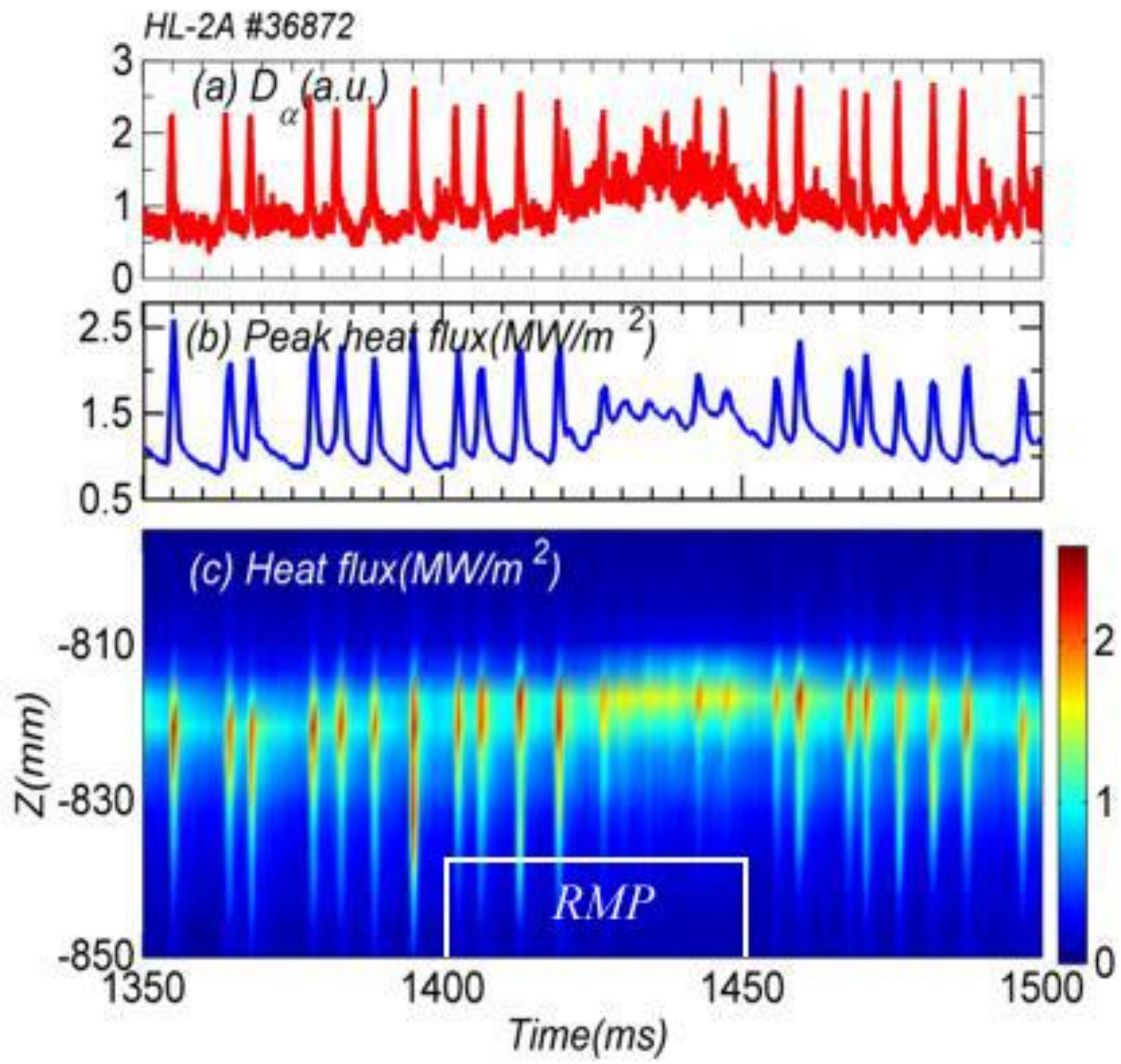
$$\rho\frac{\partial \mathbf{V}}{\partial t} = -\rho\mathbf{V}\cdot\nabla\mathbf{V} - \nabla p + \mathbf{J}\times\mathbf{B} + \nabla\cdot\bar{\tau} + \mathbf{S}_{\mathrm{V}}$$

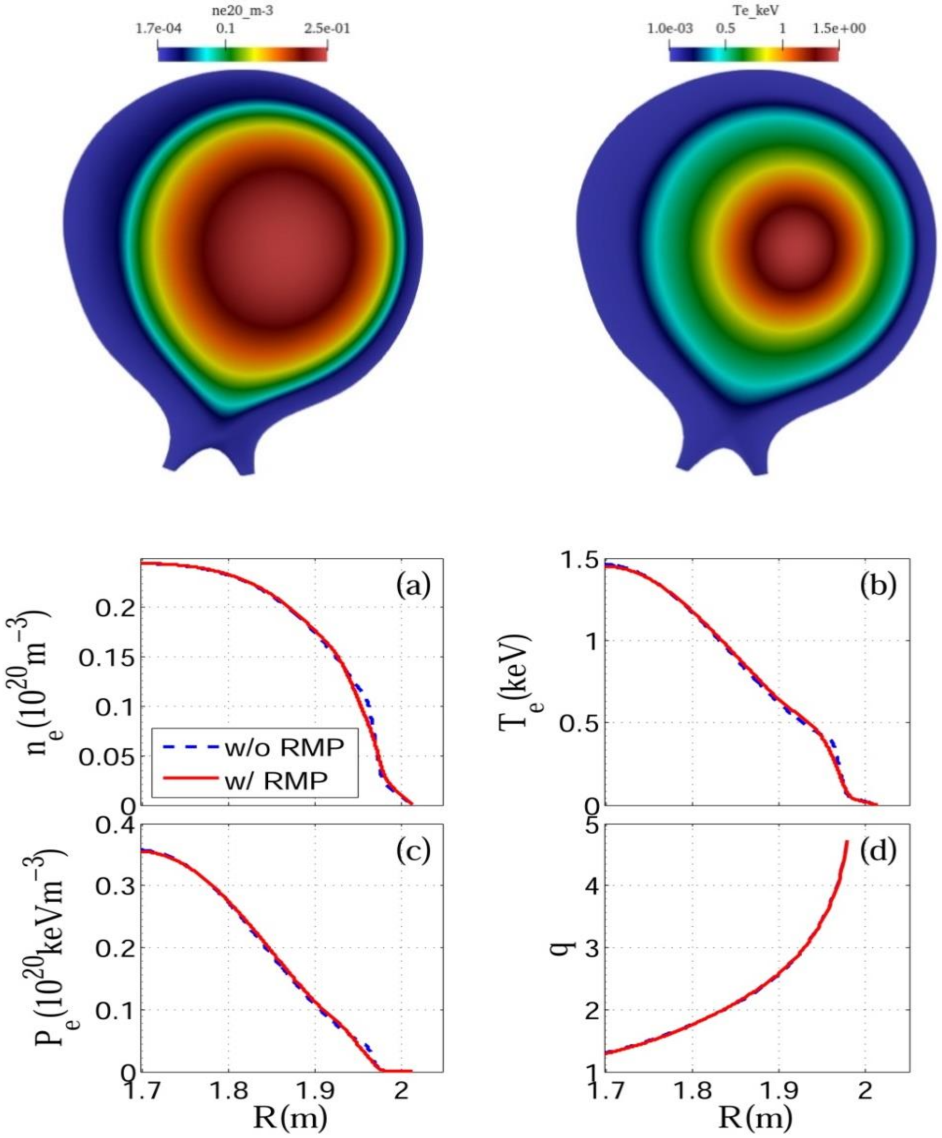
$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\mathbf{V}) + \nabla\cdot(\bar{D}\nabla\rho) + S_\rho$$

$$\frac{\partial p}{\partial t} = -\mathbf{V}\cdot\nabla p - \gamma p\nabla\cdot\mathbf{V} + \nabla\cdot(\bar{\kappa}\nabla T) + (\gamma-1)\bar{\tau}:\nabla\mathbf{V} + S_p$$

