托卡马克等离子体输运动理学理论选题

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概要

I. Vlasov 理论

II. 回旋动理学

III. 离子温度梯度模

IV. 新经典极化与带状流

I. Vlasov 理论

I.1. 基础

$$\partial_t f + \partial_{\vec z}\cdot(\dot{\vec z} f) = 0 , \quad \text{Vlasov 方程}$$

Maxwell 方程组， $(\rho, \vec J) = \sum_s (\rho_s, \vec J_s)$.

$$\rho_s = \int d^3\vec v\, e_s f_s , \quad \vec J_s = \int d^3 \vec v\, e_s \vec v f_s .$$

非正则 Hamiltonian 力学

$$L\,dt = [e\vec A(\vec x, t) + m\vec v]\cdot d\vec x - H\,dt + dS .$$

$$H = \frac{1}{2} m v^2 + e\phi(\vec x, t)$$

$$\begin{cases} \dot{\vec x} = \vec v , \\ \dot{\vec v} = \frac{e}{m}(\vec E + \vec v \times \vec B) . \end{cases}$$

$\partial_{\vec z}\cdot(\dot{\vec z}) = 0$

相空间守恒

$$\partial_t f + \vec v\cdot\partial_{\vec x} f + \frac{e}{m}(\vec E + \vec v\times\vec B)\cdot\partial_{\vec v} f = 0 .$$

I.2. 一维静电问题的Landau理论

$$\partial_t f + v\partial_x f + \frac{e}{m} E \partial_v f = 0$$ ，电子Vlasov方程

$$\frac{dE}{dx} = \frac{e}{\varepsilon_0} n_0 \left(1 - \int dv f\right)$$ ，Poisson方程

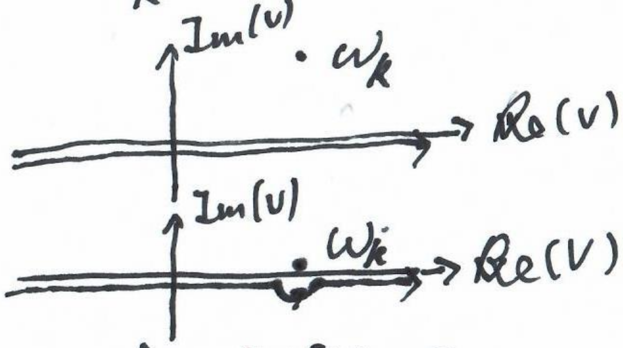
线性化. $f = f_0 + f_1$ ， $f_1 = \sum_k f_k e^{i(kx - \omega_k t)}$

$$f_k = \frac{e}{m} \frac{E_k \partial_v f_0}{i(kv - \omega_k)}$$

$$\gamma = \frac{\pi}{2} \frac{\omega_{pe}^2}{k^2} \omega_{pe} \partial_v f_0 \Big|_{v=\frac{\omega_r}{k}}$$

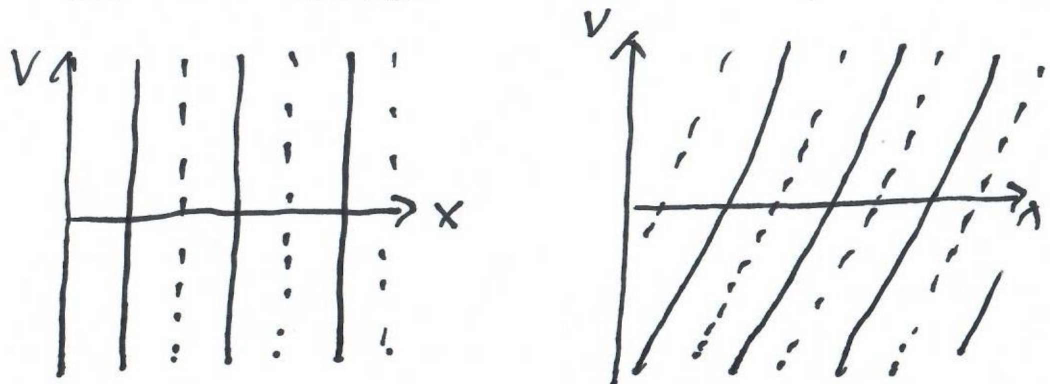
色散关系 $$1 - \frac{\omega_{pe}^2}{k^2} \int dv \frac{\partial_v f_0}{v - \omega_k / k} = 0.$$