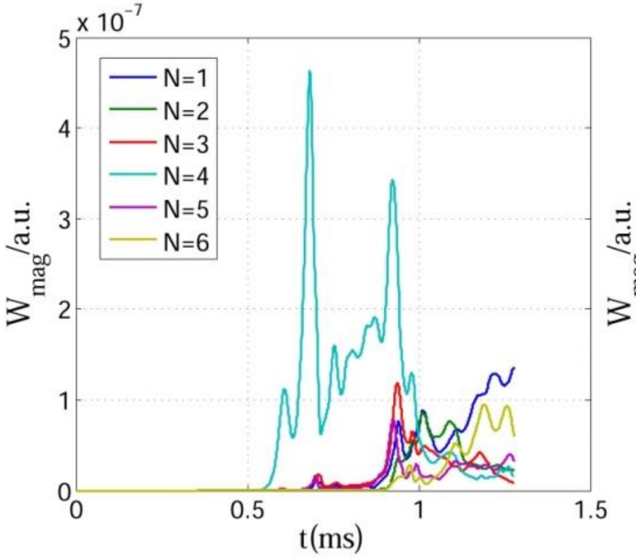


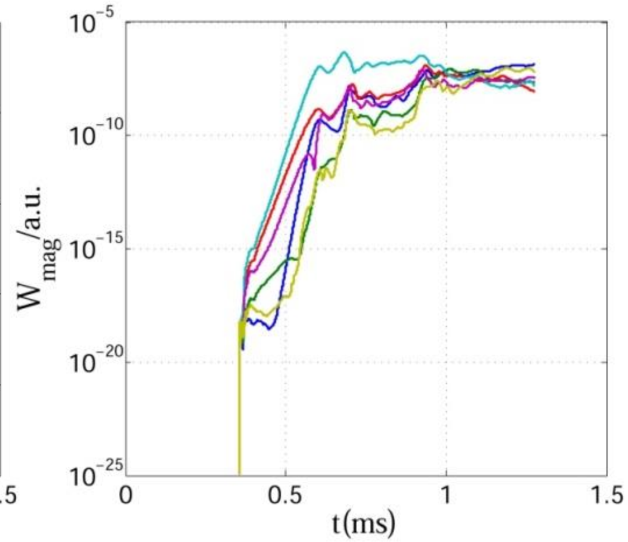
ELM control by n=1 RMP

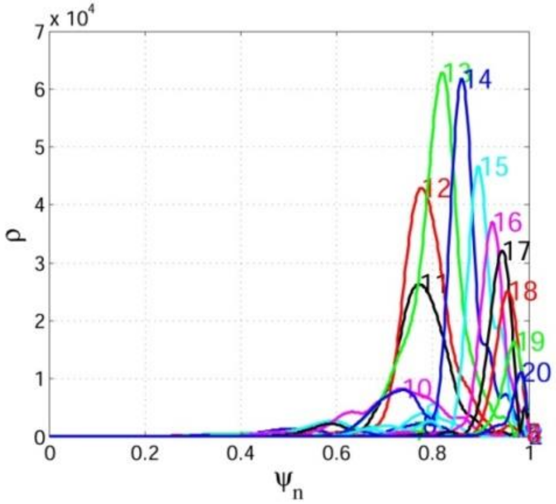


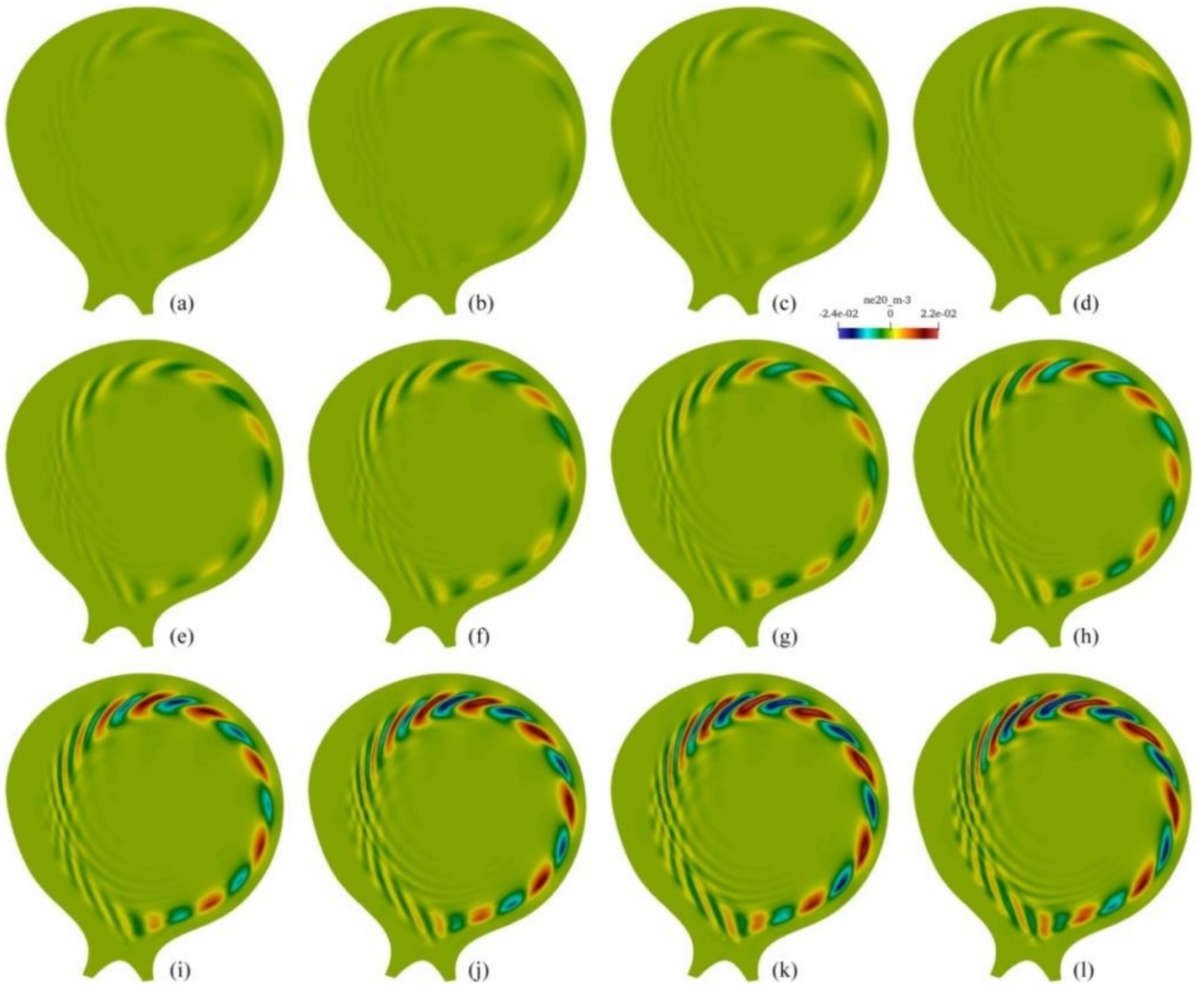


Neutral evolution of ELM without RMP



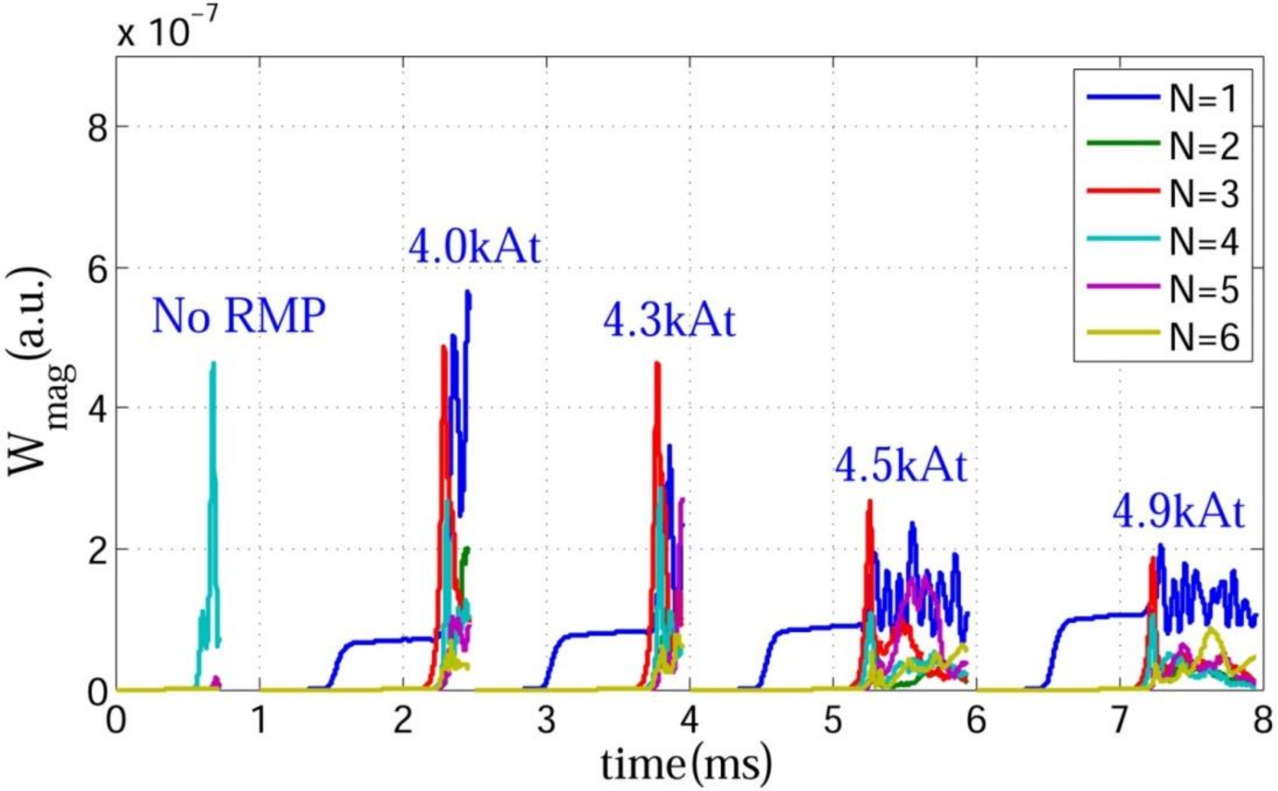


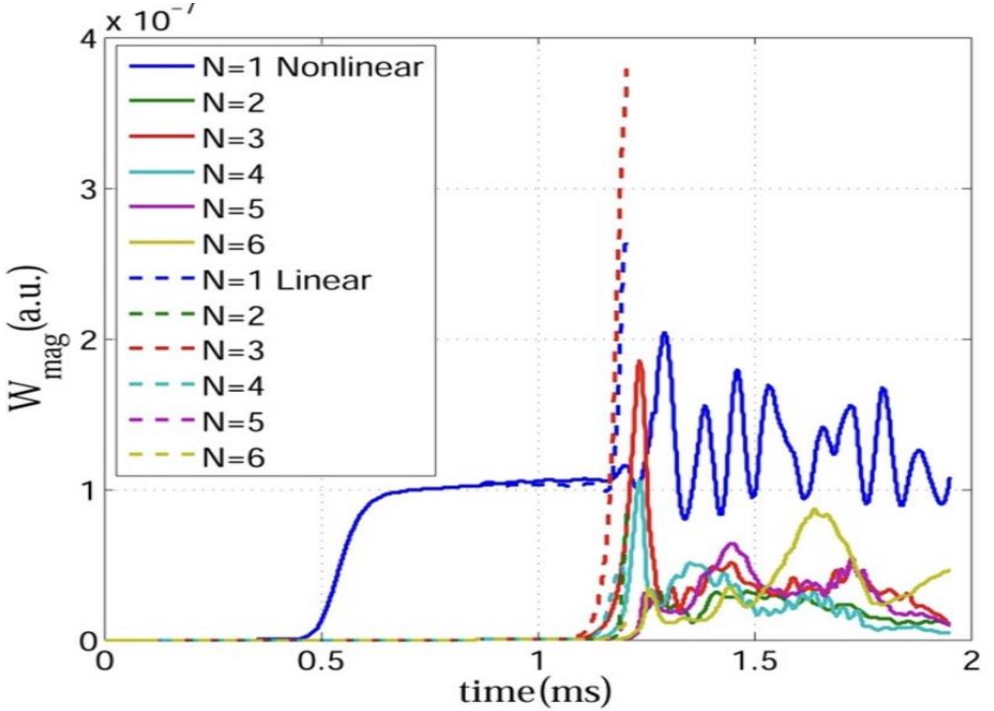




- Most unstable toroidal mode component: $n = 4$