$\omega_k = \omega_r + i\gamma$



Landau 积分路径

Landau 阻尼的 phase mixing 图象



$f_1(t=0)$ $f_1(t)$

I.3. 一维静电问题的准线性理论

$$\partial_t f_0 + \left\langle \partial_v \left(\frac{e}{m} E f_1 \right) \right\rangle = 0$$

↖ 系综平均

$$\partial_t f_0 - \partial_v \left(D_v \partial_v f_0 \right) = 0$$

$$D_v = \sum_k \pi \left(\frac{e}{m} \right)^2 |E_k|^2 e^{2\gamma_k t} \delta(kv - \omega_{pe})$$

↑ 准线性扩散系数

[1] A. Hasegawa, Plasma Instabilities and Nonlinear Effects

[2] R. Goldstein, P. Rutherford, Introduction to Plasma Physics.

II. 回旋动理学

II.1 导心运动的 Hamiltonian 力学

$\rho \ll L \sim |1/\partial_x|$，$\Omega \ll \partial_t$.　　$\vec{x} = \vec{X} + \vec{\rho}$

ρ：回旋半径　Ω：回旋频率　$\vec{x}$：粒子位置　$\vec{X}$：导心坐标

$$\mathcal{L}\,dt = [\vec{A}(\vec{X}) + v_\parallel \vec{b}]\cdot d\vec{X} - H_0\,dt.$$

$$H_0 = \frac{1}{2}v_\parallel^2 + \mu B + \phi(\vec{X})$$　　$m=1=e$

μ：磁矩守恒

$$\vec{B}^* = \nabla\times(\vec{A} + v_\parallel \vec{b}),\quad B_\parallel^* = \vec{b}\cdot\vec{B}^*,\quad \vec{b} = \vec{B}/B$$

$$\{f,g\} = -\frac{1}{B_\parallel^*}\vec{b}\cdot(\nabla f\times\nabla g) + \frac{1}{B_\parallel^*}\vec{B}^*\cdot\left(\nabla f\frac{\partial g}{\partial v_\parallel} - \nabla g\frac{\partial f}{\partial v_\parallel}\right) + (\partial_\xi f\,\partial_\mu g - \partial_\mu f\,\partial_\xi g)$$

泊松括号.

$$\begin{cases} \dot{\vec{X}} = \{\vec{X}, H_0\} = \dfrac{1}{B_\parallel^*}(\vec{b}\times\nabla H_0 + v_\parallel \vec{B}^*) \\ \dot{v}_\parallel = \{v_\parallel, H_0\} = -\dfrac{\vec{B}^*}{B_\parallel^*}\cdot\nabla H_0 \end{cases}$$

导心运动方程

$$\frac{1}{B_\parallel^*}\partial_{\vec{X}}\cdot(B_\parallel^*\dot{\vec{X}}) + \frac{1}{B_\parallel^*}\partial_{v_\parallel}(B_\parallel^*\dot{v}_\parallel) = 0.$$

相空间守恒

II.2 回旋动理学

低频 $\omega \ll \Omega$，有限Larmor半径 $k_\perp \rho \sim O(1)$.

$$\underbrace{\mathcal{L}\,dt}_{\hat{\Gamma}} = \underbrace{[\vec{A}(\vec{X}) + u_\parallel \vec{b}] \cdot d\vec{X} + \mu\, d\xi}_{\Gamma_0 \to \text{未扰动漂移指导}} - \underbrace{[\tfrac{1}{2}u_\parallel^2 + \mu B + \phi_0(\vec{X})]}_{H_0}\, dt$$

$$+ \underbrace{\delta\vec{A}(\vec{X}+\vec{\rho}_0, t)\, d(\vec{X}+\vec{\rho})}_{\Gamma_1} - \underbrace{\delta\phi(\vec{X}+\vec{\rho}_0, t)}_{H_1}\, dt .$$

Γ_1 与 H_1 依赖于回旋相位角. 回旋运动与湍流运动耦合.