- Perturbations grow in unfaborable curvature region (LFS)

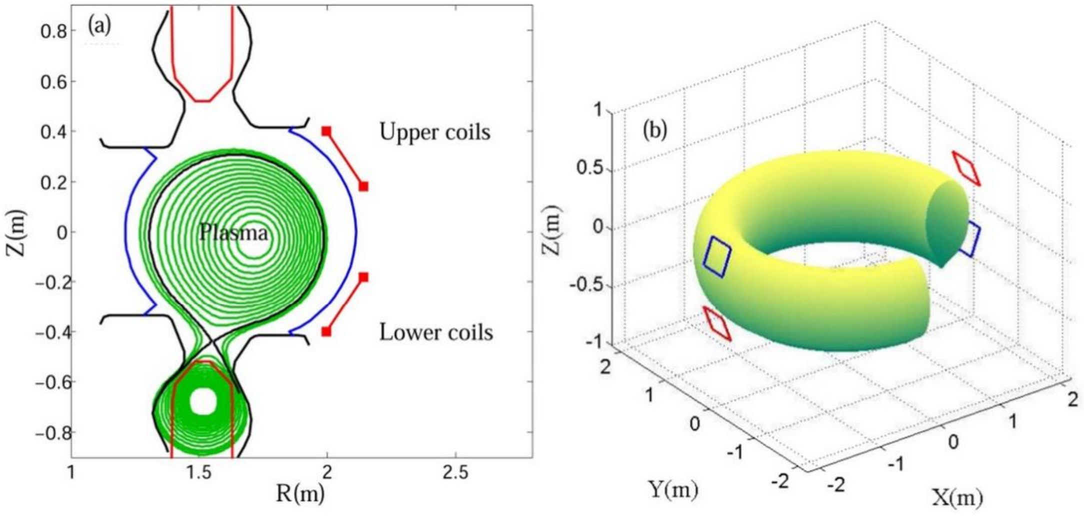
Predicted RMP coil current threshold consistent with Exp.





- The threshold of RMP coil current for ELM mititation ~4.5 kAt, agree with exp. value ~4.9 kAt
- Mechanism: RMP field (n=1) induces the coupling of toroidal components (1,4,3), which results in the re-distribution of perturbed energy in different *n*.

Another control method: biasing



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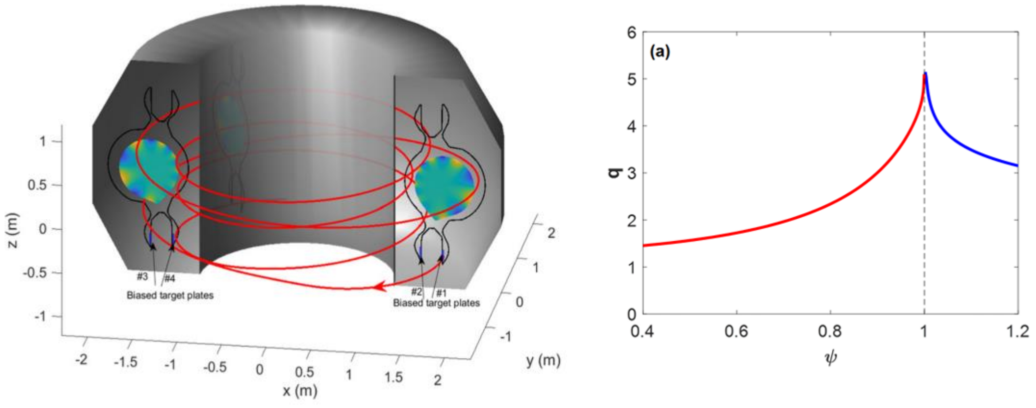
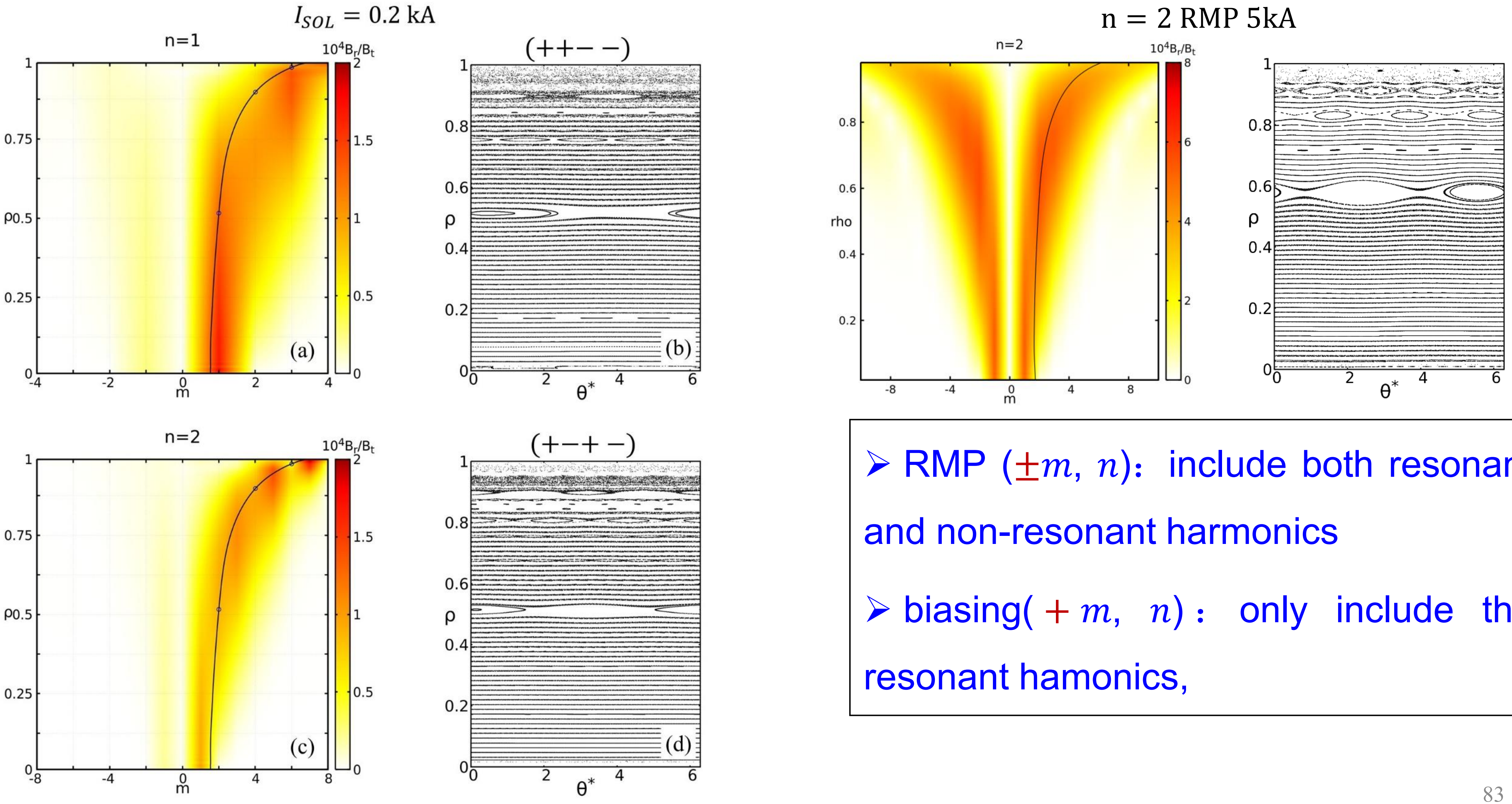


G. Z. Hao *Nucl. Fusion* 2023

- Extensive studies
- Demonstrated in many devices(DIII-D, JET, KSTAR, AUG, EAST, HL-2A/HL-3,...)
- Available for fusion reactor?

- Biased targets: occupied smaller area
- Control echanism similar with RMP coil
- ELM control is chieved in HL-2A

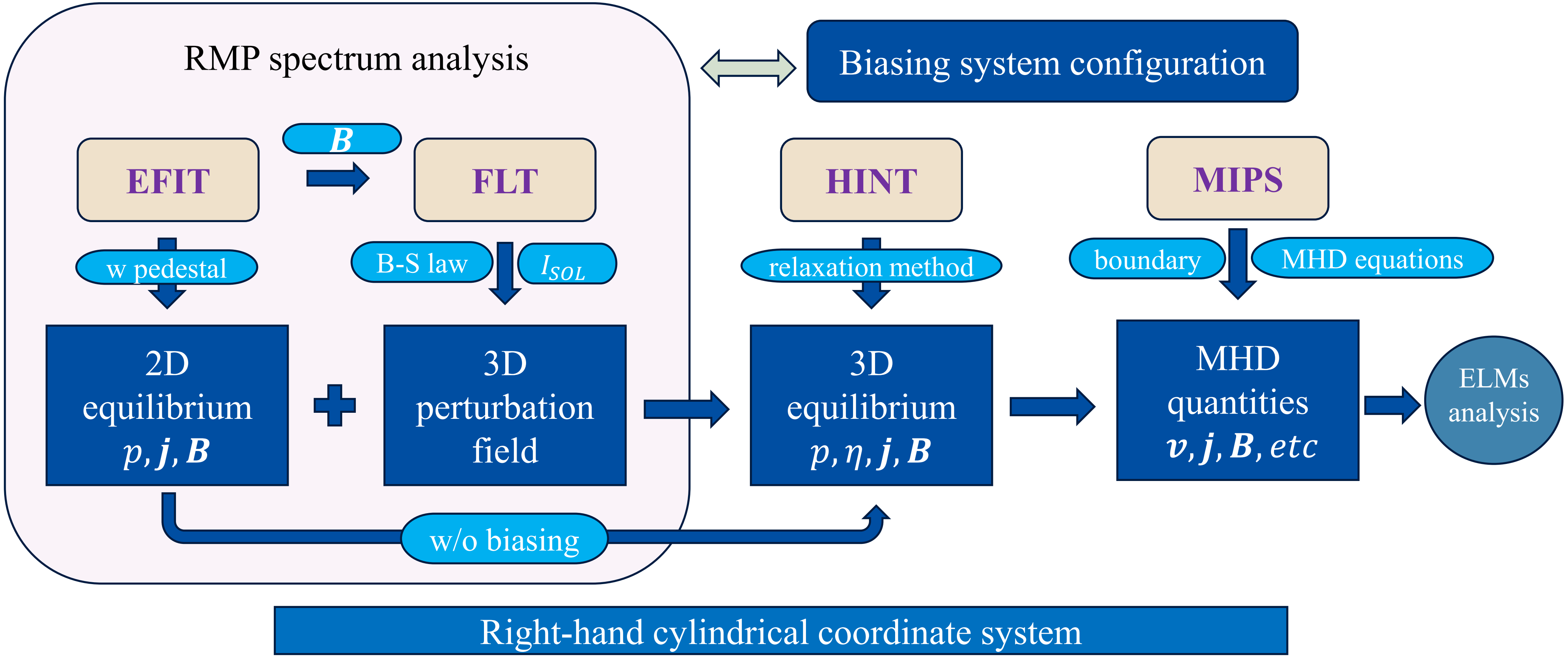
Comparison of spectrum of field perturbation for Biasing & RMP



➢ RMP ($\pm m$, n): include both resonant and non-resonant harmonics

➢ biasing($+m$, n): only include the resonant hamonics,

Workflow of studying effect of biasing on ELM

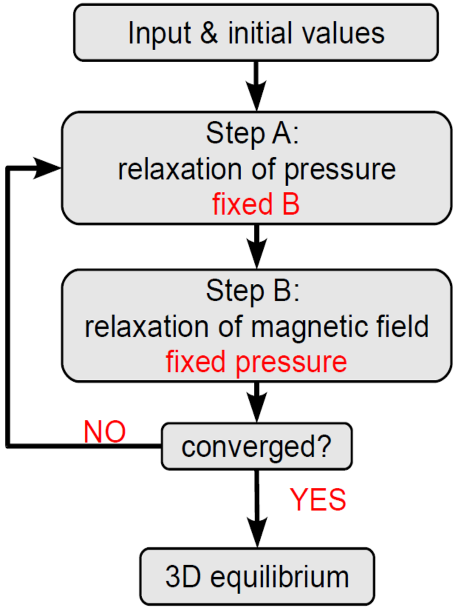


Modeling of 3D equilibrium for tokamak

- two steps of relaxation iteration； HINT code

Basic eqs

$$\begin{cases} \nabla p = \vec{J} \times \vec{B} \\ \vec{B} \cdot \nabla p = 0 \\ \nabla \times (\vec{v} \times \vec{B} - \eta \vec{J}) = 0 \end{cases}$$



Step A: pressure force free along the perturbed field lin

$$p^{i+1} = p = \frac{\int_{-L_{in}}^{L_{in}} \mathcal{F} p^i \frac{dl}{B}}{\int_{-L_{in}}^{L_{in}} \frac{dl}{B}} \qquad \mathcal{F} = \begin{cases} 1: for\ L_C \geq L_{in} \\ 0: for\ L_C < L_{in} \end{cases}$$