II.2(a). Lie Transform Hamiltonian 力学

1-form
$$\hat{\Gamma} = \Gamma_{0i}\, dZ^i - H_0\, dt + \Gamma_{1i}\, dZ^i - H_1\, dt$$

Hamiltonian 力学 $\quad d\hat{\Gamma} = 0$

↑外微分

Lagrangian 2-form

$$\omega = d\Gamma \,, \quad \omega_{ji} = \partial_j \Gamma_i - \partial_i \Gamma_j$$

(a) 导向中心 z 至回旋中心 $\bar{Z}$ 的 Lie 变换.

$$\begin{cases} \bar{Z}^i = z^i + G_1^i(z) + G_2^i(z) + \frac{1}{2} G_1^j \partial_j G_1^i \\ z^i = \bar{Z}^i - G_1^i(\bar{Z}) - G_2^i(\bar{Z}) + \frac{1}{2} G_1^j \partial_j G_1^i \end{cases}$$

(b) 标量不变性 $\bar{F}(\bar{Z}) = F(z) \Rightarrow$

Pull-back $\quad F = \bar{F} + G_1^i \partial_i \bar{F} + G_2^i \partial_i \bar{F} + \frac{1}{2} G_1^i \partial_i (G_1^j \partial_j \bar{F})$

Push-forward $\quad \bar{F} = F - G_1 \cdot dF - G_2 \cdot dF + \frac{1}{2} G_1 \cdot d(G_1 \cdot dF)$

(c) Push-forward of the fundamental 1-form

$$\bar{\bar{\Gamma}} \equiv \bar{\Gamma} - \bar{H}\, dt$$

Symplectic transform $\bar{\Gamma} = \Gamma_0$ (Poisson 括号不变)

$$\bar{\Gamma}_{0i} = \Gamma_{0i} \,, \quad \bar{\Gamma}_{1i} = 0 = \bar{\Gamma}_{2i} \,.$$

$$
\begin{cases}
\bar{H}_0 = H_0(\bar{z}) \\
\bar{H}_1 = -\partial_t S_1 - \{S_1, H_0\} + \delta\Psi_1 \\
\bar{H}_2 = -\partial_t S_2 - \{S_2, H_0\} + \delta\Psi_2
\end{cases}
$$

$$\delta\Psi_1 = H_1 - \{z^i, H_0\}\Gamma_{1i}$$

$$\delta\Psi_2 = H_2 - \{z^i, H_0\}\left(\Gamma_{2i} - \tfrac{1}{2} G_1^j \omega_{1ji}\right) - \tfrac{1}{2} G_1^j\left[\partial_j(H_1 + \bar{H}_1) + \partial_t \Gamma_{1j}\right]$$

$$
\begin{cases}
G_1^i = (\partial_j S_1 + \Gamma_{1j}) J_0^{ji} \\
G_2^i = (\partial_j S_2 + \Gamma_{2j} - \tfrac{1}{2} G_1^k \omega_{1kj}) J_0^{ji}
\end{cases}
$$

$$J_0^{ij} = \{z^i, z^j\}$$

辛变换的解.