Step B:resolve the distribution of 3D magnetic fields

$$\frac{\partial \vec{v}_1}{\partial t} = -(\vec{v}_0 \cdot \nabla)\vec{v}_0 - \nabla p + \vec{j}_1 \times (\vec{B}_0 + \vec{B}_1) + \nu \Delta \vec{v}_1$$

$$\frac{\partial \vec{B}_1}{\partial t} = \nabla \times [(\vec{v}_0 + \vec{v}_1) \times (\vec{B}_0 + \vec{B}_1) - \eta(\vec{j}_1 - \vec{j}_{net})] + \kappa_{divB} \nabla\nabla \cdot \vec{B}_1$$

$$\vec{j}_1 = \nabla \times \vec{B}_1$$

MIPS applied to study MHD instabilities based on 3D equilibrium

MHD Infrastructure for Plasma Simulation code （MIPS模型）

Cylindrical coordinate

Input initial 3D equilibrium

↓

perturbation

↓

time evolution of MHD quantities

↓

Analysis of MHD stability

mass:
$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{v})$$

momentum:
$$\rho \frac{\partial \vec{v}}{\partial t} = -\rho(\vec{v} \cdot \nabla)\vec{v} + \vec{J} \times \vec{B} - \nabla p + \frac{4}{3}\nabla[\nu\rho(\nabla \cdot \vec{v})] - \nabla \times [\nu\rho\vec{\omega}]$$

energy:
$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\vec{v}) - (\gamma - 1)p\nabla \cdot \vec{v} + \chi_\perp \nabla_\perp^2 (p - p_{eq}) + \chi_\parallel \nabla \cdot \left(\frac{\vec{B}}{B^2}\vec{B} \cdot \nabla p\right)$$

Ohm's law:
$$\vec{E} = -\vec{v} \times \vec{B} + \eta(\vec{J} - \vec{J}_{eq})$$

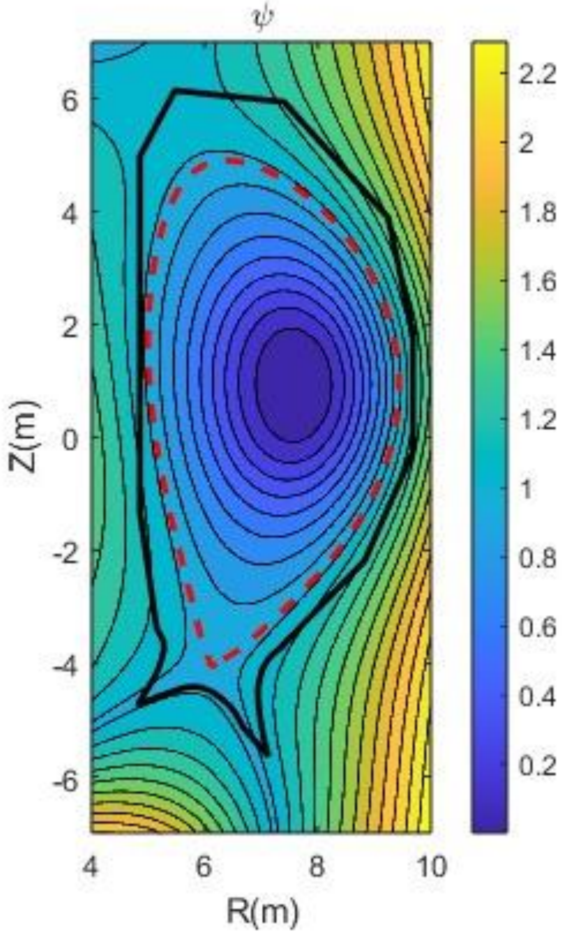
Faraday's law:
$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

Ampere's law:
$$\vec{J} = \frac{1}{\mu_0}\nabla \times \vec{B}$$

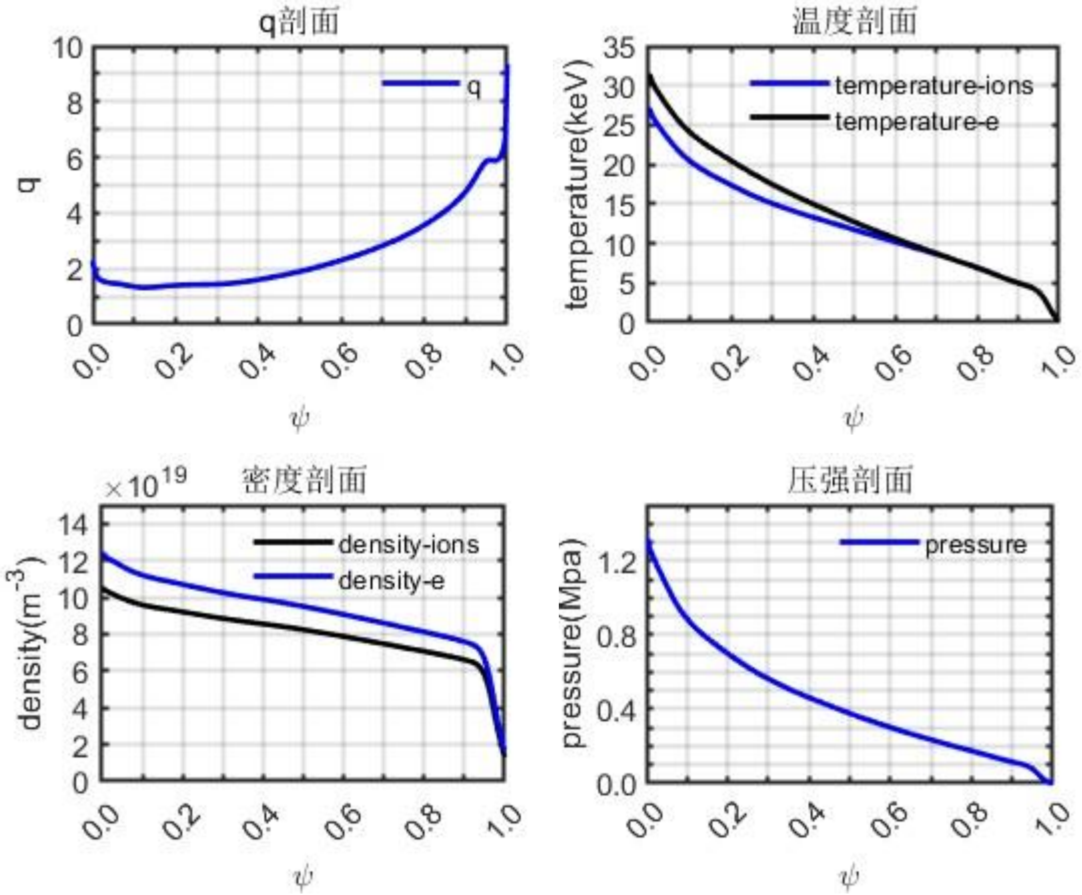
Where the vorticity: $\vec{\omega} = \nabla \times \vec{v}$

Hybrid scenario of CFETR 13MA

➢poloidal flux



➢profiles of equilibrium quantities



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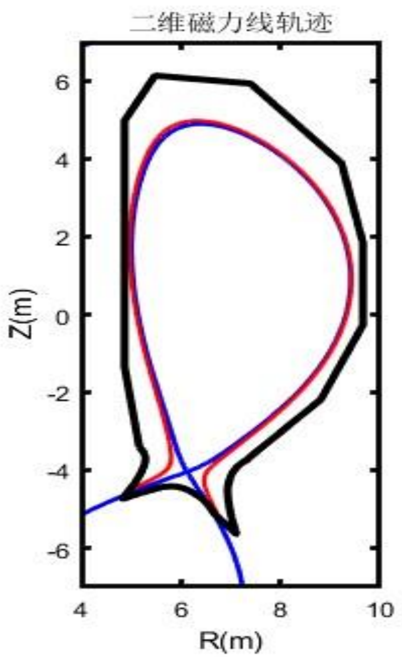
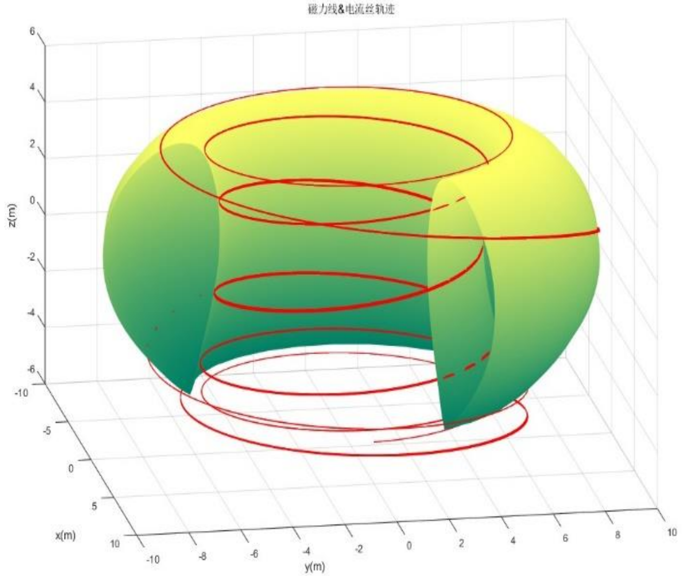
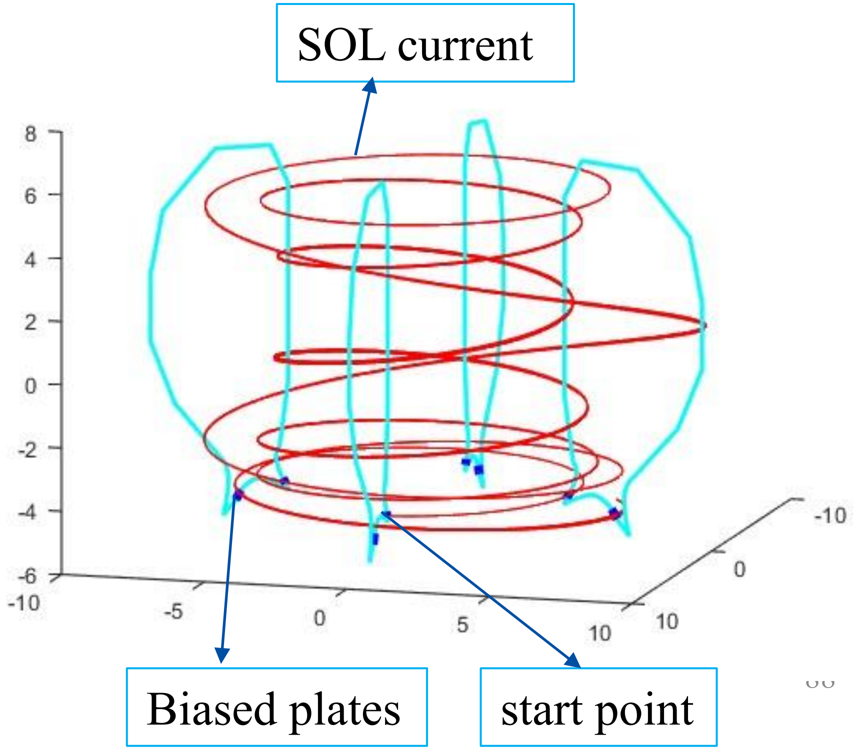
Modeling of SOL current driven by biased plates

- Assume 2x4 biased targets in low divertor surface
- Assume the SOL current along the magnetic field line

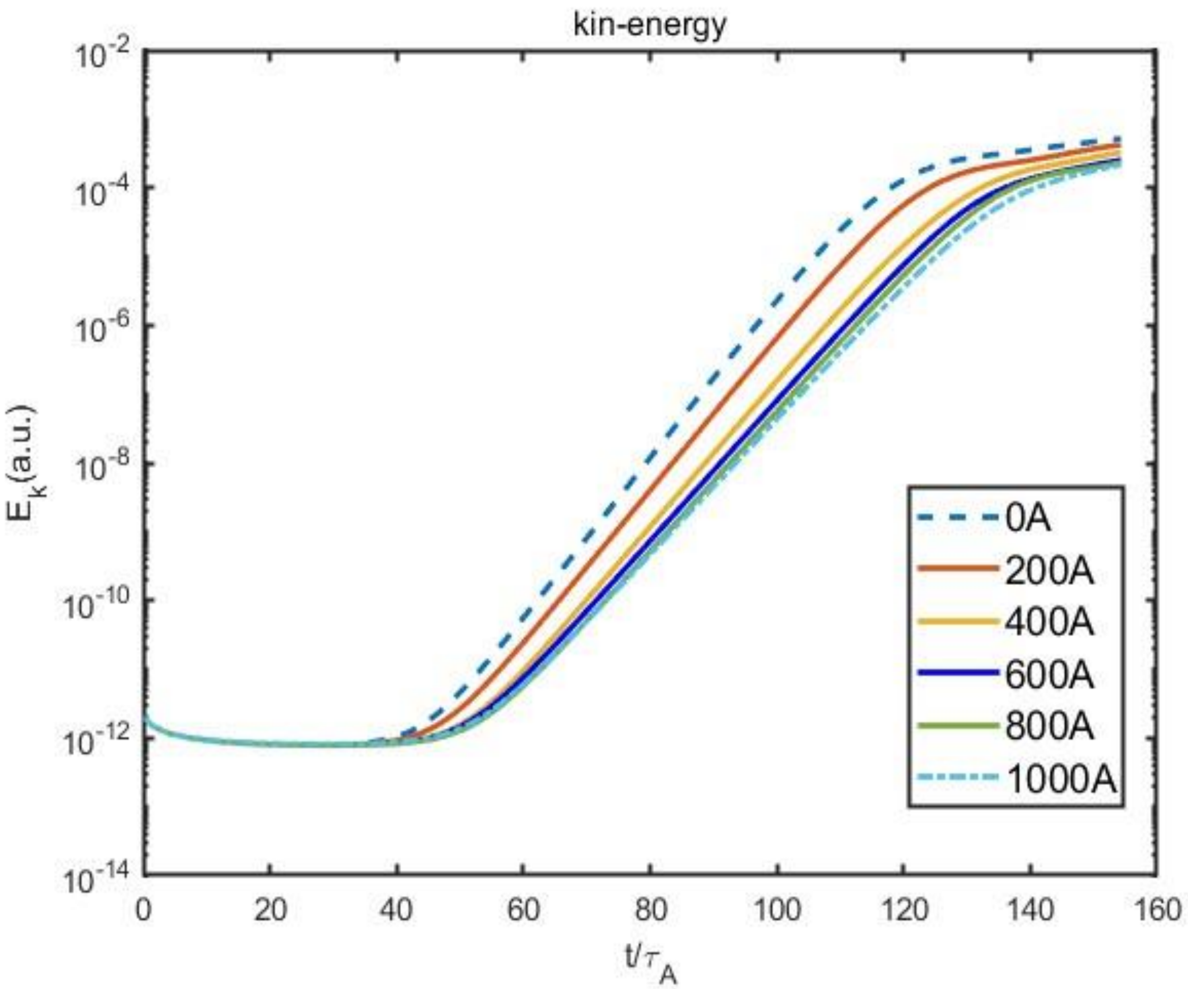


$$\frac{B}{dl} = \frac{\vec{B}\cdot\nabla u^1}{du^1} = \frac{\vec{B}\cdot\nabla u^2}{du^2} = \frac{\vec{B}\cdot\nabla u^3}{du^3}$$

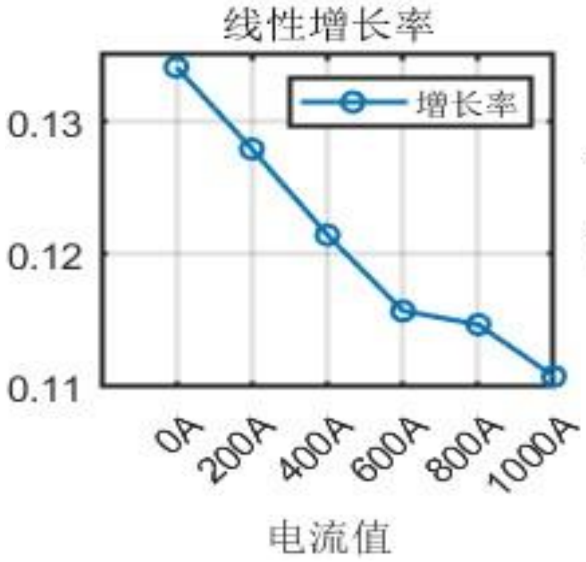


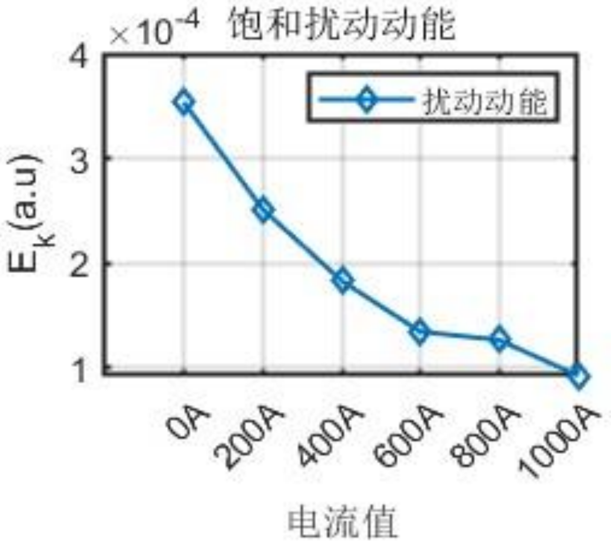


Step3: ELM behaviors with increasing of I_{SOL}



- linear phase： growth rate is reduced by I_{SOL}
- Saturation level is reduced by I_{SOL}