回旋运动的解耦

$$\bar{H}_0 = H_0(\vec{\bar{X}}, \bar{v}_\parallel, \bar{\mu})$$

$$\bar{H}_1 = \langle \delta\Psi_1 \rangle_g \leftarrow \text{回旋平均}, \quad \bar{H}_2 = \langle \delta\Psi_2 \rangle_g .$$

$$\delta\Psi_n = \langle \delta\Psi_n \rangle + \widetilde{\delta\Psi_n}$$

$$\partial_t S_n + \{ S_n, H_0 \} = \widetilde{\delta\Psi_n}, \quad S_n \simeq \frac{1}{B_0} \int d\xi \, \widetilde{\delta\Psi_n} .$$

$$\begin{cases} G_n^{\vec{X}} = -\dfrac{\vec{b}_0}{B_\parallel^*} \times [\delta\vec{A}_n + \nabla S_n] - \dfrac{\vec{B}^*}{B_\parallel^*} \dfrac{\partial S_n}{\partial v_\parallel} \\ G_n^{v_\parallel} = \dfrac{\vec{B}^*}{B_\parallel^*} \cdot \delta\vec{A}_n + \dfrac{\vec{B}^*}{B_\parallel^*} \cdot \nabla S_n \end{cases}$$

$$\begin{cases} G_n^{\xi} = \delta\vec{A}_n \cdot \partial_\mu \vec{\rho}_0 - \partial_\mu S_n \\ G_n^{\mu} = \delta\vec{A}_n \cdot \partial_\xi \vec{\rho}_0 + \partial_\xi S_n \end{cases}$$

$$G_n^{\vec{r}} = G_n^{\vec{X}} + (G_n^{\xi}\partial_\xi + G_n^{\mu}\partial_\mu)\vec{\rho}_0 - \{ \vec{X} + \vec{\rho}, S_n \} .$$

$$\delta\Psi_n = \delta\psi_n - (\dot{\vec{X}} + \dot{\vec{\rho}})_0 \cdot \delta\vec{A}_n$$

$$[\delta\vec{A}_1, \delta\psi_1] = [\delta\vec{A}, \delta\phi],$$

$$[\delta\vec{A}_2, \delta\psi_2] = \frac{1}{2} [G_1^{\vec{r}} \times \delta\vec{B}, G_1^{\vec{r}} \cdot \delta\vec{E} - G_1^{j} \partial_j \bar{H}_1]$$

题.2 (b) 回旋平均 Vlasov 方程

$$\partial_t \bar F + \{\bar F, \bar H\} = 0$$

$\dot{Z} = \{Z, \bar H\}$, $\partial_Z \cdot (\dot{Z}) = 0$, 相空间守恒.

静电扰动情形. $H_1 = \delta\phi(\vec X + \vec\rho_0, t)$

长波近似 $\widetilde{\delta\phi} = \vec\rho_0 \cdot \nabla\delta\phi$, $\tilde S_1 = \frac{1}{B_0}\int d\xi\, \widetilde{\delta\phi}$

$$G_1^\mu = \partial_\xi S_1 = \frac{1}{B_0}\vec\rho \cdot \nabla\delta\phi(\bar{X}, \bar v_\parallel, \bar\mu, \xi)$$

II. 2 (c) 回旋动力学 Poisson 方程

粒子分布

粒子密度 $n_{part.}(\vec{r}) = \int d^3\vec{v}\, f(\vec{r},\vec{v})$

$$= \int d^3 v\, d^3\vec{x}\, \delta^3(\vec{r}-\vec{x})\, f(\vec{x},\vec{v})$$

$$= \int B_\parallel^* d^3\vec{X}\, du_\parallel\, d\mu\, d\xi\, \delta^3(\vec{X}+\vec{\rho}_0-\vec{r})\, F(\vec{X},u_\parallel,\mu)$$

导心分布

Pull-back of F

极化密度 $n_{pol.}(\vec{r}) = \int \underline{B_\parallel^*}\, d^3\vec{X}\, \underline{du_\parallel\, d\mu\, d\xi}\, \delta^3(\vec{X}+\vec{\rho}_0-\vec{r})\, G_1^{\mu}\, \partial_\mu F_0$

Maxwellian

$$n_{pol}(\vec{r}) = \int d^3\vec{v}\, \frac{1}{B_0}\vec{\rho}_0\cdot\nabla\delta\phi\,(\vec{r}-\vec{\rho}_0)\left(-\frac{B_0}{T}\right)F_0$$

温度

$$= \int d^3\vec{v}\, \frac{F_0}{T}\vec{\rho}_0\vec{\rho}_0 : \nabla\nabla\delta\phi$$

$$= \nabla\cdot\int d^3\vec{v}\, \frac{F_0}{T}\vec{\rho}\vec{\rho}\cdot\nabla\delta\phi$$

$$= \nabla\cdot\frac{n_0 m}{eB^2}\nabla_\perp\delta\phi.$$

$n_{part}(\vec{r}) = n_{pol.} + n_{gc}$, $n_{gc} = \int d^3\vec{v}\, F.$

[1] J. Cary, A. Brizard. RMP 09 综述

[2] R. Littlejohn, J. Plasma Phys. 1983, PF 81

[3] P. Catto, W. Tang, D. Baldwin. Plasma Phys. 81

[4] T. Antonsen, B. Lane. PF 80 推导

[5] R. Hazeltine, J. Meiss, Plasma Confinement

[6] E. Frieman, L. Chen. PF 82

[7] T. S. Hahm, PF 88 现代

[8] A. Brizard J. Plasma Phys. 89 ← 概念突破

[9] W. W. Lee. PF 83

[10] J. R. Cary. Phys. Rep. 79 李一变换

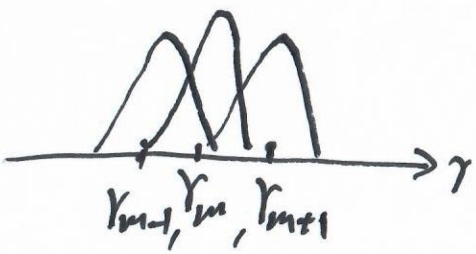
[11] R. G. Littlejohn. J. Math. Phys. 82

III. 离子温度梯度模 (ITG)

漂移波近似 $k_\parallel \ll k_\perp$ ，$k_\perp \rho \sim O(1)$

对于不稳定的模，平行Landau阻尼应尽可能地小，因此每个模 (n, m) ← 极向模数
$\uparrow$ 环向模数

都尽可能地局域在其有理面 r_m $[q(r_m) = m/n]$ 附近



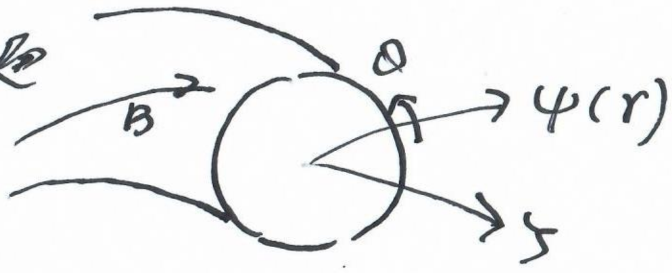
III.1 气球模变换

托卡马克位型

$\vec{B} = I(\psi)\nabla\zeta + \nabla\zeta \times \nabla\psi$ ← 极向磁通

$d\psi_T/d\psi = q(\psi)$ 安全因子

$d\zeta/d\theta = q(\psi)$



$$\delta\phi_n = \underbrace{F(\psi,\theta)}_{\text{缓变}}\, e^{in\overbrace{(\zeta - \int d\theta\, q)}}$$

相位沿磁力线不变
垂直于磁力线快变，$n \gg 1$.

此一种表示的缺点：磁剪切 $dq/dr \neq 0$ 时.

$\delta\phi_n(\theta) = \delta\phi(\theta+2\pi)$ 与 $F(\psi,\theta)$ 缓变矛盾！

环向 fourier 分解 $\propto e^{in\zeta}$ 后，通常对于线性本征值问题

$$\mathcal{L}(\theta,\psi)\,\delta\phi_n(\theta,\psi) = \lambda\,\delta\phi_n(\theta,\psi).$$

运用关于 θ 的周期性条件！

(a) $\delta\phi(\theta,\psi) = \sum_m a_m(\psi)\, e^{-im\theta}$， $a_m(\psi) = \frac{1}{2\pi}\oint d\theta\, e^{im\theta}\,\delta\phi(\theta,\psi)$ 整数

(b) 令 $a(x,\psi) = \frac{1}{\pi}\sum_m a_m \frac{\sin[(m-x)\pi]}{(m-x)}$， x 为实数！

$a(m,\psi) = a_m(\psi)$.