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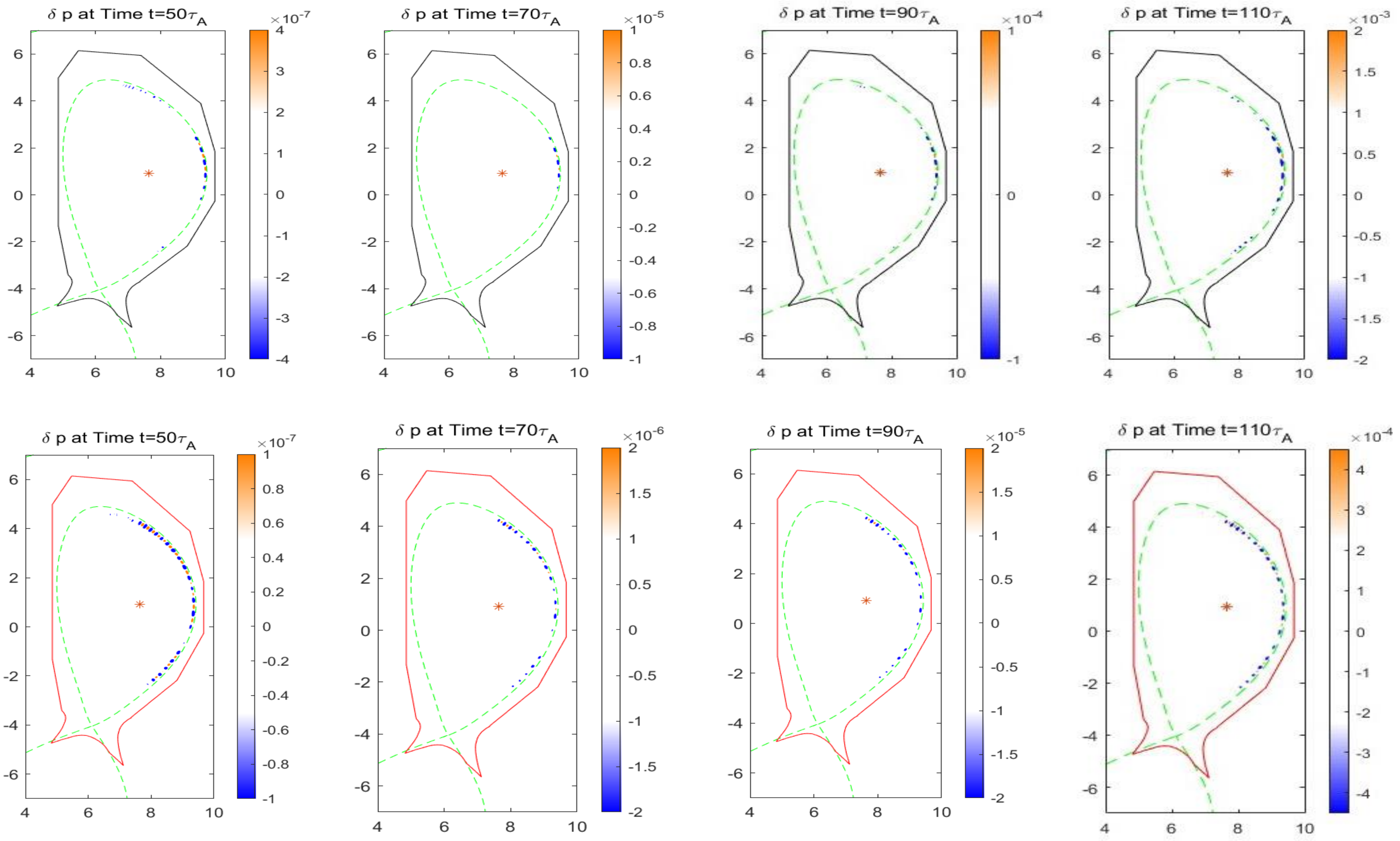
Effect of biasing on ELM structure

w/o biasing

$I_{SOL} = 1$ kA



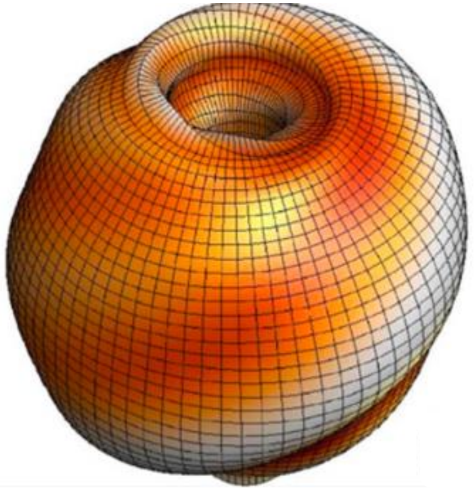
Summary & outlook

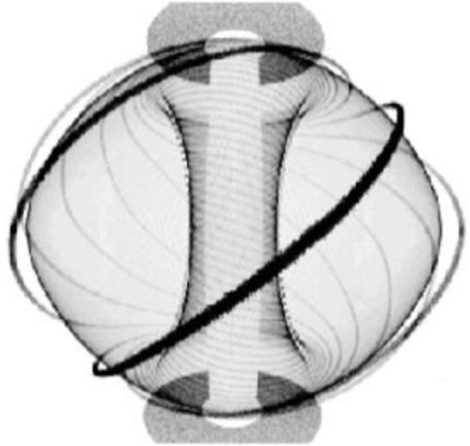
Fusion Gain:

$$Q \propto nT\tau_E \propto \beta_N H B^3 a^3 / q_{95}^2$$

RWM ELM

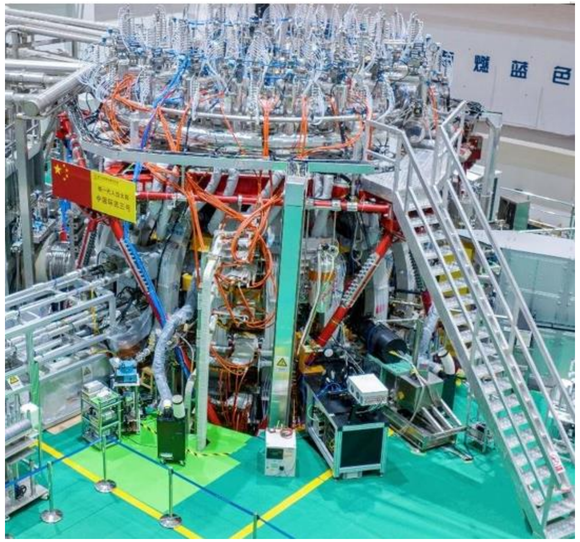
- RWM:
  - dispersion relation
  - three damping mechanism
- ELM:
  - E-L equation (s-alpha equation)
  - unstable boundary in s-alpha space
  - Physical picture of ELM control by 3D field perturbations