(c) $a(x,\psi) = \int_{-\infty}^{+\infty} d\eta\, e^{-ix\eta}\, \delta\hat{\phi}(\eta,\psi)$ 不必此周期！

$\delta\hat{\phi}(\eta,\psi) \to 0$，当 $|\eta| \to \infty$ 时.

求解 $\mathcal{L}(\eta,\psi)\,\delta\hat{\phi}(\eta,\psi)=\lambda\,\delta\hat{\phi}(\eta,\psi)$

在 $\eta\in(-\infty,+\infty)$ 上的本征函数与空间周期性函数

$\delta\phi(\theta,\psi)$，且本征值相同！

气球模变换

$$\delta\phi(\theta,\psi)=e^{-i\omega t}\sum_m e^{i(n\zeta-m\theta)}\int_{-\infty}^{+\infty}\underbrace{dx\,\delta\hat{\phi}(x,\psi)\,e^{i(m-nq)x}}_{\text{傅变}}$$

[1] J. W. Connor, R. J. Hastie, J. B. Taylor, PRL 78

Proc. R. Soc. Lond. A. 365, 1-17 (1979)

[2] L. Chen's Lecture Note

III.2 线性 ITG 模

绝热电子模型.

线性回旋动理学 离子分布函数

$$\delta f = \underbrace{-\frac{e}{T}\delta\phi F_0}_{\text{绝热}} + \underbrace{h}_{\text{非绝热}}$$

气球模表象下的回旋动理学 Vlasov 方程

$$i\frac{v_\parallel}{qR}\partial_x h + (\omega-\omega_d)h = (\omega-\omega_{*T})F_M J_0 \delta\hat{\phi}(x),$$

$$J_0 = J_0(k_\perp \rho)$$

$$\omega_d = 2\epsilon_n \omega_* (\mu B + v_\parallel^2)(\cos x + \underset{\uparrow\,\text{shear}}{s}\, x \sin x), \quad \epsilon_n = \underset{\substack{\uparrow \\ \text{密度标长}}}{L_n}/R$$

$$\omega_* = -k_\theta T_i / eBL_n$$

$$\omega_{*T} = \omega_* [1+\eta_i(2\mu B + v_\parallel^2 - 3/2)], \quad \eta_i = L_n/\underset{\substack{\uparrow \\ \text{温度标长}}}{L_T}$$

设定 $k_\parallel v_\parallel << \omega$，微扰法解出高阶 δf 项，代入准中性方程。

$$\left(1+\frac{1}{\tau}\right)\delta\hat{\phi} - \int d^3v\, F_M J_0 \left[1-\left(\frac{v_\parallel}{qR}\frac{1}{\omega-\omega_d}\partial_x\right)^2\right]\frac{\omega-\omega_{*T}}{\omega-\omega_d} J_0\,\delta\hat{\phi} = 0$$

$\tau = T_e/T_i$

III.2(a) 流体近似

保留 ω_d/ω，$k^2\rho^2$，$k_\parallel v_{thi}/\omega$ 的首阶

$$\frac{1}{2}\frac{v_{thi}^2}{\omega^2}\frac{1}{qR}\partial_x\frac{1}{qR}\partial_x\delta\hat{\phi} + \left[\left(\frac{1}{\tau}+\frac{\omega_*}{\omega}\right)\frac{1}{A} + b_i - \frac{\omega_d}{\omega}\right]\delta\hat{\phi} = 0$$

① ② ③

$A = 1-(1+\eta_i)\omega_*/\omega$，$b_i = \frac{1}{2}k^2 v_{thi}^2/\Omega_i^2$

① $\sim \frac{k_\parallel^2 c_s^2}{\omega^2}\frac{\omega_*}{\omega}\eta_i$，② $\sim O(1)$，③ $\sim \frac{\omega_d\omega_*}{\omega^2}\eta_i$.

(1) ① $\sim O(\varepsilon) \sim$ ③ $\Rightarrow \omega = -\tau\omega_*$ electron drift mode

(2) ① $\sim O(1) \sim$ ③

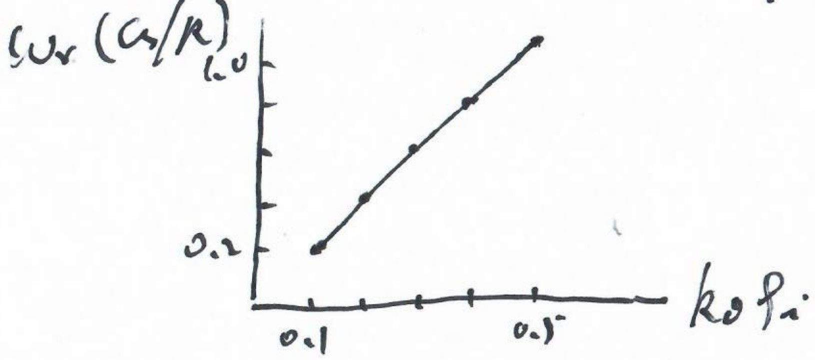
(2.1) ①, ② balance ⇒ $\omega^3 \sim k_\parallel^2 c_s^2 \omega_* \eta_i$

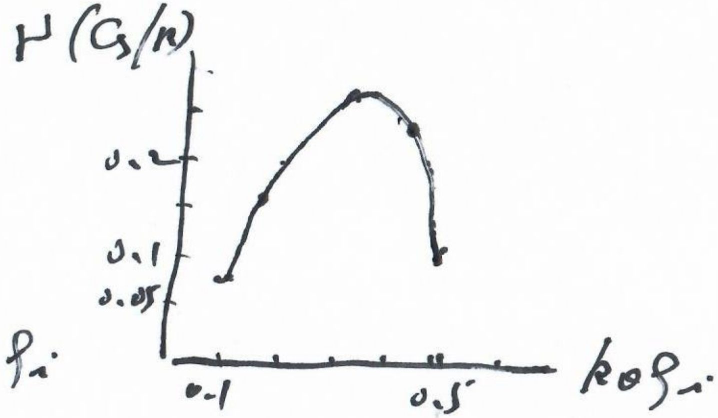
环耦合效应弱, slab ITG

(2.2) ①, ③ balance ⇒ $\omega^2 \sim \omega_d \omega_* \eta_i$

环耦合效应强 toroidal ITG

III.2(b) 回旋动力学模拟 (GENE, NLT)

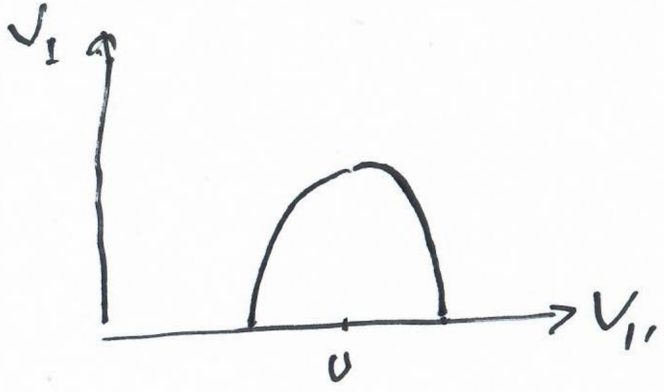




[1] F. Romanelli, PFB 89, 90

[2] L. Ye, et al. JCP 16

Ⅲ. 2(c) 相空间共振结构 (NLT)



$$\omega_d = \omega_r$$

$$\omega_d = k_\theta V_d, \quad V_d = \frac{m v_\parallel^2 + \mu B}{R e B}.$$

[3] S. Sun, et al., NF 22, 23

IV. 新经典极化与带状流

IV.1 Ware Pinch

捕获粒子受环向电场 E_ζ 作用，但其在环向运动不自由。故环向动量守恒要求

$$\Gamma^r_{\text{Ware}} \sim \sqrt{\varepsilon}\, n \frac{eE_\zeta}{eB_p}, \quad \varepsilon = r/R.$$