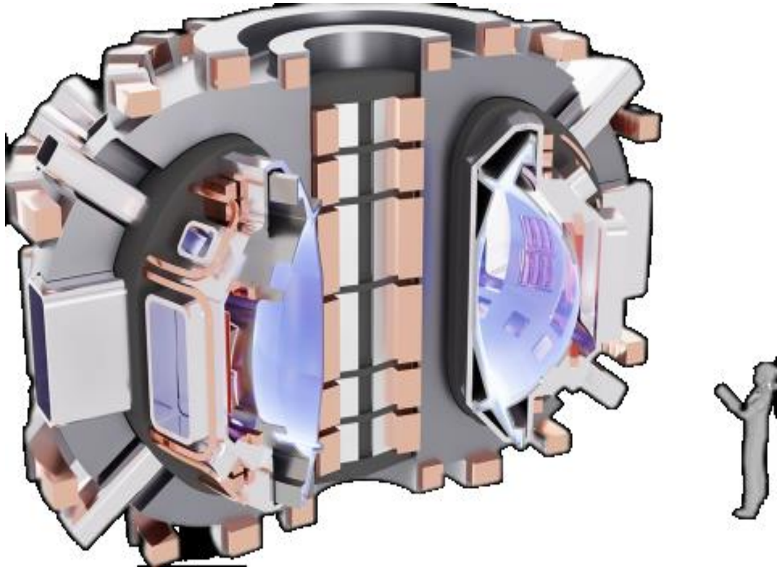


Summary & outlook

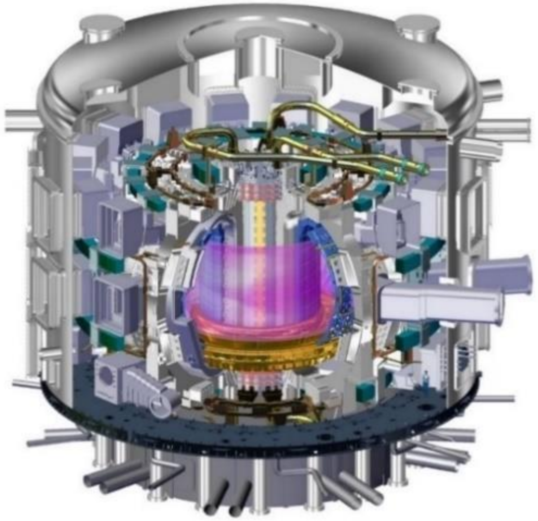
HL-3



SPARC



ITER



➢ MHD instabilities in DT plasma:

- effect of alpha particles on MHD equilibrium, self-consistent (reactor)
- non-linear interaction between alpha particles and MHD instabilities
- integration control instabilities (RWM. ELM, NTM)
- Avaliable and robust of control method in reactor environment
- ……

感谢聆听！

边缘局域模的分类及特点

序号	分类	特征	相关MHD模式	能量约束性能	释放的能量
1	I类ELM	加热功率超过阈值时就会发生，典型的ELM，频率随加热功率增加而增加	剥离气球模或高n气球模	好	大
2	II类ELM	在高三角度、高拉长比、高安全因子的等离子体中发生	接近气球模第二稳定区时发生	较I类ELM低约10%	约为I类ELM的10%
3	III类ELM	加热功率超过阈值时就会发生，与I类ELM相反，频率随加热功率增加反而降低	电阻气球模，电阻交换模	较I类ELM低约10-30%	约为I类ELM的10%
4	Grassy ELM	在高三角度、高安全因子、高极向比压的等离子体中发生	剥离气球模	与I类ELM相同	约为I类ELM的10%
5	EDA H模	在高安全因子、高三角度的等离子体中发生，D_α谱形的辐射增强	Quasi coherent模	与I类ELM相同	无能量放出
6	Quiescent H模（QH模）	当等离子体与壁之间间隔较大时发生	发生机制尚不明了	与I类ELM相同	无能量放出

$$\delta\varphi(nq,\,\theta,\,\zeta) = \sum_{m=-\infty}^{+\infty} \delta\varphi_m(nq)\exp\{-i(n\zeta - m\theta)\}.$$

$$\delta\varphi_m(nq) = \delta\varphi(nq - m)$$

Using this invariance property, the Fourier decomposition in (2.72) can be expressed as

$$\delta\varphi(nq,\,\theta,\,\zeta) = \sum_{m=-\infty}^{+\infty} \delta\varphi(nq - m)\,\exp\{-in\zeta + m\theta\}. \tag{2.74}$$

One can further introduce the Laplace transform

$$\delta\varphi(nq) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \delta\varphi(\eta)\,\exp\{inq\eta\}\,d\eta. \tag{2.75}$$

Using this transform 2.74 can be written as

$$\delta\varphi(nq,\,\theta,\,\phi) = \frac{1}{2\pi}\exp\{-in\zeta\}\int_{-\infty}^{+\infty} \delta\varphi(\eta)\sum_m \exp\{im(\theta - \eta)\}\,d\eta. \tag{2.76}$$

Noting that

$$\frac{1}{2\pi}\sum_{m=-\infty}^{+\infty} \exp\{im(\theta - \eta)\} = \sum_{j=-\infty}^{+\infty} \delta(\eta - \theta - j2\pi),$$

equation (2.76) can be transformed to

$$\delta\varphi(nq,\,\theta,\,\zeta) = \sum_{j=-\infty}^{+\infty} \delta\varphi(\theta + j2\pi)\,\exp\{-in(\zeta - q(\theta + j2\pi))\}. \tag{2.77}$$

Advanced Tokamak Stability Theory

Linjin Zheng

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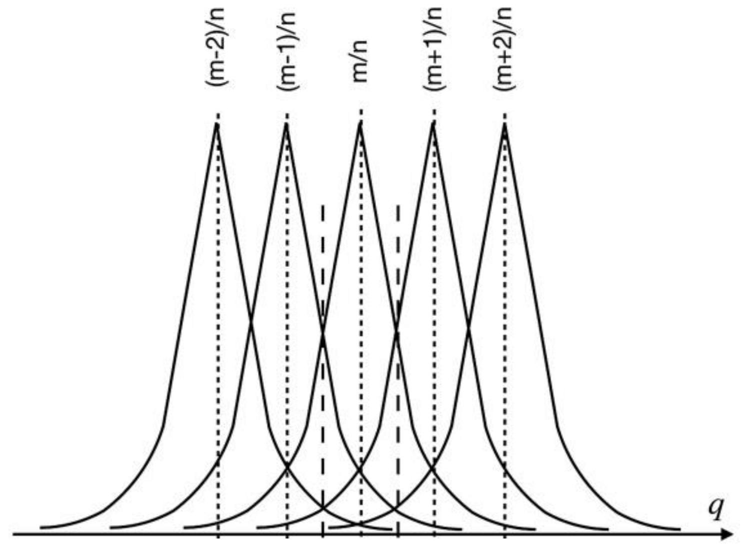

$$\delta W_2 = \frac{\pi}{\mu_0}\int_0^{\psi_a} W\,d\psi$$

$$\begin{aligned}
W &\propto \sum_{j=-\infty}^{\infty}\int_0^{2\pi} \bar{\xi}^*(\psi,\theta+2j\pi)F(\psi,\theta+2j\pi)\bar{\xi}^2(\psi,\theta+2j\pi)d\theta \\
W &\propto \int_{-2j\pi}^{2\pi+2j\pi} \bar{\xi}^*(\psi,\theta+2j\pi)F(\psi,\theta+2j\pi)\bar{\xi}(\psi,\theta+2j\pi)d(\theta+2j\pi) \\
W &\propto \int_{-\infty}^{\infty} \bar{\xi}^*(\psi,\Theta)F(\psi,\Theta)\bar{\xi}(\psi,\Theta)d(\Theta)
\end{aligned} \tag{2.22}$$

To evaluate continuity of the tangential magnetic field observe that the condition $\nabla\cdot\mathbf{B}=0$ within the wall implies that $i\mathbf{k}\cdot\hat{\mathbf{B}}_w=-\partial\hat{B}_{wr}/\partial x$ with $\mathbf{k}=(m/b)\mathbf{e}_\theta+k\mathbf{e}_z$. Similarly, in the vacuum regions $i\mathbf{k}\cdot\hat{\mathbf{B}}_1=-k_b^2\hat{V}$, where $k_b^2=k^2+m^2/b^2$. Thus continuity of the tangential fields requires

$$\hat{V}_I\Big|_{b^-}=\frac{1}{k_b^2}\frac{\partial\hat{B}_{wr}}{\partial x}\Big|_{x=0}\qquad \hat{V}_{II}\Big|_{b^+}=\frac{1}{k_b^2}\frac{\partial\hat{B}_{wr}}{\partial x}\Big|_{x=w}\qquad(11.161)$$

$$\begin{aligned}
\tau_w &= \nu g\\
&= \nu\frac{K_b'}{K_b}\left(\frac{1-K_b'I_a'/I_b'K_a'}{1-K_b'I_b/I_b'K_b}\right)\\
&= \nu\frac{K_b'}{K_b}\frac{I_b'K_a'-K_b'I_a'}{I_b'K_a'}\frac{I_b'K_b}{I_b'K_b-K_b'I_b}\\
&= \nu\frac{I_b'K_a'K_b'-I_a'K_b'^2}{I_b'K_a'K_b-I_bK_b'K_a'}\\
&= \nu\frac{K_b'(I_b'K_a'-I_a'K_b')/K_a'}{(I_b'K_b-K_b'I_b)}\\
&= \nu\frac{K_b'(I_b'-I_a'K_b'/K_a')}{(I_b'K_b-K_b'I_b)}
\end{aligned}$$