IV.2 环向动量输入或环向 Renold's stress 导致的"等效力"作用于捕获粒子之上

$$F_{\zeta,M} = m\,\partial_t u \,\frac{m v_\parallel^2}{T} \sim m(\partial_t u)\,\varepsilon$$

$\uparrow$ 流体平行速度

推广地（根据 Ware Pinch 理论）导致 Pinch 流

$$\Gamma^r_M \sim -\varepsilon^{3/2} n \frac{m(\partial_t u)}{eB_p} \sim \frac{q}{\sqrt{\varepsilon}} \frac{nm}{eB_T^2} B_p \frac{1}{nm} \partial_r \underbrace{\Pi^r_\zeta}_{\text{环向 Renold's stress}}$$

IV.3 加热或湍动能流诱导的粒子流.

粘性-纵通变梯度导致的平行动量

$$M_\zeta \sim -\varepsilon^{3/2} \frac{m\,\partial_t P}{eB_p} \leftarrow \text{压强} \qquad \text{(J. Wesson, Tokamaks)}$$

加热或热输运引起 $\partial_t P$，从而诱导环向惯性力

$$F_\zeta = -\partial_t M_\zeta$$

按 Ware Pinch 理论有粒子流

$$\Gamma^r_H \sim -\varepsilon^{3/2} \frac{m\,\partial_r \partial_t P}{e^2 B_p^2} \sim \frac{q^2}{\sqrt{\varepsilon}} \frac{nm}{eB_T^2} \frac{1}{ne} \partial_r^2 \underset{\text{湍动能流}}{\underset{\uparrow}{Q^r}}$$

IV.4 变化的径向电场 $(\partial_t \phi(r))$ 诱导的对流热粒子

$\partial_t \phi(r)$ 的加热效应导致

$$\partial_t P \sim ne\,\partial_t \phi(r)$$

按加热诱导的粒子流得新经典极化流

$$\Gamma^r_E \sim \epsilon^{3/2}\frac{m\partial_t E_r}{e^2 B_p^2}n_e \sim \frac{q^2}{\sqrt{\epsilon}}\frac{nm}{eB_T^2}\partial_t E_r$$

Rosenbluth-Hinton 由回旋动理学理论得到的

$$\epsilon_r = \underbrace{1.6q^2/\sqrt{\epsilon}}_{\text{新经典极化}} + \underbrace{1}_{\text{经典极化}}$$

IV.5 ZF 的驱动机制

$$\Gamma^r_E + \Gamma^r_M + \Gamma^r_H = 0 \quad \Rightarrow$$

$$n e_i \partial_t E_r - \underbrace{\partial_t \partial_r P_{r\theta}}_{\substack{\text{加热}\\ \text{或能量输运}}} - n e_i \underbrace{\partial_t u_\zeta}_{\substack{\text{环向动量注入}\\ \text{或输运}}} B_p = 0$$

极向 Reynold's 协强张量分量：通过 u_θ 体现

柱几何

$$n m_i \partial_t u_\theta + \begin{bmatrix} 1 \\ 1/\varepsilon_r \end{bmatrix} \frac{1}{r}\partial_r\left(r \Pi^r_\theta\right) = 0$$

环几何

IV.6 广义 RH Test

$$\underbrace{k^2 \delta\phi_k}_{\text{残余}} = \frac{1}{\varepsilon_r} k^2 \delta\phi_k(0) + i k u_{ik}(0) B_p$$

$$- k^2 \frac{1}{e_i} \underbrace{T_{ik}(0)}$$

初始条件对湍流仅起作用！
(新经典极化)
(极向雷诺协强(PRS)起作用)
(湍流能量起作用)

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PRS in 初始阶段 屏蔽